

VARIABILITY AND SUPPRESSION OF VARIABILITY IN THE PHOTORESPONSE IN THE MOUSE ROD

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THE BIOLOGICAL PHENOMENON

THE MATHEMATICAL MODELS

CV IN THE DIFFERENT MODELS

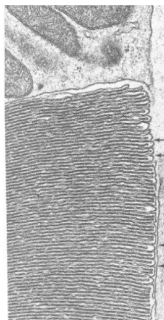
OUTLINE

THE BIOLOGICAL PHENOMENON

THE MATHEMATICAL MODELS

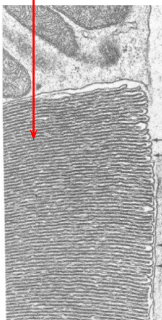
CV IN THE DIFFERENT MODELS

THE ROD OUTER SEGMENT (ROS)



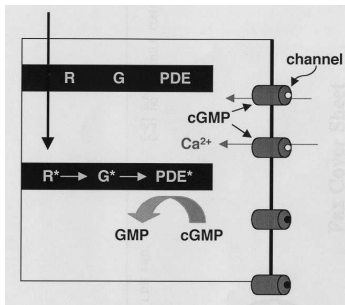
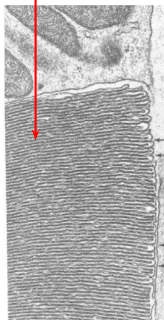
THE ROD OUTER SEGMENT (ROS)

photon



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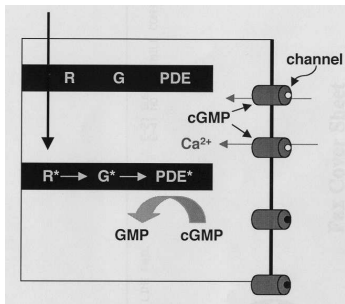
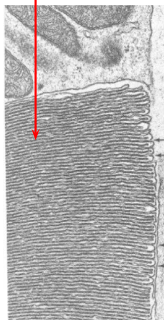
photon



On disc: $R^* \rightarrow G^* \rightarrow PDE^*$

THE ROD OUTER SEGMENT (ROS)

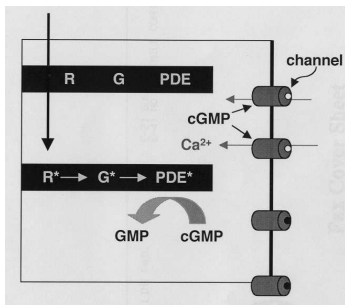
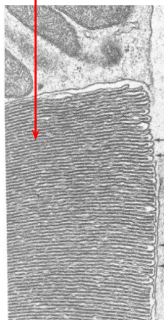
photon



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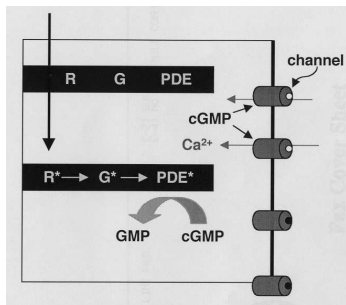
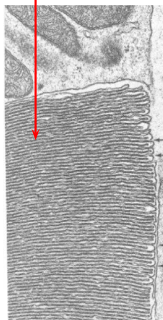
photon



In cytosol: PDE* depletes cyclic guanosine monophosphate (cGMP) ... the lowering of concentration of cGMP causes the closure of ion channels in the cell membrane.

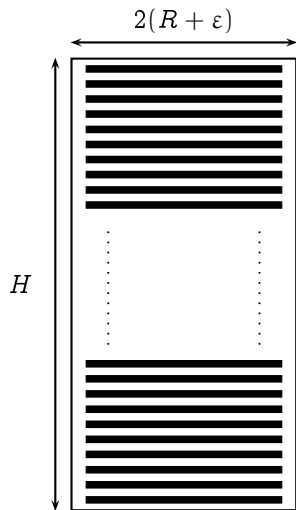
THE ROD OUTER SEGMENT (ROS)

photon

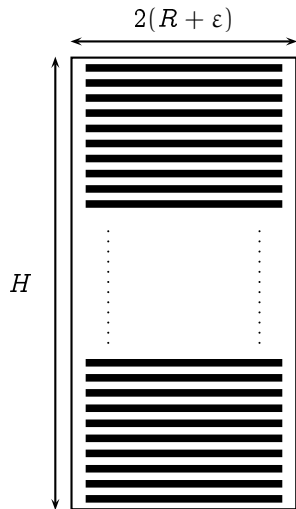


In cytosol: PDE* depletes cyclic guanosine monophosphate (cGMP) ... the lowering of concentration of cGMP causes the closure of ion channels in the cell membrane. This unbalance in the flux of Ca^{2+} ions causes the current drop which triggers the electrical signal.

THE MOUSE ROD OUTER SEGMENT: GEOMETRY



THE MOUSE ROD OUTER SEGMENT: GEOMETRY



Dimensions:

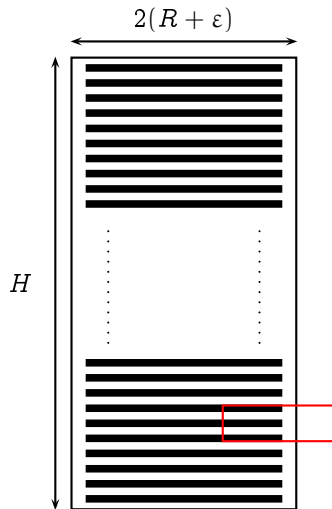
$H = 23.6\mu\text{m}$, $R = 0.7\mu\text{m}$;

$\epsilon = 0.014\mu\text{m}$ width of:

- ▶ disc;
- ▶ gap between discs;
- ▶ outer shell;

Number of discs: $n_{discs} = 800\text{--}1000$.

THE MOUSE ROD OUTER SEGMENT: GEOMETRY



Dimensions:

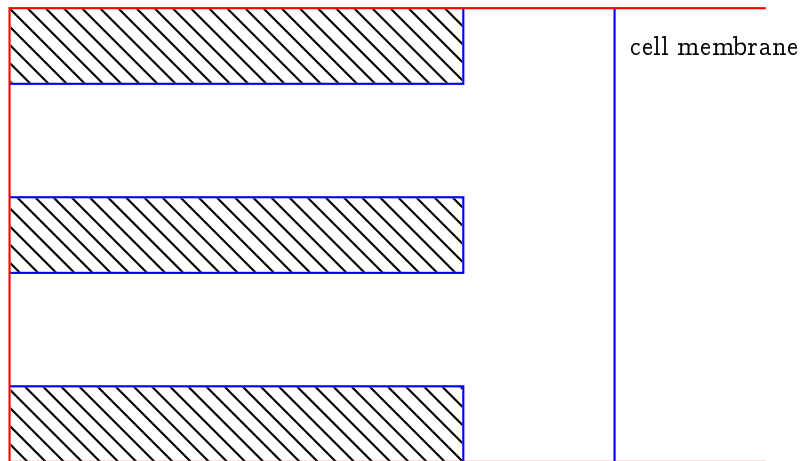
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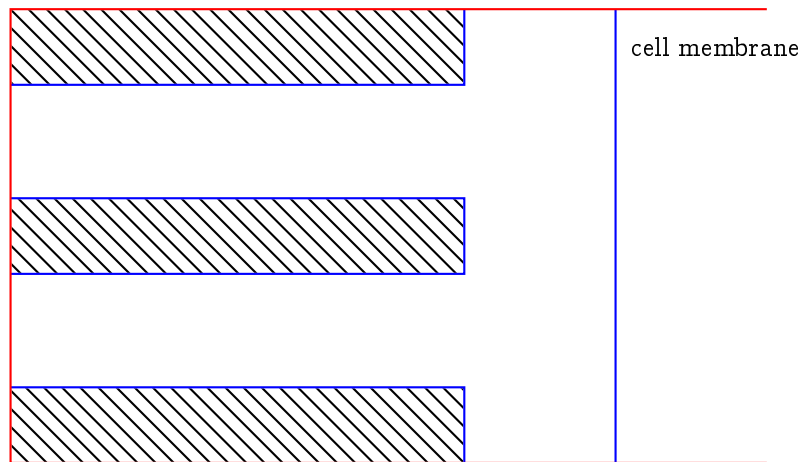
- ▶ disc;
- ▶ gap between discs;
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THE MOUSE ROD OUTER SEGMENT: FUNCTIONS

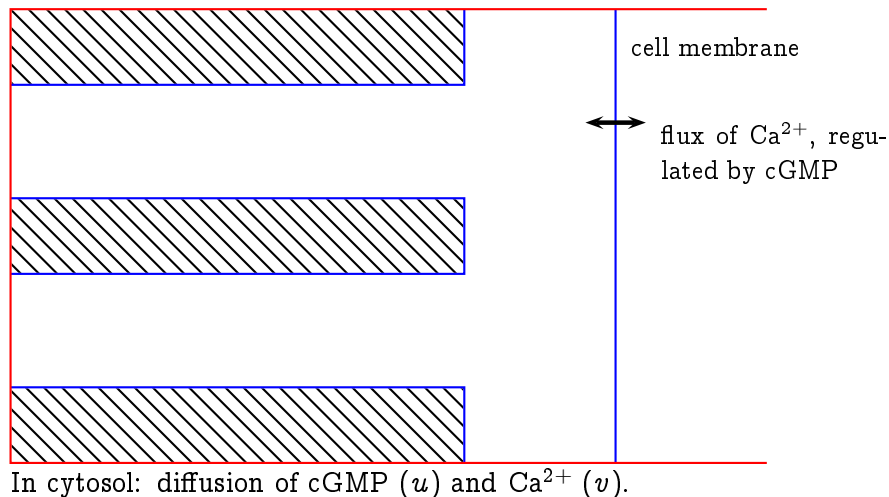


THE MOUSE ROD OUTER SEGMENT: FUNCTIONS

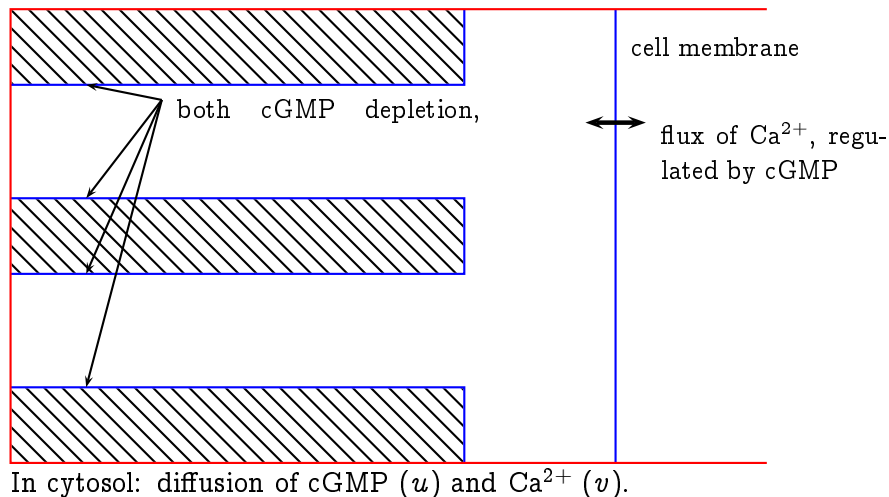


In cytosol: diffusion of cGMP (u) and Ca^{2+} (v).

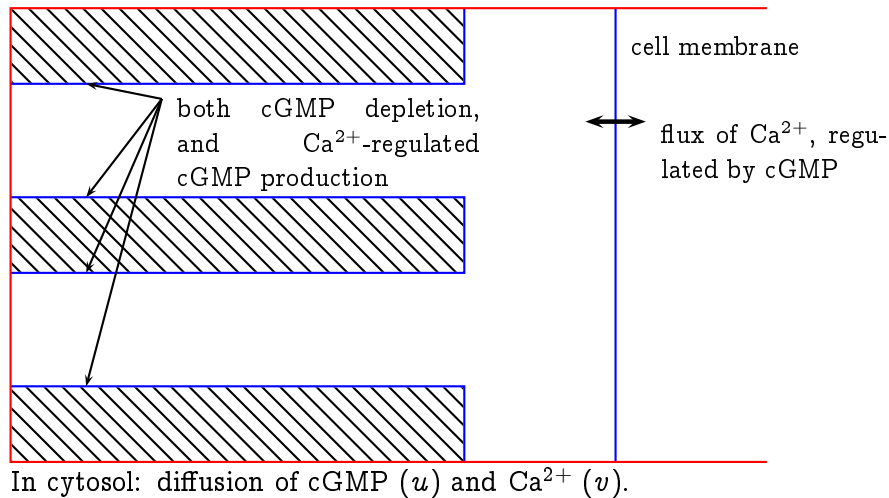
THE MOUSE ROD OUTER SEGMENT: FUNCTIONS



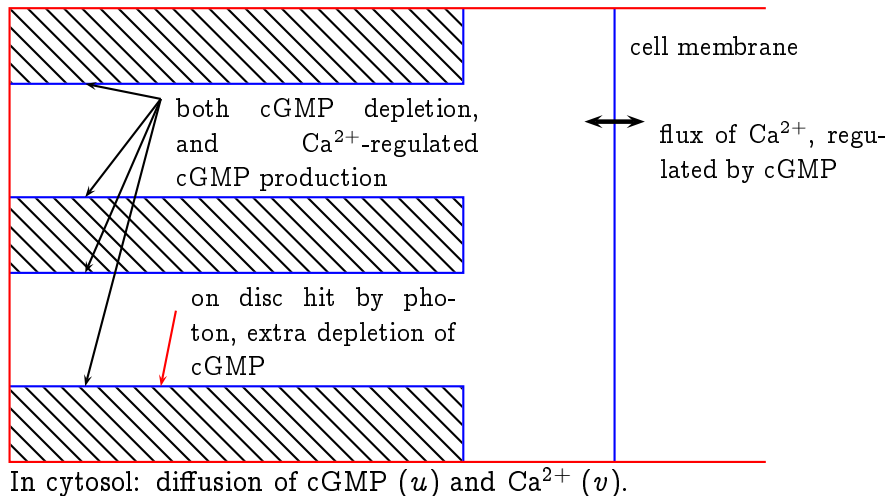
THE MOUSE ROD OUTER SEGMENT: FUNCTIONS



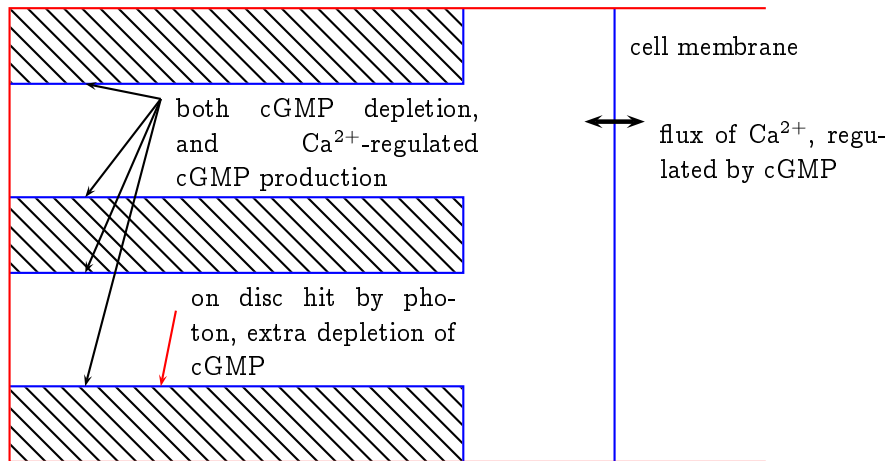
THE MOUSE ROD OUTER SEGMENT: FUNCTIONS



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THE MOUSE ROD OUTER SEGMENT: FUNCTIONS



Assume for simplicity that the photon hits the center of the rod outer segment.

THE PROBLEM: THE CYTOSOL COMPARTMENT

The part of the cascade taking place on the **disk**, is a *source of variability*: the shut off time of the activated rhodopsin is random.

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It has been long known a mechanism of *variability suppression* relying upon a multistep deactivation (and final switch off) of the activated molecule.

Here, we investigate the effect of the **cytosol** compartment.

OUTLINE

THE BIOLOGICAL PHENOMENON

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CV IN THE DIFFERENT MODELS

THE COMPLETE MODEL: DIFFUSION EQUATIONS

In the case of Mouse, we adopt a transversally well stirred model (only the longitudinal spatial dependence is preserved).

$$\begin{aligned}\frac{\partial u}{\partial t} - D_u \frac{\partial^2 u}{\partial z^2} &= -\beta u + \frac{\alpha_{\max} K^m + \alpha_{\min} v^m}{K^m + v^m}, & 0 < z < H/2, \\ \frac{\partial v}{\partial t} - D_v \frac{\partial^2 v}{\partial z^2} &= -\frac{j_1}{V_{\text{cyto}}} \frac{v}{K_1 + v} + \frac{j_2}{V_{\text{cyto}}} \frac{u^n}{K_2^n + u^n}, & 0 < z < H/2.\end{aligned}$$

u : [cGMP]; v : [Ca^{2+}].

The model has been obtained from the 3-D scheme through homogenization, concentration of capacity ... E.g., D_u , D_v are *effective* diffusivities, taking into account the geometry of the domain.

THE COMPLETE MODEL: INITIAL AND BOUNDARY DATA

INITIAL DATA: At time $t = 0$, we have

$$\begin{aligned}u(z, 0) &= u_{\text{dark}} & 0 < z < H/2, \\v(z, 0) &= v_{\text{dark}} & 0 < z < H/2.\end{aligned}$$

Here u_{dark} , v_{dark} are the equilibrium concentrations at dark.

THE COMPLETE MODEL: INITIAL AND BOUNDARY DATA

BOUNDARY DATA at $z = 0, H/2$: null flux conditions at $z = H/2$; at $z = 0$ we have for u the photon-activated depletion:

$$\begin{aligned}D_u \frac{\partial u}{\partial z}(0, t) &= K^*[\text{PDE}]_{\sigma}^* u(0, t), \\D_v \frac{\partial v}{\partial z}(0, t) &= 0.\end{aligned}$$

Here $[\text{PDE}]_{\sigma}^*$ is the surface concentration of activated Phosphodiesterase ...

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Here $[\text{PDE}]_{\sigma}^*$ is the surface concentration of activated Phosphodiesterase ... and the source of variability.

THE COMPLETE MODEL: CURRENT DROP FUNCTIONALS

The response of the ROS is evaluated essentially through the variation of the current exchanged through its boundary.

THE COMPLETE MODEL: CURRENT DROP FUNCTIONALS

We define the total averaged current $J(t)$, and the drop $I(t)$

$$J(t) = \frac{2}{H} \int_0^{H/2} \left[j_{\text{ex}}^{\text{sat}} \frac{v(z, t)}{K_1 + v(z, t)} + j_{\text{cG}}^{\text{max}} \frac{u(z, t)^n}{K_2^n + u(z, t)^n} \right] dz ,$$
$$I(t) = 1 - \frac{J(t)}{J_{\text{dark}}} , \quad J_{\text{dark}} = J(0) .$$

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$$I(t) = 1 - \frac{J(t)}{J_{\text{dark}}} , \quad J_{\text{dark}} = J(0) .$$

We look at the $\text{CV} = \text{s.d.}/\text{mean}$ of two functionals

$$I_{\text{int}} = \int_0^{\infty} I(t) dt , \quad \text{CV}_{\text{exp}}(I_{\text{int}}) \simeq .25 - .35 ,$$
$$I_{\text{peak}} = \max_{t \geq 0} I(t) , \quad \text{CV}_{\text{exp}}(I_{\text{peak}}) \simeq .20 .$$

THREE FACTORS SUPPRESSING VARIABILITY

We investigate the effects of three factors:

$$\begin{aligned}\frac{\partial u}{\partial t} - D_u \frac{\partial^2 u}{\partial z^2} &= -\beta u + \frac{\alpha_{\max} K^m + \alpha_{\min} v^m}{K^m + v^m}, & 0 < z < H/2, \\ \frac{\partial v}{\partial t} - D_v \frac{\partial^2 v}{\partial z^2} &= -\frac{j_1}{V_{\text{cyto}}} \frac{v}{K_1 + v} + \frac{j_2}{V_{\text{cyto}}} \frac{u^n}{K_2^n + u^n}, & 0 < z < H/2.\end{aligned}$$

THREE FACTORS SUPPRESSING VARIABILITY

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- D: diffusion

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THREE FACTORS SUPPRESSING VARIABILITY

We investigate the effects of three factors:

- ▶ D: diffusion
- ▶ F: feedback

$$\begin{aligned}\frac{\partial u}{\partial t} - D_u \frac{\partial^2 u}{\partial z^2} &= -\beta u + \frac{\alpha_{\max} K^m + \alpha_{\min} v^m}{K^m + v^m}, & 0 < z < H/2, \\ \frac{\partial v}{\partial t} - D_v \frac{\partial^2 v}{\partial z^2} &= -\frac{j_1}{V_{\text{cyto}}} \frac{v}{K_1 + v} + \frac{j_2}{V_{\text{cyto}}} \frac{u^n}{K_2^n + u^n}, & 0 < z < H/2.\end{aligned}$$

THREE FACTORS SUPPRESSING VARIABILITY

We investigate the effects of three factors:

- ▶ D: diffusion
- ▶ F: feedback
- ▶ N: nonlinearity

$$\begin{aligned}\frac{\partial u}{\partial t} - D_u \frac{\partial^2 u}{\partial z^2} &= -\beta u + \frac{\alpha_{\max} K^m + \alpha_{\min} v^m}{K^m + v^m}, & 0 < z < H/2, \\ \frac{\partial v}{\partial t} - D_v \frac{\partial^2 v}{\partial z^2} &= -\frac{j_1}{V_{\text{cyto}}} \frac{v}{K_1 + v} + \frac{j_2}{V_{\text{cyto}}} \frac{u^n}{K_2^n + u^n}, & 0 < z < H/2.\end{aligned}$$

THREE FACTORS SUPPRESSING VARIABILITY

We investigate the effects of three factors:

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- ▶ F: feedback
- ▶ N: nonlinearity

The complete DFN model:

$$\begin{aligned}\frac{\partial u}{\partial t} - D_u \frac{\partial^2 u}{\partial z^2} &= -\beta u + \frac{\alpha_{\max} K^m + \alpha_{\min} v^m}{K^m + v^m}, & 0 < z < H/2, \\ \frac{\partial v}{\partial t} - D_v \frac{\partial^2 v}{\partial z^2} &= -\frac{j_1}{V_{\text{cyto}}} \frac{v}{K_1 + v} + \frac{j_2}{V_{\text{cyto}}} \frac{u^n}{K_2^n + u^n}, & 0 < z < H/2.\end{aligned}$$

The DF_n model, where equations are linearized:

$$\begin{aligned}\frac{\partial u}{\partial t} - D_u \frac{\partial^2 u}{\partial z^2} &= A_{11}(u - u_{\text{dark}}) + A_{12}(v - v_{\text{dark}}), \\ \frac{\partial v}{\partial t} - D_v \frac{\partial^2 v}{\partial z^2} &= A_{21}(u - u_{\text{dark}}) + A_{22}(v - v_{\text{dark}}).\end{aligned}$$

The dFN model, stipulating a globally well-stirred assumption:

$$\begin{aligned}\frac{du}{dt} &= -\beta u + \frac{\alpha_{\max} K^m + \alpha_{\min} v^m}{K^m + v^m} - \frac{2K^*}{H} [\text{PDE}]_{\sigma}^* u, \\ \frac{dv}{dt} &= -\frac{j_1}{V_{\text{cyto}}} \frac{v}{K_1 + v} + \frac{j_2}{V_{\text{cyto}}} \frac{u^n}{K_2^n + u^n}.\end{aligned}$$

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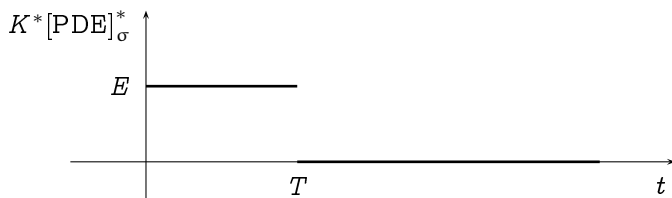
CV IN THE DIFFERENT MODELS

THE RANDOM SIMPLIFIED INPUT

We simplify the random behaviour of $[\text{PDE}]_{\sigma}^*$ as follows:

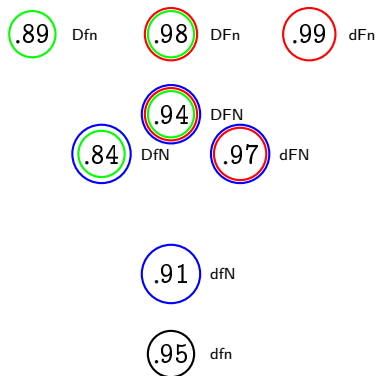
$$K^*[\text{PDE}]_{\sigma}^* = \begin{cases} E & 0 < t < T, \\ 0 & t > T, \end{cases}$$

where $E > 0$, and T an exponentially distributed random time, with average $1/\lambda$.

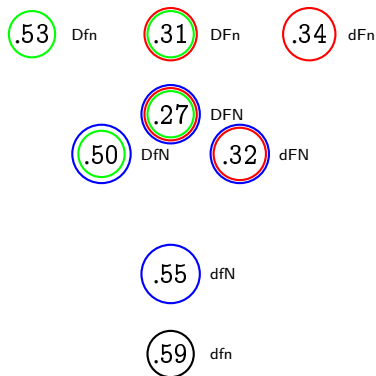


THE EFFECTS ON CV: SIMPLIFIED INPUT

Integral functional



Peak functional

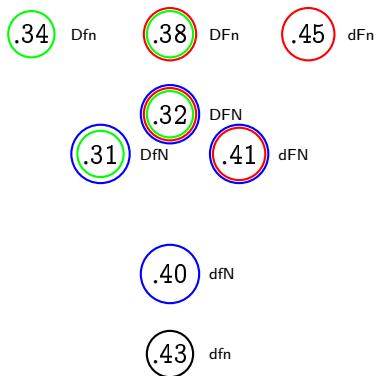


● Diffusion; ● Feedback; ● Nonlinearity.

THE EFFECTS ON CV: REALISTIC INPUT

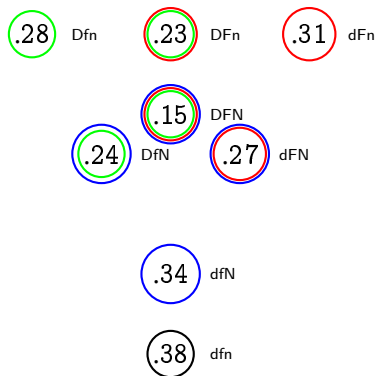
Integral functional

input $CV(\int K^*[PDE]_{\sigma}^*) = .46$



Peak functional

input $CV(\max K^*[PDE]_{\sigma}^*) = .31$



● Diffusion; ● Feedback; ● Nonlinearity.

CONCLUSIONS

- ▶ We examined the influence of three factors on the variability of the rod response: diffusion, feedback and nonlinearity, with the aim of singling out the contributions of each. We considered only the cytosol part of the cascade.

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- ▶ Diffusion and nonlinearity have a comparable effect in reducing variability. This comparable role is played in both the functionals.

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 - ▶ We considered both an idealized activation mechanism, and (just numerically) a realistic model of the biochemistry: they reveal the same trend.
- ▶ Diffusion and nonlinearity have a comparable effect in reducing variability. This comparable role is played in both the functionals.
- ▶ On the contrary the effects of feedback are opposite in the two functionals: feedback reduces variability of the supremum, and increases variability of the integral.

COLLABORATORS & REFS

Joint work with: P.Bisegna, G.Caruso, E.DiBenedetto, H.Hamm, L.Shen.

Andreucci, Bisegna, DiBenedetto, *C.R.Acad.Sci. Paris Sér.I*, 2002.

Khanal, Alexiades, DiBenedetto, Hamm, in *Proc. UPNF* 2002.

Andreucci, Bisegna, DiBenedetto, *Annali Mat. Pura Appl.*, 2003.

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Funding: NIH

OUTLINE

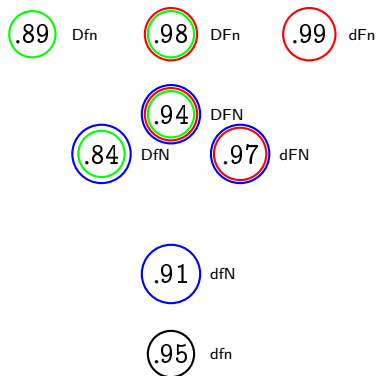
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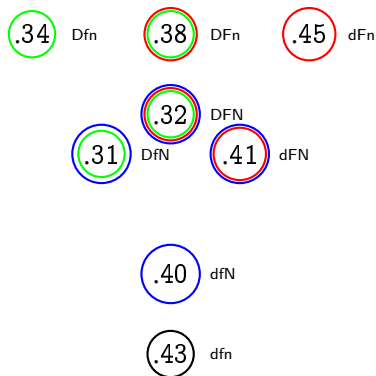
CV IN THE DIFFERENT MODELS

THE EFFECTS ON $CV(I_{\text{int}})$: SIMPLIFIED VS REALISTIC

Integral functional
simplified



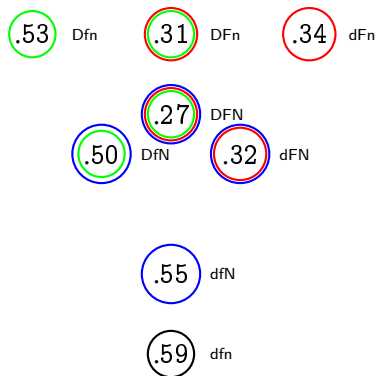
Integral functional
realistic



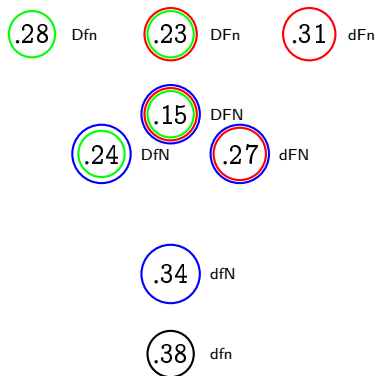
● Diffusion; ● Feedback; ● Nonlinearity.

THE EFFECTS ON $CV(I_{\text{peak}})$: SIMPLIFIED VS REALISTIC

Peak functional
simplified



Peak functional
realistic



● Diffusion; ● Feedback; ● Nonlinearity.

DIFFUSION FEEDBACK NONLINEARITY

The DFN model:

$$\frac{\partial u}{\partial t} - D_u \frac{\partial^2 u}{\partial z^2} = -\beta u + \frac{\alpha_{\max} K^m + \alpha_{\min} v^m}{K^m + v^m}, \quad 0 < z < H/2,$$

$$\frac{\partial v}{\partial t} - D_v \frac{\partial^2 v}{\partial z^2} = -\frac{j_1}{V_{\text{cyto}}} \frac{v}{K_1 + v} + \frac{j_2}{V_{\text{cyto}}} \frac{u^n}{K_2^n + u^n}, \quad 0 < z < H/2,$$

plus initial and boundary conditions, e.g.,

$$D_u \frac{\partial u}{\partial z}(0, t) = K^*[\text{PDE}]_{\sigma}^* u(0, t),$$

$$D_v \frac{\partial v}{\partial z}(0, t) = 0.$$

The DFn model:

$$\begin{aligned}\frac{\partial u}{\partial t} - D_u \frac{\partial^2 u}{\partial z^2} &= A_{11}(u - u_{\text{dark}}) + A_{12}(v - v_{\text{dark}}), \\ \frac{\partial v}{\partial t} - D_v \frac{\partial^2 v}{\partial z^2} &= A_{21}(u - u_{\text{dark}}) + A_{22}(v - v_{\text{dark}}),\end{aligned}$$

and the current functional is also linearized.

The dFN model:

$$\begin{aligned}\frac{du}{dt} &= -\beta u + \frac{\alpha_{\max} K^m + \alpha_{\min} v^m}{K^m + v^m} - \frac{2K^*}{H} [\text{PDE}]_{\sigma}^* u, \\ \frac{dv}{dt} &= -\frac{j_1}{V_{\text{cyto}}} \frac{v}{K_1 + v} + \frac{j_2}{V_{\text{cyto}}} \frac{u^n}{K_2^n + u^n}.\end{aligned}$$

The DfN model:

$$\frac{\partial u}{\partial t} - D_u \frac{\partial^2 u}{\partial z^2} = -\beta u + \frac{\alpha_{\max} K^m + \alpha_{\min} v_{\text{dark}}^m}{K^m + v_{\text{dark}}^m},$$

plus initial and boundary conditions, e.g.,

$$D_u \frac{\partial u}{\partial z}(0, t) = K^*[\text{PDE}]_{\sigma}^* u(0, t).$$

The differential equation is actually linear but the current functional is not linearized.

The Dfn model:

$$\frac{\partial u}{\partial t} - D_u \frac{\partial^2 u}{\partial z^2} = -\beta u + \frac{\alpha_{\max} K^m + \alpha_{\min} v_{\text{dark}}^m}{K^m + v_{\text{dark}}^m},$$

plus initial and boundary conditions, e.g.,

$$D_u \frac{\partial u}{\partial z}(0, t) = K^*[\text{PDE}]_{\sigma}^* u(0, t),$$

and the current functional is also linearized.

The dFn model:

$$\begin{aligned}\frac{du}{dt} &= -\frac{2K^*}{H}[\text{PDE}]_{\sigma}^* u + A_{11}(u - u_{\text{dark}}) + A_{12}(v - v_{\text{dark}}), \\ \frac{dv}{dt} &= A_{21}(u - u_{\text{dark}}) + A_{12}(v - v_{\text{dark}}).\end{aligned}$$

The current functionals are also linearized.

The dfN model:

$$\frac{du}{dt} = -\frac{2K^*}{H}[\text{PDE}]_{\sigma}^* u - \beta u + \frac{\alpha_{\max} K^m + \alpha_{\min} v_{\text{dark}}^m}{K^m + v_{\text{dark}}^m}.$$

The differential equation is actually linear but the current functional is not linearized.

The dfn model:

$$\frac{du}{dt} = -\frac{2K^*}{H}[\text{PDE}]_{\sigma}^* u - \beta u + \frac{\alpha_{\max} K^m + \alpha_{\min} v_{\text{dark}}^m}{K^m + v_{\text{dark}}^m}.$$

The current functionals are also linearized.

$$\text{CV}\left(\frac{H}{2} \int_0^{\infty} (u(t) - u_{\text{dark}}) dt\right) = \left[\frac{1 + \frac{2EH^{-1}}{|A_{11}|(1+\lambda(2|A_{11}|)^{-1})(1+\lambda|A_{11}|^{-1})^2}}{1 + \frac{2EH^{-1}}{|A_{11}|(1+\lambda(2|A_{11}|)^{-1})}} \right]^{\frac{1}{2}},$$

$$\text{CV}\left(\sup_{0 < t < \infty} |u(t) - u_{\text{dark}}|\right) = \frac{1}{\sqrt{1 + 2|A_{11}|\lambda^{-1} + 4EH^{-1}\lambda^{-1}}}.$$

MULTISTEP SHUTTING OFF

Denote by $\chi_{(t_{j-1}, t_j]}$ the characteristic function of the interval $(t_{j-1}, t_j]$. Then the rate equations for G (G-protein) and E (Phosphodiesterase) are

$$\begin{aligned}G_t &= \sum_{j=1}^n \nu_j \chi_{(t_{j-1}, t_j]}(t) - k_1 G E, \\E_t &= k_1 G E - k_2 E,\end{aligned}$$

with initial data

$$G(0) = E(0) = 0.$$

PARAMETERS

Symbol	Units	Definition	Value
a_{int}	μm^2	interior transversal area of the ROS	$\pi R^2 \nu / (1 + \nu)$
a_{inc}	μm^2	area of the incisure	0.0367
a_{tot}	μm^2	total transversal area of the ROS	$2\pi R \sigma \varepsilon + a_{\text{int}} + a_{\text{inc}}$
a_{pat}	μm^2	patent transversal area of the ROS	$2\pi R \sigma \varepsilon + a_{\text{inc}}$
α_{max}	μMs^{-1}	maximum rate of cGMP synthetization	76.5
$\alpha_{\text{max}}/\alpha_{\text{min}}$		suppression rate of cGMP synthetization	13.9
β	s^{-1}	rate of cGMP hydrolysis by dark act. PDE2.9	
B_{Ca}		buffering power of cytoplasm for Ca^{2+}	20
D_{cG}	$\mu\text{m}^2 \text{s}^{-1}$	diffusivity of cGMP	150
D_u	$\mu\text{m}^2 \text{s}^{-1}$	effective axial diffusivity of cGMP	$D_{\text{cG}} \frac{a_{\text{pat}}}{a_{\text{tot}}}$
D_{Ca}	$\mu\text{m}^2 \text{s}^{-1}$	diffusivity of Ca^{2+}	15
D_v	$\mu\text{m}^2 \text{s}^{-1}$	effective axial diffusivity of Ca^{2+}	$D_{\text{Ca}} \frac{a_{\text{pat}}}{a_{\text{tot}}}$
E	μms^{-1}	amplitude of act. in simplified model Dfn	6.96
ε	μm	disk thickness	0.0145
\mathcal{F}	Cmol^{-1}	Faraday's constant	96500
f_{Ca}		frac. of current carried by Ca^{2+}	0.06
H	μm	height of ROS	23.6

PARAMETERS

Symbol	Units	Definition	Value
j_1	$\mu\text{Ms}^{-1}\mu\text{m}^3$	coefficient in eq. for v	$j_{\text{ex}}^{\text{sat}}/(B_{\text{Ca}}\mathcal{F})$
j_2	$\mu\text{Ms}^{-1}\mu\text{m}^3$	coefficient in eq. for v	$j_{\text{cG}}^{\text{max}}f_{\text{Ca}}/(2B_{\text{Ca}}\mathcal{F})$
$j_{\text{cG}}^{\text{max}}$	pA	maximum cGMP-gated channel current	3550
$j_{\text{ex}}^{\text{sat}}$	pA	saturated exchanger current	1.8
K_{hyd}	$\mu\text{m}^3\text{s}^{-1}$	surface hydrolysis rate by dark-act. PDE	$2.8 \cdot 10^{-5}$
k^*	$\mu\text{m}^3\text{s}^{-1}$	surface hydrolysis rate by light-act. PDE	0.9
K	μM	half-saturating v for GC activity	0.129
K_2	μM	u for half-max cGMP-gated channel opening	20
K_1	μM	v for half-max exchanger channel opening	1.6
λ	s^{-1}	parameter in the simplified model Dfn	3.60
n_{discs}		number of discs	814
m		Hill coefficient for the effect of GC	2.45
n		Hill coefficient for the cGMP-gated channels	3
R	μm	radius of disk	0.7
V_{cyto}	μm^3	cytoplasmic volume	$a_{\text{tot}}H$
u_{dark}	μM	concentration of cGMP in the dark	3.0750
v_{dark}	μM	concentration of Ca^{2+} in the dark	0.4363