Optimal operation of feedback flashing ratchets

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Feedback flashing ratchets are thermal rectifiers that use information on the state of the system to operate switching on and off a periodic potential in order to induce a directed flux. We discuss different strategies for this operation with the aim of maximizing the net flux of particles in the collective version of the ratchet consisting in N overdamped Brownian particles. We show the optimal protocols for one-particle feedback flashing ratchets and for collective feedback flashing ratchets with an infinite number of particles. Finally we comment on the unsolved problem of the general optimal strategy for any number of particles.

Introduction

▶ Ratchets: Controllers that act on stochastic systems with the aim of inducing directed motion through the rectification of fluctuations.

▶ Flashing ratchets: Operate switching on and off an asymmetric periodic potential

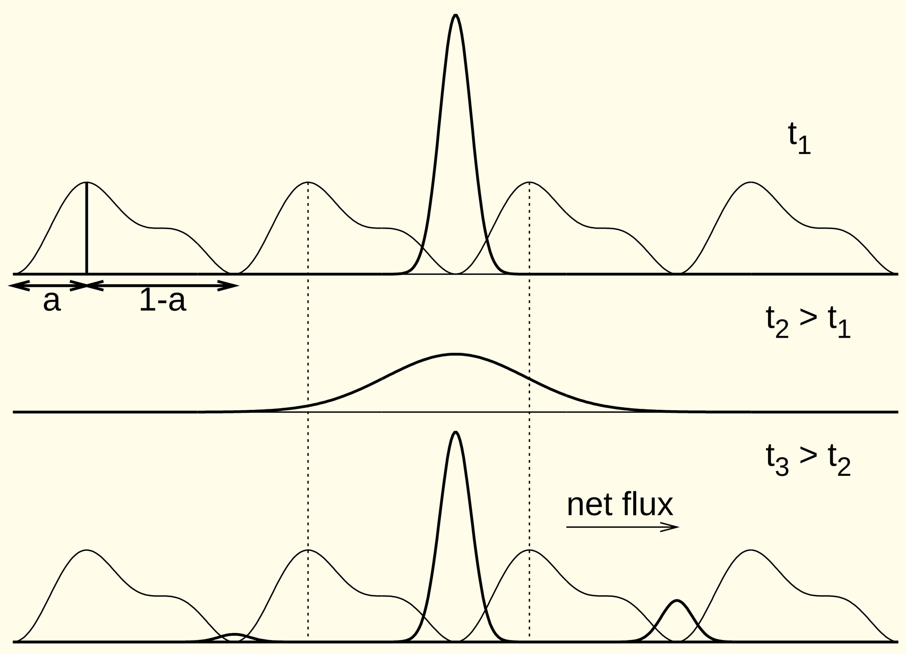

Fig. 1: Operation of a flashing ratchet

▶ Control in collective flashing ratchets

N Brownian particles (Langevin equations)

$$\gamma \dot{x}_i(t) = \alpha(t) F(x_i(t)) + \xi_i(t) \quad (i = 1, \ldots, N)$$

$\alpha(t)$ control: $\alpha = 0$ (off) or $\alpha = 1$ (on)

$\xi_i(t)$ thermal fluctuations (white noise)

$f(t) = \frac{1}{N}\sum_{i=1}^{N} F(x_i(t))$ net force

- Open-loop control (e.g. periodic: switch on and off periodically)
- Feedback or closed-loop control (information about the state of the system)

▶ Motivation

- Relevance in Theoretical Physics [1]
- Applications to Biology and Nanotechnology [1]

▶ Objetives

- Maximum flux in collective feedback ratchets
- Optimal open-loop vs. closed-loop controls
- Maximum performance of Maxwell's demons

Periodic switching protocol [2]

▶ Operation

$$\alpha(t) = \alpha(t + t_{\text{on}} + t_{\text{off}}) = \begin{cases} 1 & \text{if } 0 \le t < t_{\text{on}} \\ 0 & \text{if } t_{\text{on}} \le t < t_{\text{on}} + t_{\text{off}} \end{cases}$$

▶ Comments

- Open-loop protocol
- Independent of N
- Optimal values for the on and off semiperiods [3]
  t_{on}: characteristic time to 'slide down'
  t_{off}: characteristic time to diffuse
- Optimal protocol (maximum flux) for $N \to \infty$

Instant maximization protocol [4]

▶ Operation

$$\alpha(t) = \Theta(f(t)) \equiv \begin{cases} 1 & \text{if } f(t) \ge 0 \\ 0 & \text{if } f(t) < 0 \end{cases}$$

▶ Comments

- Exact solution for one particle (effective potential + Fokker-Planck)
- Optimal (maximum flux) for one particle
- Flux goes to zero as N increases

Threshold protocol [5]

▶ Operation

$$\alpha(t) = \begin{cases} 1 & \text{if } f(t) \ge u_{\text{on}}, \\ 1 & \text{if } u_{\text{off}} < f(t) < u_{\text{on}} \text{ and } \dot{f}_{\text{exp}}(t) \ge 0, \\ 0 & \text{if } u_{\text{off}} < f(t) < u_{\text{on}} \text{ and } \dot{f}_{\text{exp}}(t) < 0, \\ 0 & \text{if } f(t) \le u_{\text{off}}. \end{cases}$$

$$\dot{f}_{\text{exp}} \equiv \frac{1}{\gamma N} \sum_i [\alpha(t) F(x_i(t)) F'(x_i(t)) + kT F''(x_i(t))]$$

▶ Comments

- The thresholds $u_{\text{on}} \ge 0$ and $u_{\text{off}} \le 0$ prevent the dynamics to get trapped.
- The instant maximization protocol is recovered by taking $u_{\text{on}} = u_{\text{off}} = 0$ (optimum for one particle)
- Nonzero thresholds: Flux goes to a constant as N increases
- For large number of particles the optimal thresholds are related with optimal periods [5]

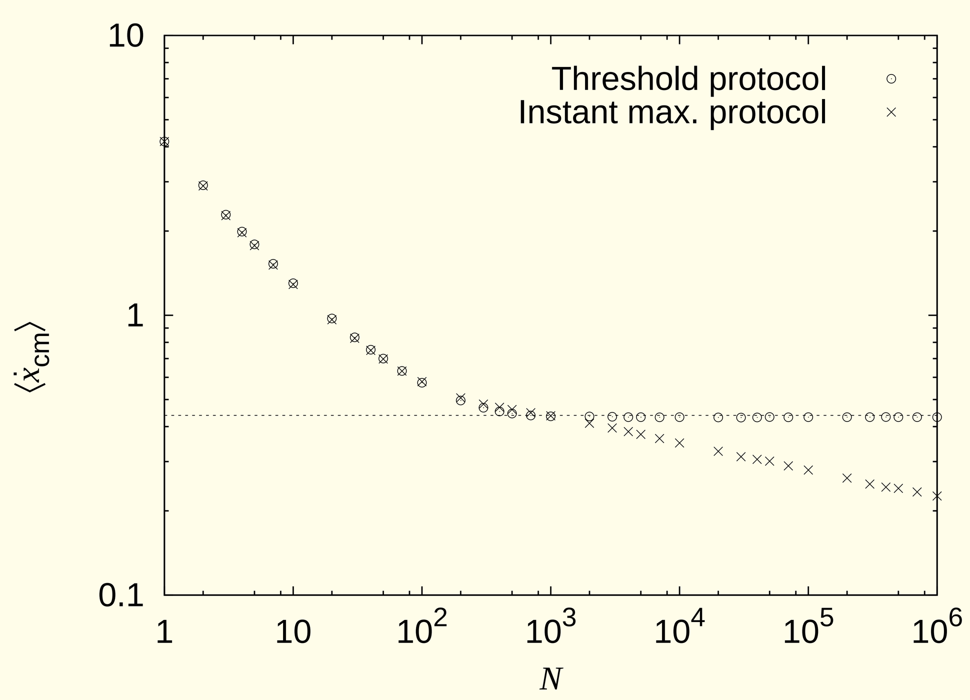

Fig. 2: Flux vs N for the instant maximization protocol and the threshold protocol. The dashed line stands for the best optimal periodic switching (open-loop).

Maximal displacement protocol [6]

▶ Operation

$$\alpha(t) = \Theta(d(t))$$

$$d(t) = \sum_{i=1}^{N} (x_i(t) - x_0)$$

▶ Comments

- It bases its control in the net displacement, not in the net force
- Numerical simulations show that it only performs better than the instant maximization protocol in the cases of 2 and 3 particles for potential heights greater than $30 k_B T$ [6]. For other cases it performs worse.

Conclusions and open questions

▶ Conclusions

- Optimal strategy for $N = 1$ (instant maximization protocol) and for $N \to \infty$ (open-loop periodic protocol)
- The threshold protocol with proper values is optimal for both $N = 1$ and $N \to \infty$
- Theoretical design of experimental set-ups for feedback flashing ratchets [4, 5, 6]

▶ Open questions

- Can the threshold protocol achieve the maximum flux with proper threshods values for a given number of particles? [7]
- A systematic study using stochastic Pontryagin maximum principle and/or stochastic Bellman's dynamics is lacking
- Experimental results for feedback ratchets [6]

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