Stationarization via Surrogates

Pierre Borgnat & Patrick Flandrin

Université de Lyon, Laboratoire de Physique de l'Ecole Normale Supérieure de Lyon, UMR CNRS 5672, 69364 Lyon Cedex 07, France

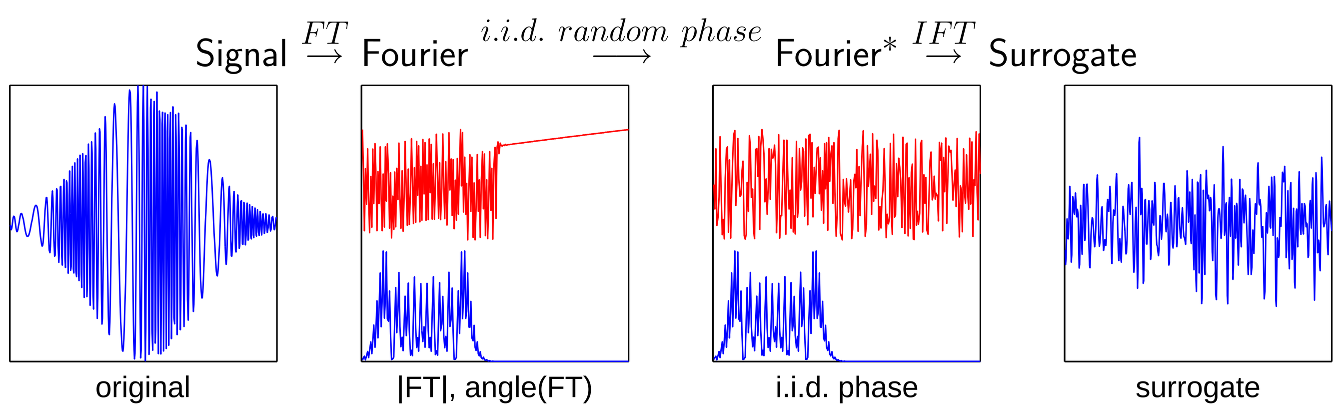
Firstname.Lastname@ens-lyon.fr

1. Revisiting Surrogates

Original context — Surrogate data were introduced in [Theiler et al., 1992] for **testing nonlinearity**.

- This is a method of Monte-Carlo sampling aiming at creating new samples from the data;
- The new samples are constrained to satisfy a given null hypothesis, here = linearity of the system.

Generate Surrogate Data — Simplest form: Fourier-based Surrogates



Use of Surrogates for Non-linearity tests. — The Non-linearity test is applied to a set of surrogates.

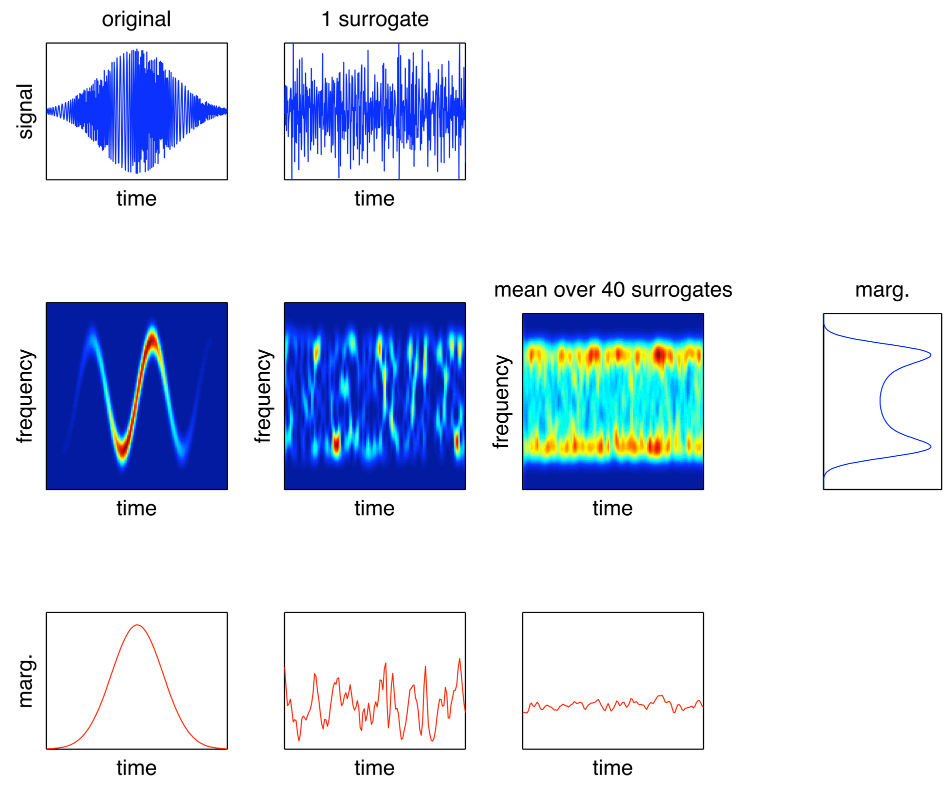
- Surrogates → set of data under the null hypothesis for the test → experimental threshold.

Variations. — Other surrogate methods with added constraints from the original data:

- Improved Surrogates having same probability distribution, see review in [Schreiber & Schmitz, 1996],
- Preservation of the mean and variance structure → preserves nonstationary evolution [Keylock, 2006].

2. Stationarization via Surrogates

New idea — Surrogates act as null hypothesis for **stationary signals**.



Rationale — For a same spectrum density, "nonstationary" signals differ from "stationary" ones by temporal *structures* encoded in the spectrum phase.

3. Stationarity Testing with Surrogates

Objective — Deal with "stationarity" in an operational sense, including the possibility of its test relatively to a given observation scale and that would both encompass stochastic and deterministic variants.

Time-Frequency framework — Natural language for nonstationary signals and processes: Time-frequency spectra (and their estimates) should undergo no evolution in "stationary" situations.

- **Multitaper spectrograms** are defined as

$$S_{x,K}(t,f) = \frac{1}{K}\sum_{k=1}^{K}\left|\int_{-\infty}^{+\infty} x(s)\, h_k(s-t)\, e^{-i2\pi fs}\, ds\right|^2,$$

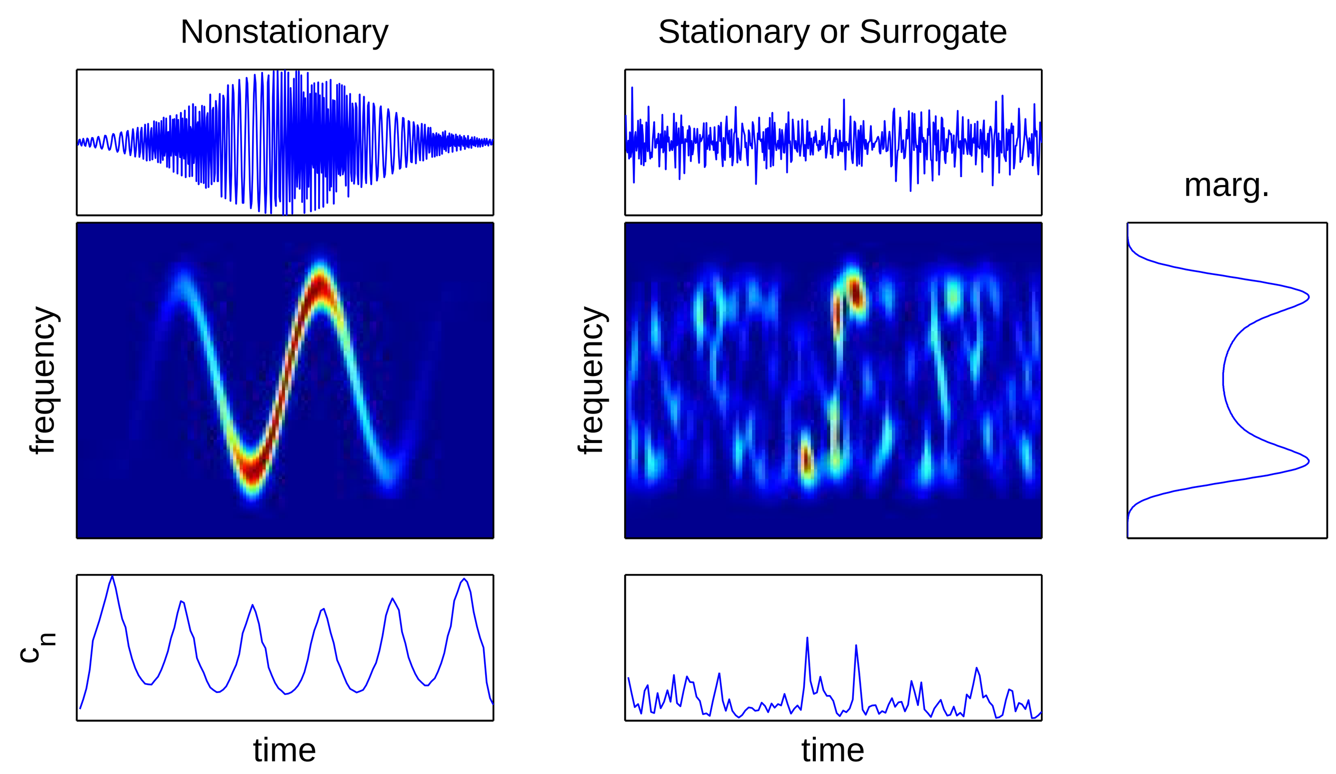
  with K short-time windows $h_k(t)$ chosen as the K first Hermite functions [Bayram & Baraniuk, 2000].
- **Advantage:** *estimates* of the Wigner-Ville Spectrum for stochastic processes and *reduced interference distributions* for deterministic signals, with reduced estimation variance without some extra time-averaging (unappropriate in a nonstationary context).
- **In practice:** spectrograms evaluated only at N time positions $\{t_n, n = 1, \ldots N\}$, with a spacing $t_{n+1} - t_n$ which is a fraction of the width of the K windows $h_k(t)$, K chosen in between 5 and 10.

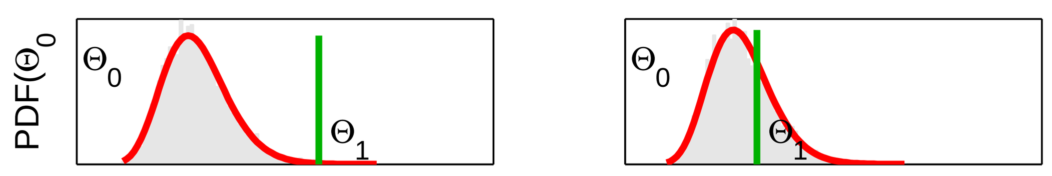
Principle of the test — Compare *local* features vs. *global* ones obtained by marginalization over time, relatively to a chosen observation scale [Xiao et al. 2007 A].

- Contrast Global/Local with some TFD distance κ: $\quad c_n^{(x)} := \kappa\left(S_{x,K}(t_n,.), \langle S_{x,K}(t_n,.)\rangle_{n=1,\ldots N}\right),$
- Fluctuations in time of these divergences $c_n^{(x)}$: $\quad L(g_n,h_n) := \frac{1}{N}\sum_{n=1}^{N}(g_n - h_n)^2,$
- Test statistics = dispersion of the fluctuations: $\quad \Theta_1 = L\left(c_n^{(x)}, \langle c_n^{(x)}\rangle_{n=1,\ldots N}\right).$

Null hypothesis — provided by the collection of Test Statistics of the surrogates s_j:

$$\left\{\Theta_0(j) = L\left(c_n^{(s_j)}, \langle c_n^{(s_j)}\rangle_{n=1,\ldots N}\right), j = 1, \ldots J\right\}.$$





$$\begin{cases} \Theta_1 > \gamma : \text{"nonstationarity"}; \\ \Theta_1 < \gamma : \text{"stationarity"}. \end{cases}$$

Stationarity Test — Form the one-sided test, with threshold γ obtained from the null hypothesis.

Variant — Use One-Class SVM to learn the distribution of stationary signals [Xiao et al. 2007 B].

4. Variations in Time-Frequency Contexts

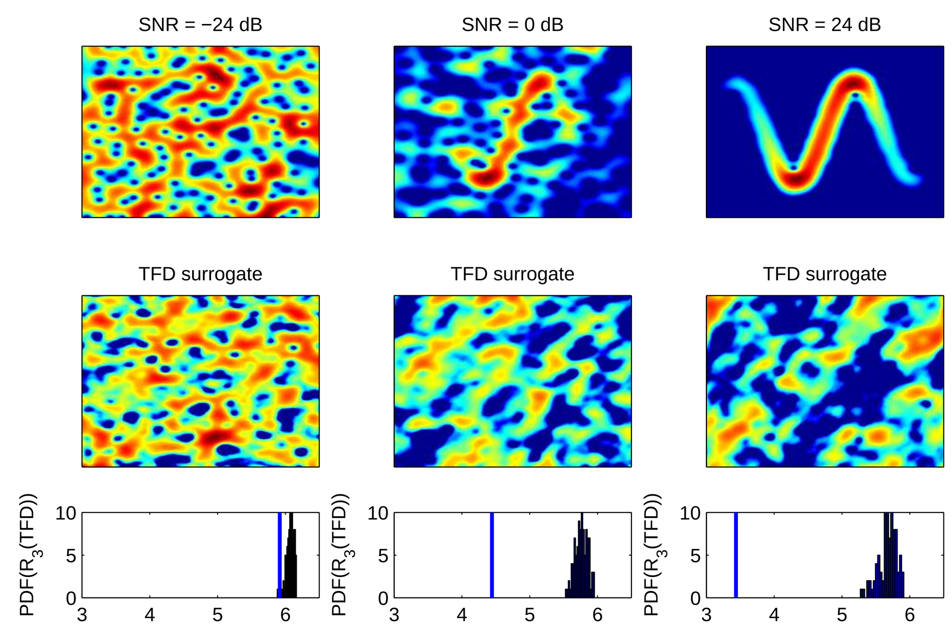
4.a For Transient Detection

Objective — Assess the level of significance of a "time-frequency" patch in a fluctuating background.

Principle — Construct a **Time-Frequency Distribution Surrogate**, directly via phase randomization of the 2D Fourier transform.

- Constraints: preserve the amplitude of the Ambiguity function; positivity of the TFD Surrogate.
- Procedure: iterative correction of the phase and thresholding negative values of the TFD.

Statistical Test — with Renyi entropy: $R_\alpha(TFD) = \frac{1}{1-\alpha}\iint\left(\frac{TFD(t,f)}{\iint TFD(t,f)dtdf}\right)^\alpha dtdf.$



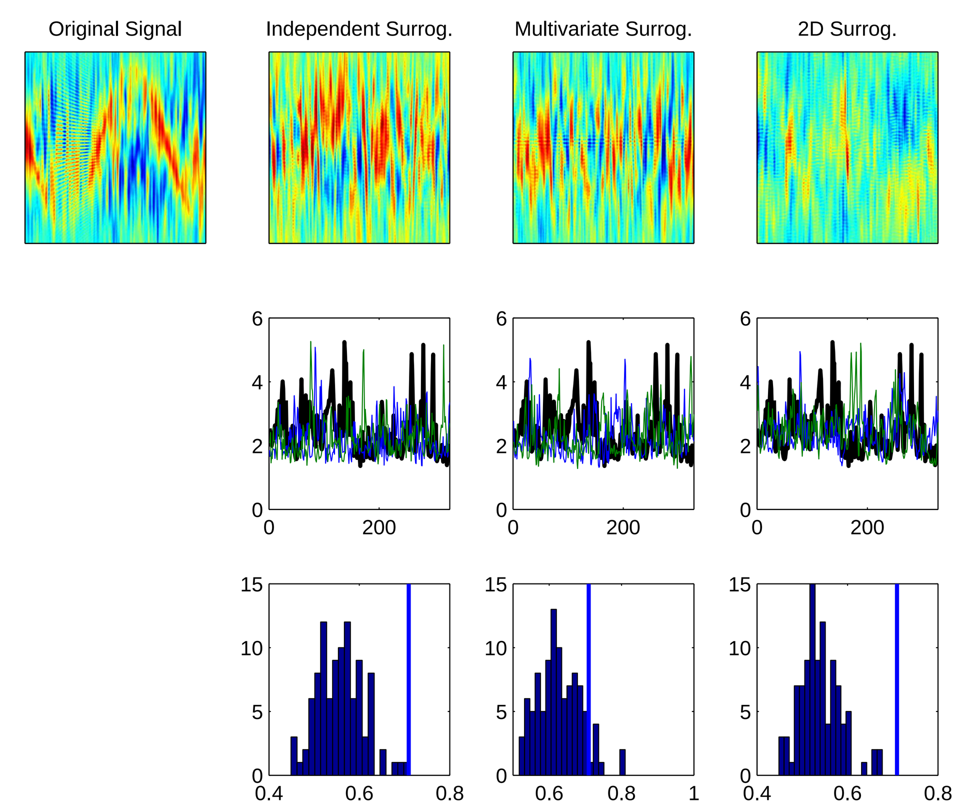
4.b For Nonstationary Correlations

Objective — Assess the existence and evolution of cross-correlations in multivariate data.

Principle — Compare the cross-correlations of original data with the one of surrogates.

- Multivariate formulation of original surrogates keeps the marginal cross-spectrum [Prichard & Theiler 1994]. But it does not keep the 'geometrical' structure of the quadratic distribution in the plane.
- **Proposition:** directly design time-lag surrogates for the cross-correlations via a Fourier phase randomization of the 2D cross-correlations.

Statistical Test — with Kurtosis of the divergence between local and averaged cross-correlations.



Open question: — Comparison to other resampling plans: Bootstrap; Jackknife; Cross-Validation [Efron, 1982].

[M. Bayram and R.G. Baraniuk, 2000] "Multiple window time-varying spectrum estimation," in *Nonlinear and Nonstationary Signal Processing* (W.J. Fitzgerald et al., eds.), pp. 292–316, Cambridge Univ. Press.

[B. Efron, 1982] *The Jackknife, the Bootstrap and Other Resampling Plans.* Philadelphia: SIAM.

[C.J. Keylock, 2006] "Constrained surrogate time series with preservation of the mean and variance structure," *Phys. Rev. E*, vol. 73, pp. 030767.1-030767.4.

[T. Schreiber and A. Schmitz, 1996] "Improved surrogate data for nonlinearity tests," *Phys. Rev. Lett.*, vol. 73, no. 4, pp. 635–638.

[J. Theiler et al., 1992] J. Theiler, S. Eubank, A. Longtin, B. Galdrikian and D. Farmer, "Testing for nonlinearity in time series: the method of surrogate data," *Physica D*, vol. 58, no. 1–4, pp. 77–94.

[Xiao et al., 2007 A] J. Xiao, P. Borgnat and P. Flandrin, "Testing stationarity with time-frequency surrogates," *Proc. EUSIPCO-07*, Poznan (PL).

[Xiao et al., 2007 B] J. Xiao, P. Borgnat, P. Flandrin and C. Richard, "Testing stationarity with surrogates – A one-class SVM approach," *Proc. IEEE Stat. Sig. Proc. Workshop SSP-07*, Madison (WI).