Activation barrier scaling and switching path distribution in micromechanical parametric oscillators

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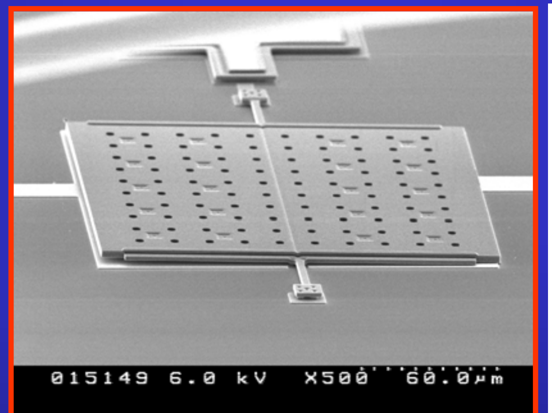
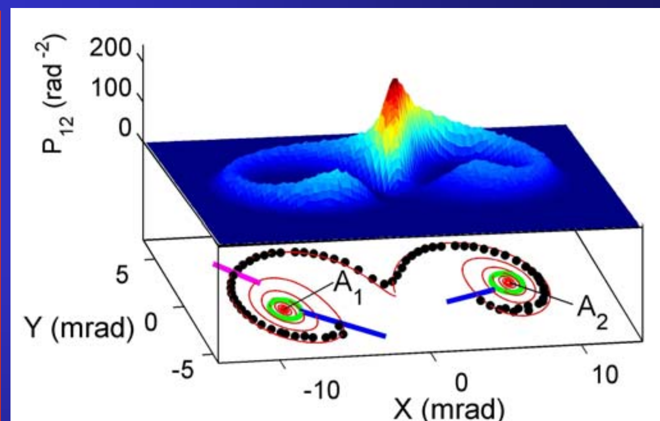
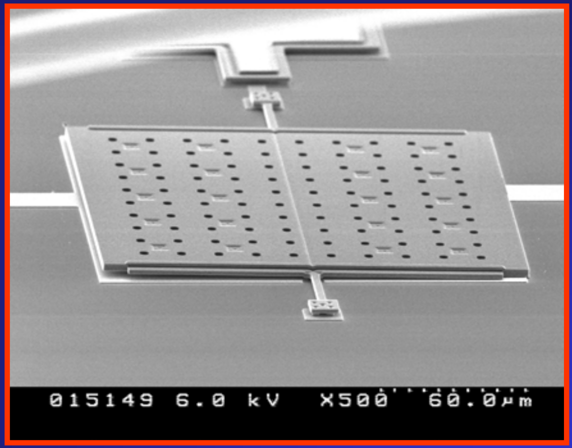
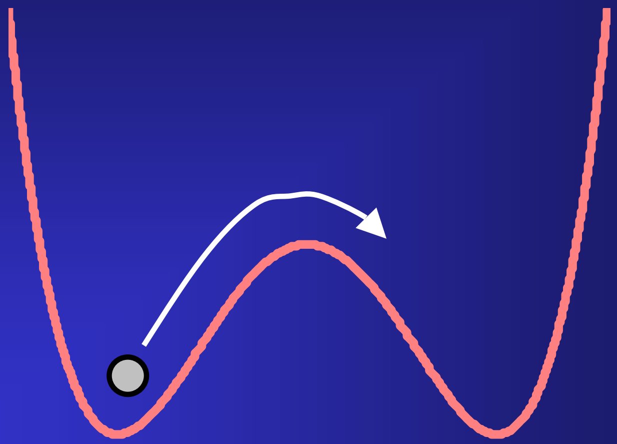




June 2nd, 2008. UPoN, Lyon, France.

Outline





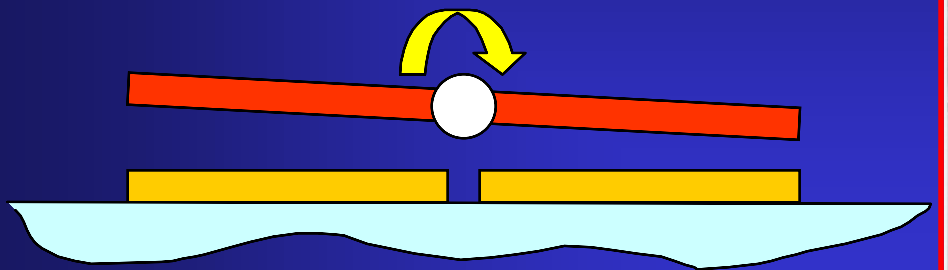
what are the generic features of fluctuations in systems far from equilibrium?

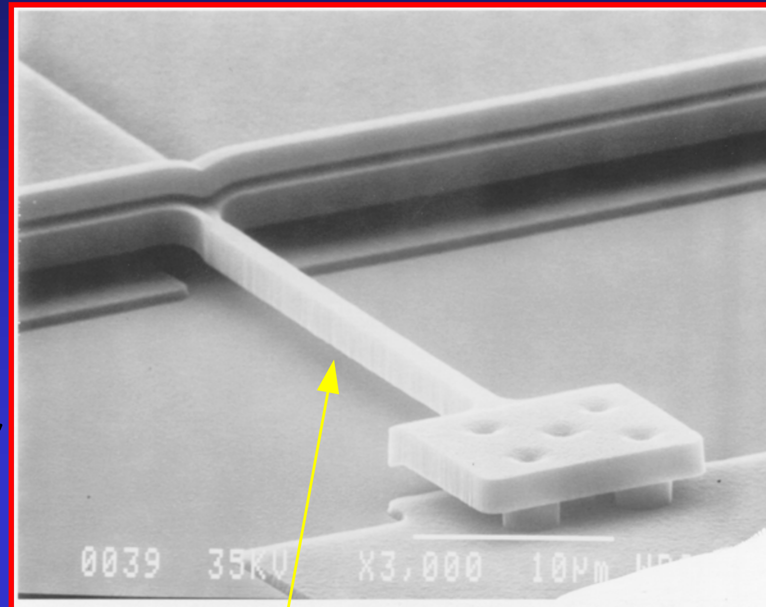
- Nonlinear micromechanical parametric oscillator: multistability and bifurcations.
- Fluctuation induced switching between dynamic states.
- Switching path distribution for multivariable system:

  Direct measurement of most probable switching path

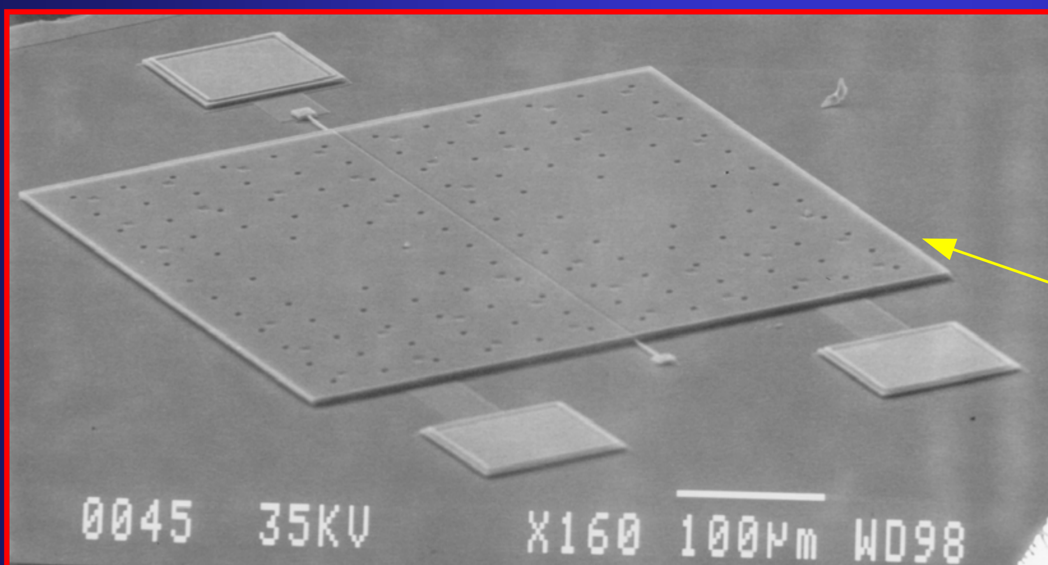
  Demonstration of the lack of time reversal symmetry.
- Universal scaling of activation barrier near the bifurcation point.

micromechanical torsional oscillator



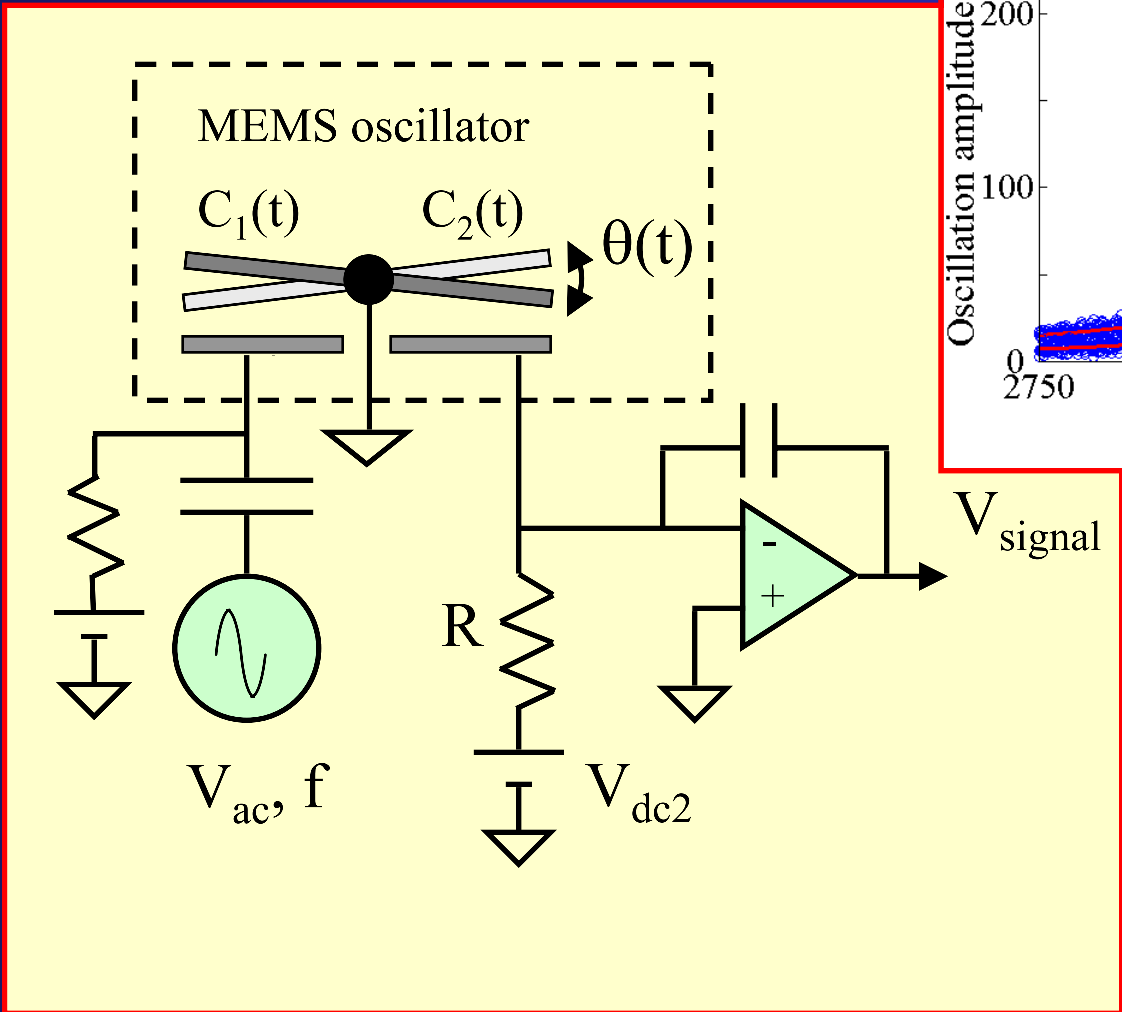


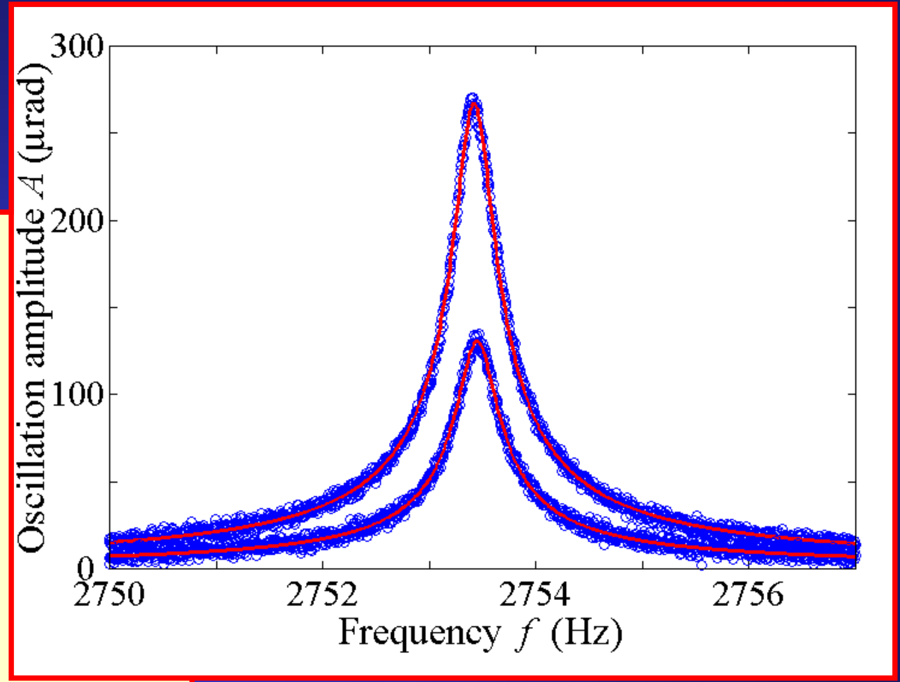
Torsional rod
cross section: 1.5 x 2 µm^2



poly-Si plate:
500 µm x 500 µm x 3.5 µm

Simple Harmonic Oscillations





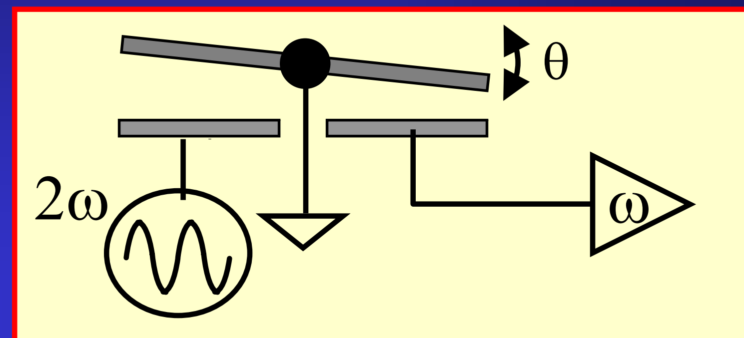
At small excitation, response depends linearly on excitation voltage

Q = 7150

Parametric resonance

$$\ddot{\theta} + 2\gamma\dot{\theta} + [\omega_0^2 + \frac{k_e}{I}\cos(\omega t)]\theta = -\alpha\theta^2 - \beta\theta^3$$

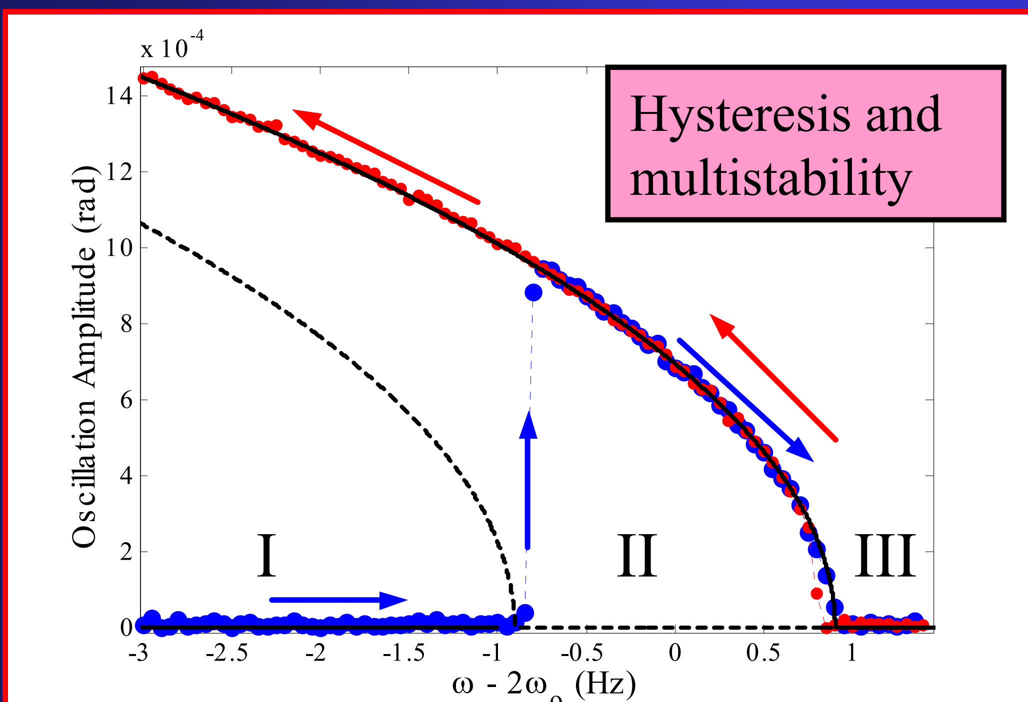
$$\omega \sim 2\omega_0$$

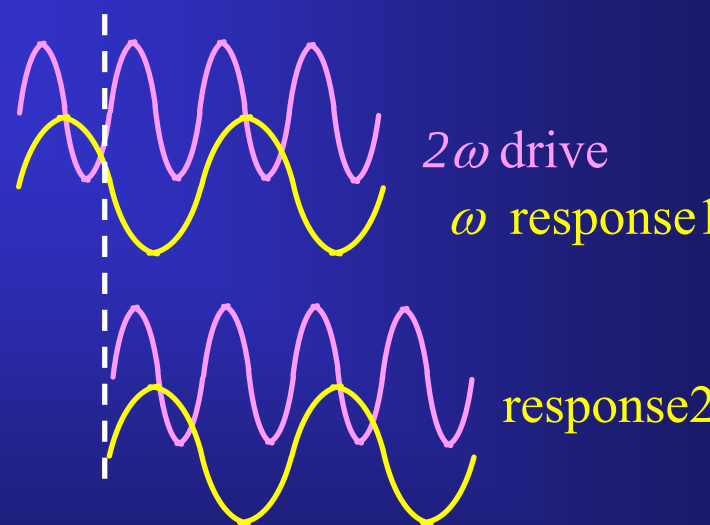


Electrostatic torque

$$\tau_E = \frac{1}{2}\frac{dC}{d\theta}V^2$$

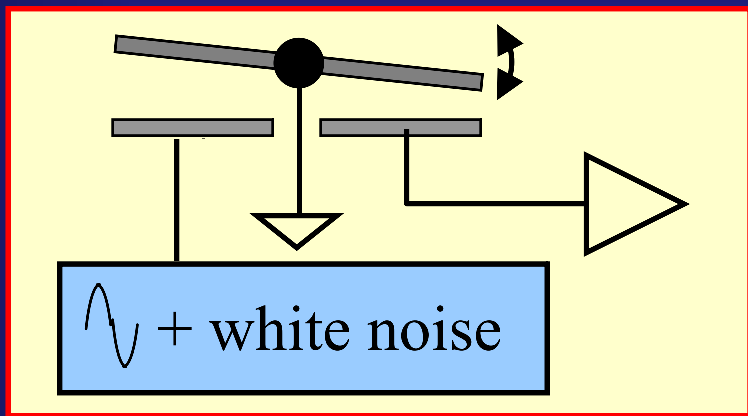
$$\tau_E(\theta) = \tau_E(0) + \frac{\partial \tau_E}{\partial \theta}\theta + \frac{1}{2}\frac{\partial^2 \tau_E}{\partial \theta^2}\theta^2 + \frac{1}{6}\frac{\partial^3 \tau_E}{\partial \theta^3}\theta^3$$

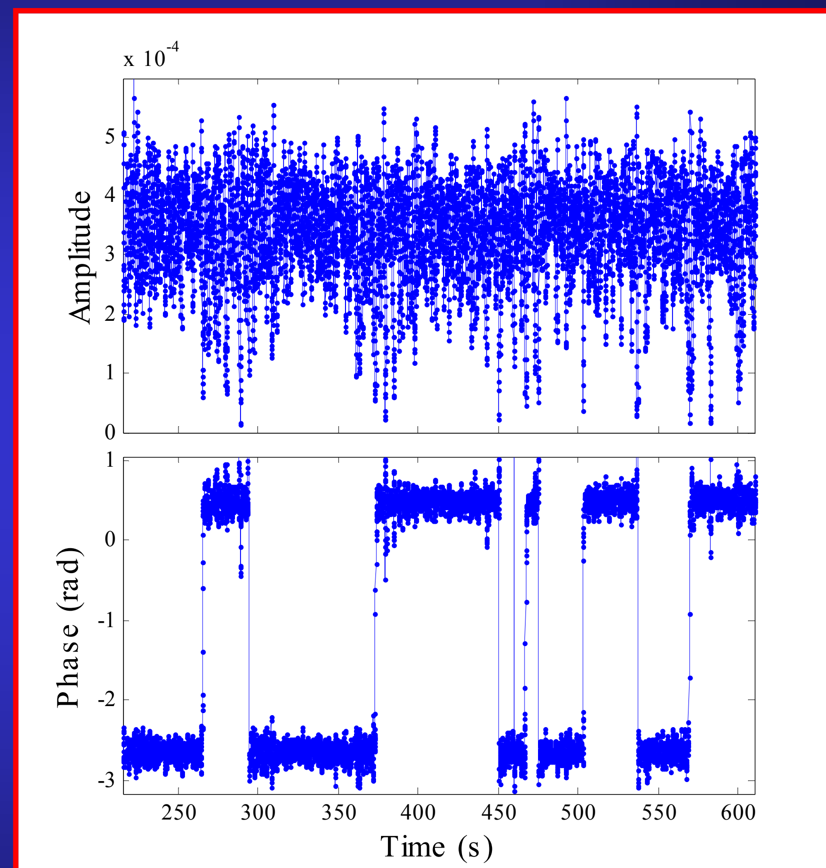


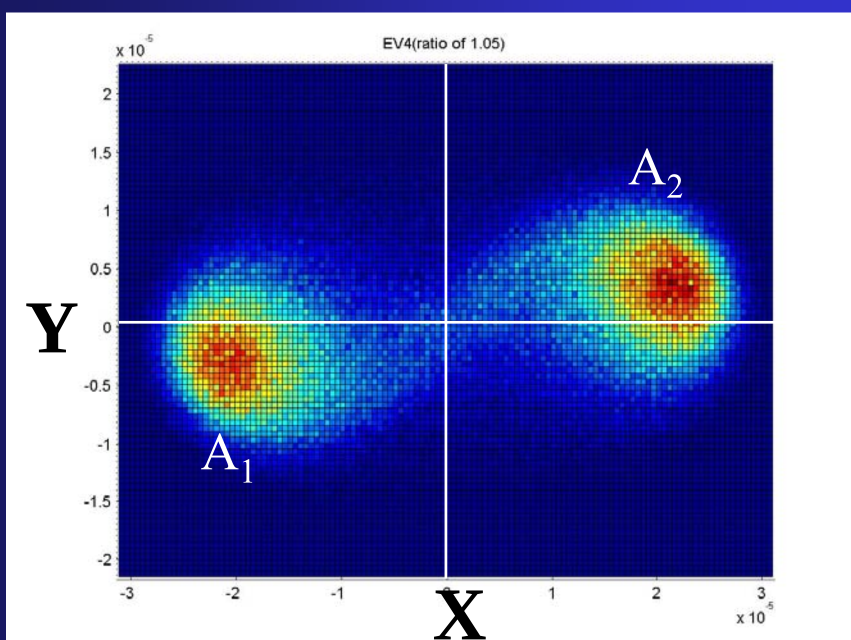


Two stable responses at ω

FLuctuation induced phase slips in a parametrically driven oscillator

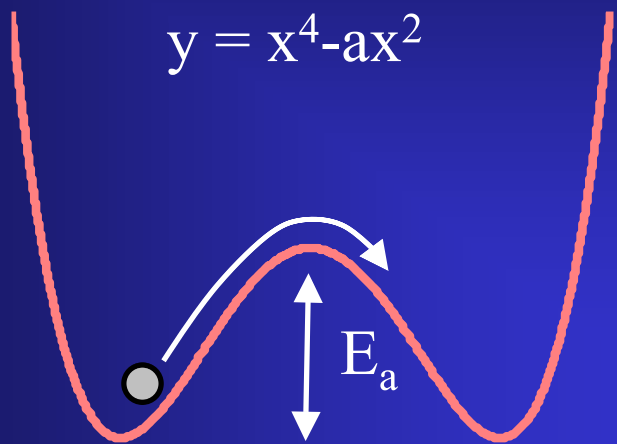






Fluctuation induced switching

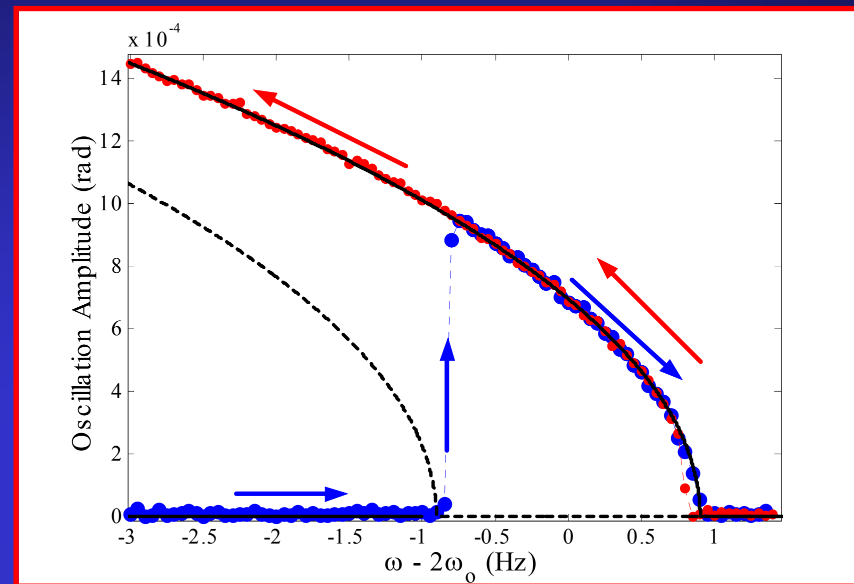
System in thermal equilibrium



Kramers $\Gamma \propto \exp\left(-E_a/T\right)$

$E_a \propto a^2$

Nonequilibrium system



$\Gamma \propto \exp\left(-R/I_n\right)$

$I_n =$ noise intensity

$R \propto \Delta\upsilon^{\alpha}$

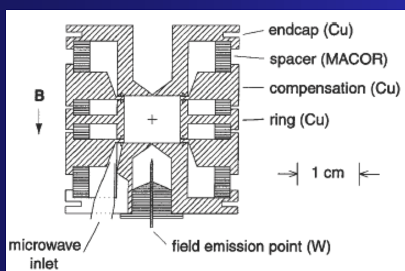
- Dykman & Krivoglaz ’79
- Lehmann, Reimann & Hanggi ’00
- Maier & Stein ’01
- Dykman, Schwartz & Shapiro ’05

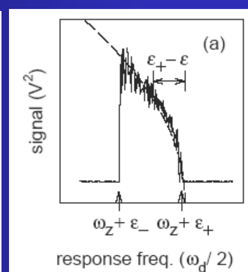
Noise activated escape in periodically driven systems

$$\Gamma \propto \exp\left(-R/I_n\right)$$

Penning traps

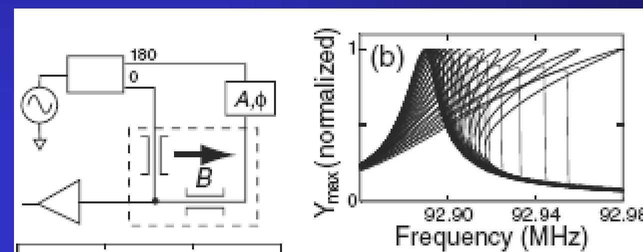
- Lapidus, Enzer and Gabrielse '99





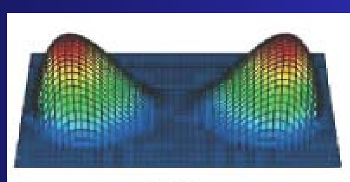
nanomechanical beams < 100MHz, Q ~ 10,000

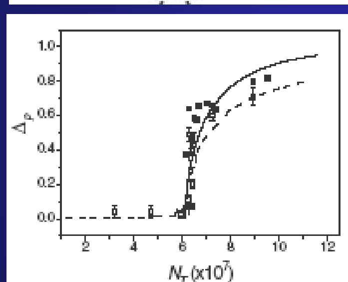
Aldridge and Cleland '05



Magneto-optical traps Many body effects

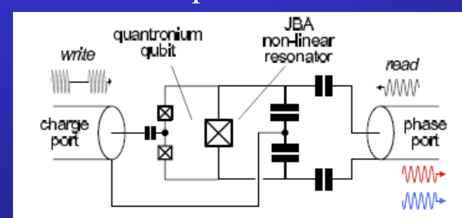
- Kim *et al.* '05

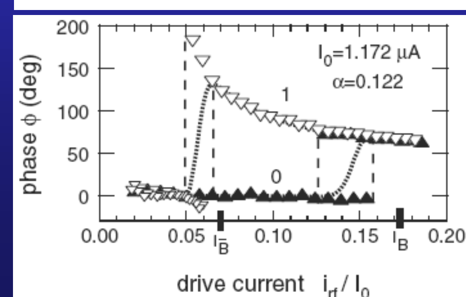


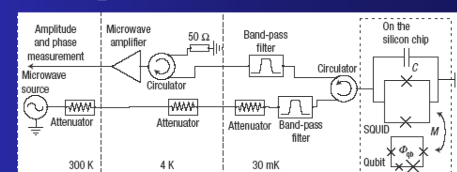


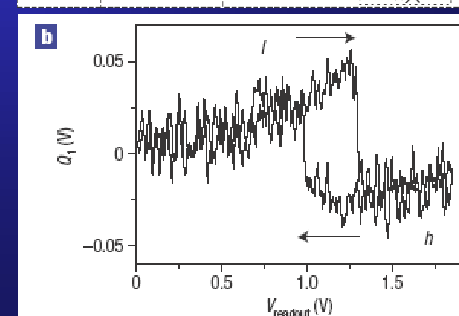
rf driven Josephson junctions > GHz, Q ~ 100

- Siddiqi *et al.* '04
- Lupascu *et al.* '07



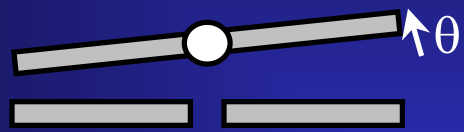






Transformation to rotating frame: 2 dynamical variables

$$\ddot{\theta} + 2\gamma\dot{\theta} + [\omega_0^2 + \frac{k_e}{I}\cos(\omega t)]\theta = -\alpha\theta^2 - \beta\theta^3 + N(t)$$



$$\theta = aX\cos\left(\frac{\omega t}{2}\right) - aY\sin\left(\frac{\omega t}{2}\right)$$

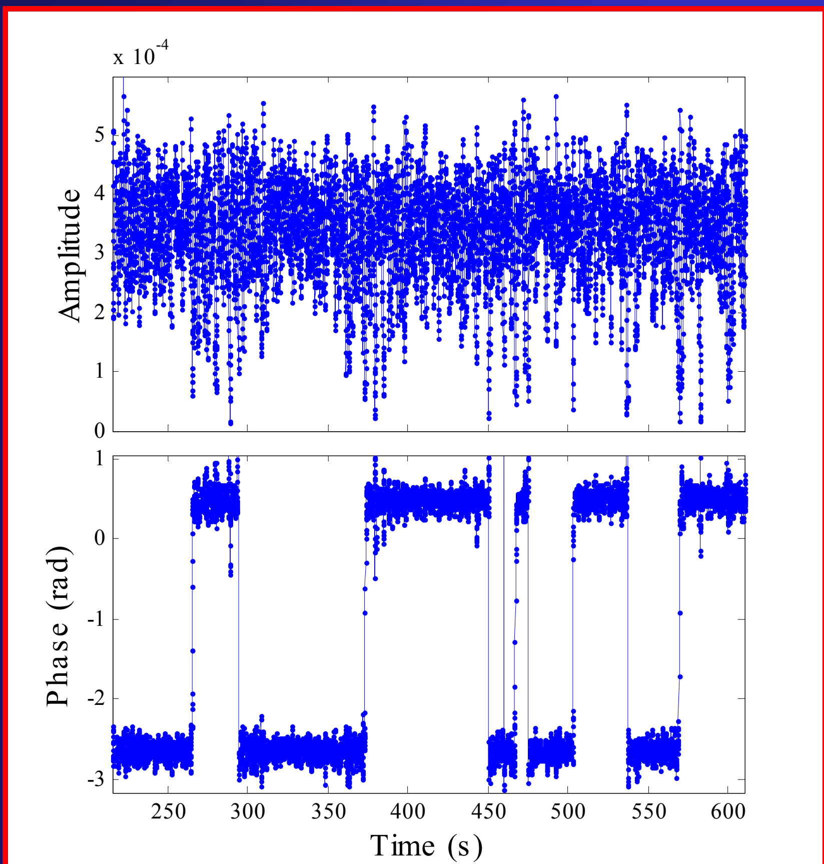
$$\frac{d\theta}{dt} = bX\sin\left(\frac{\omega t}{2}\right) + bY\cos\left(\frac{\omega t}{2}\right)$$

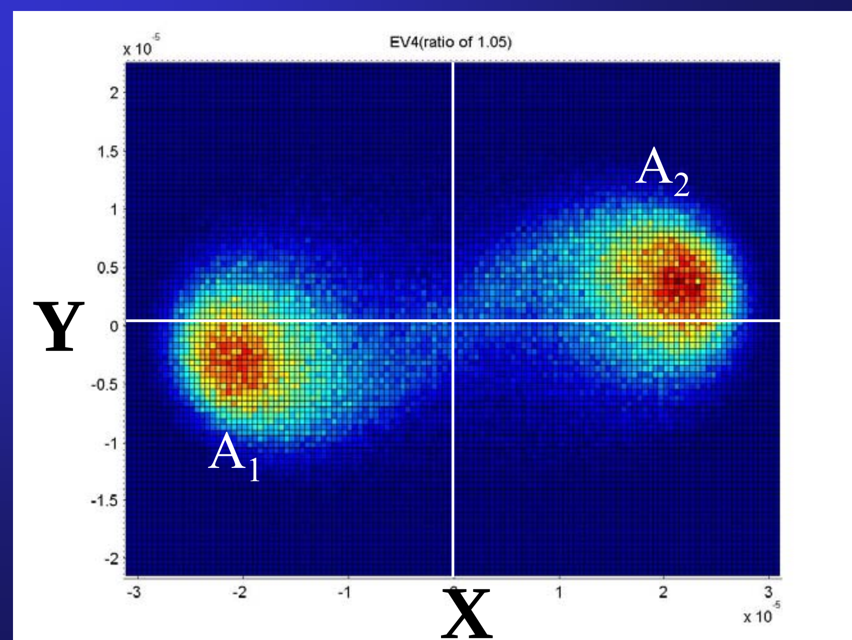
$$\dot{X} = -X + \frac{dg}{dY} + \xi_1(t)$$

$$\dot{Y} = -Y + \frac{dg}{dX} + \xi_2(t)$$

$$g(X,Y) = \tfrac{1}{2}(X^2+Y^2)[\Omega + (X^2+Y^2)] + \tfrac{1}{2}\zeta(Y^2 - X^2)$$

Dykman, Maloney, Smelyanskiy, Silverstein ’98



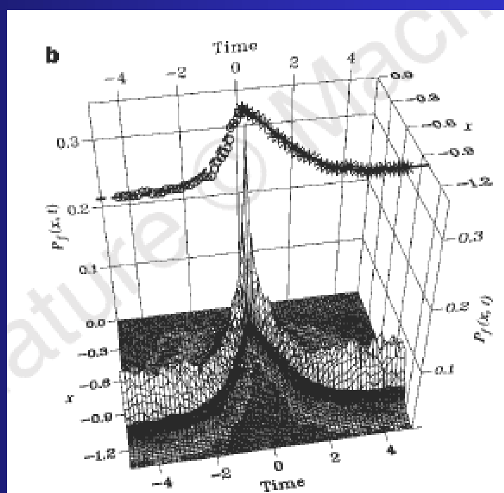


Most probable switching paths: Theory

- Linear systems: Onsager and S. Machlup ’1953
- General Markov system: Wentzel and Freidlin ’1972
- Resonantly driven Duffing oscillator (no detailed balance): Dykman and Krivoglaz ’1979
- Parametrically driven oscillator (no detailed balance): Dykman, Maloney, Smelyanskiy and Silverstein ’1998

Fluctuation Paths: experiment

Luchinksy and McCintock, Nature 1997

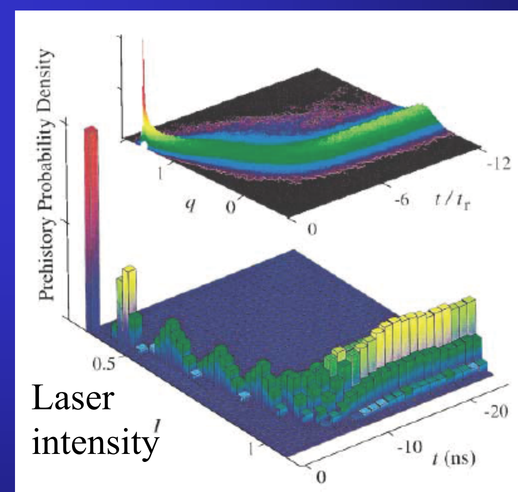


Electronic circuit simulation

Distinguishes fluctuation and relaxation motion from attractor to a specific point

Paths within the basin of an attractor

Hales, Zhukov, Roy, Dykman, PRL 2000



Dropout events in laser with feedback

1 dynamic variable: laser intensity

Paths of fluctuation induced switching

H. B. Chan, M. I. Dykman, C. Stambaugh, PRL 100, 130602 (2008).

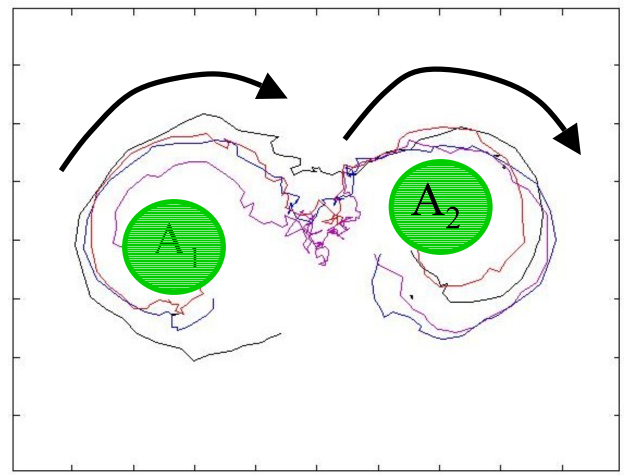
Switching Path Distribution

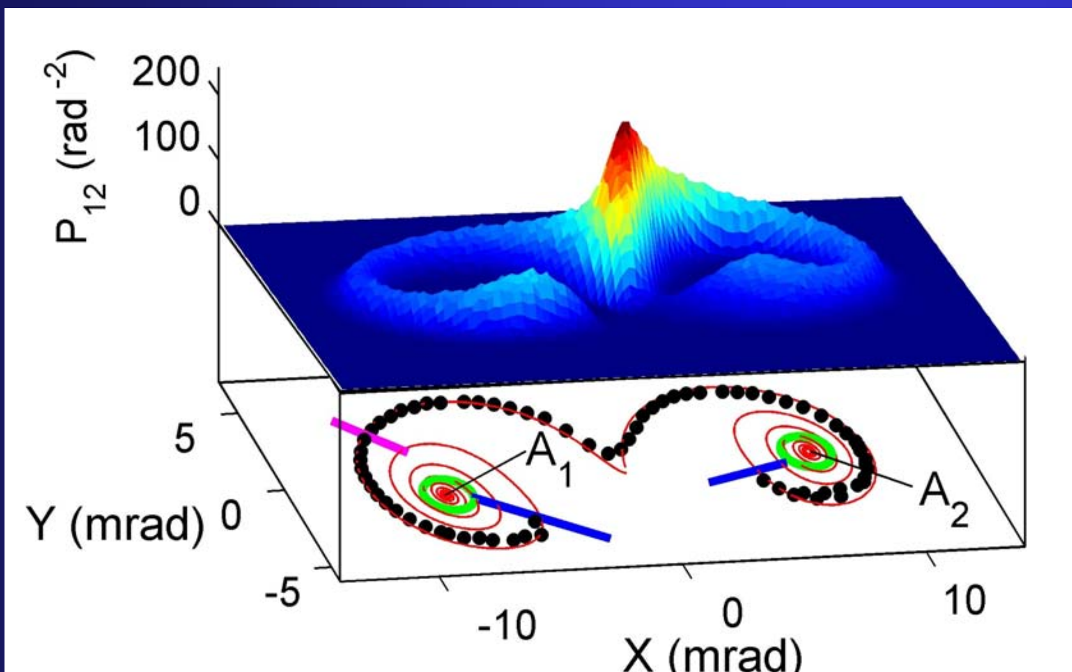
$$p_{12}(\mathbf{q},t) = \int_{\Omega_2} d\mathbf{q}_f \, \rho(\mathbf{q}_f, t_f; \mathbf{q}, t \,|\, \mathbf{q}_i, t_i)$$

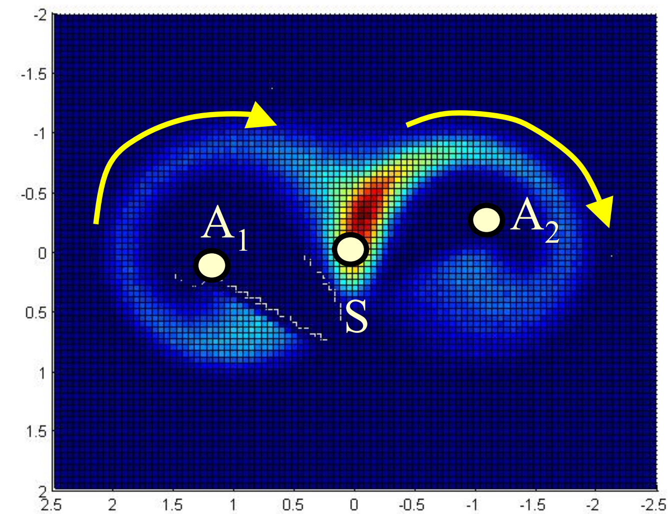
probability density at q, t, q_f, t_f, given that it was at q_i at t_i.

q_i within l_d of A_1, q_f within l_d of A_2

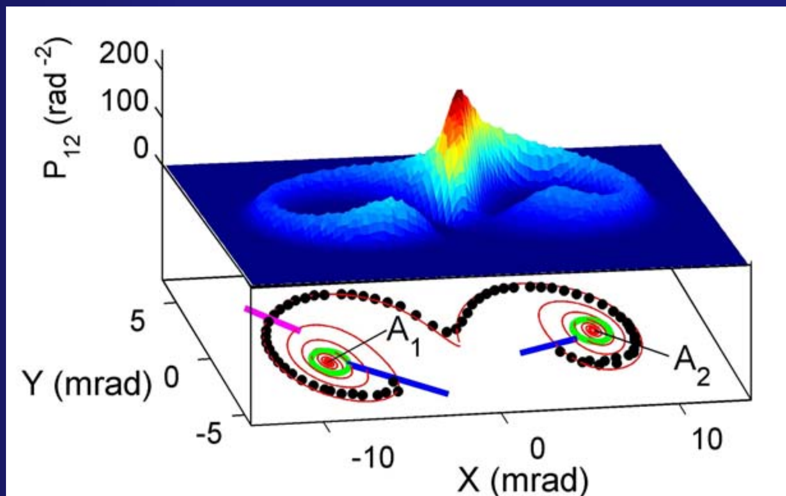
System start close to A_1 and later arrive close to A_2







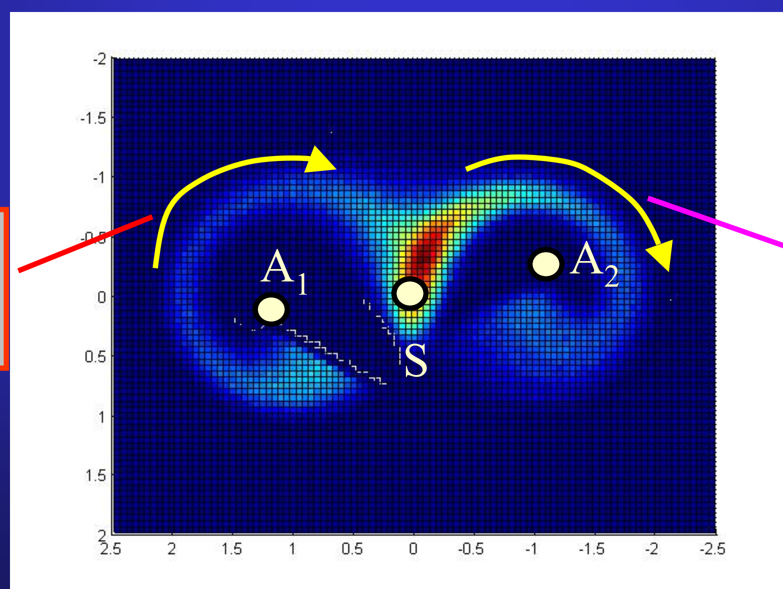
Generic features of switching path distribution



- Switching trajectories form narrow tubes centered on Most Probable Switching Path
- Diffusion motion around saddle:

  strong peak in SPD
- Gaussian cross section
- Constant probability current

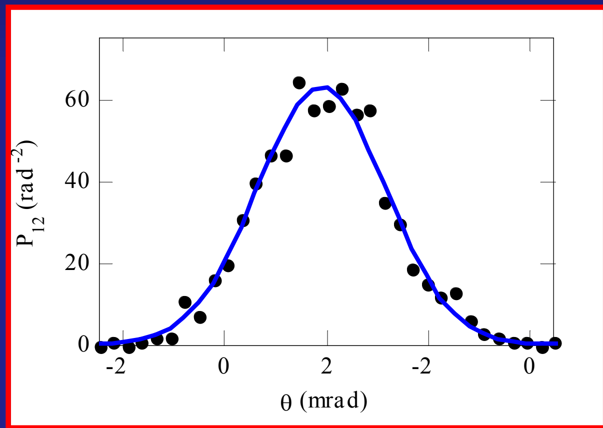
Uphill: the appropriate fluctuation brings the system from attractor A_1 to saddle S

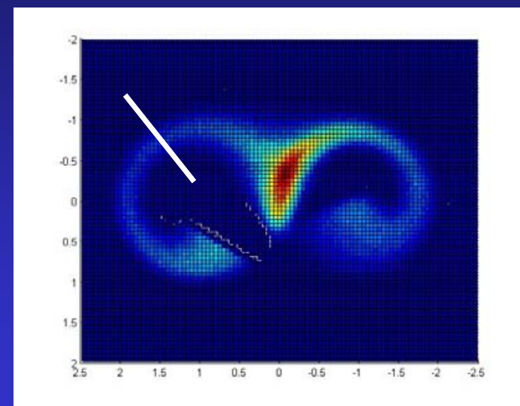


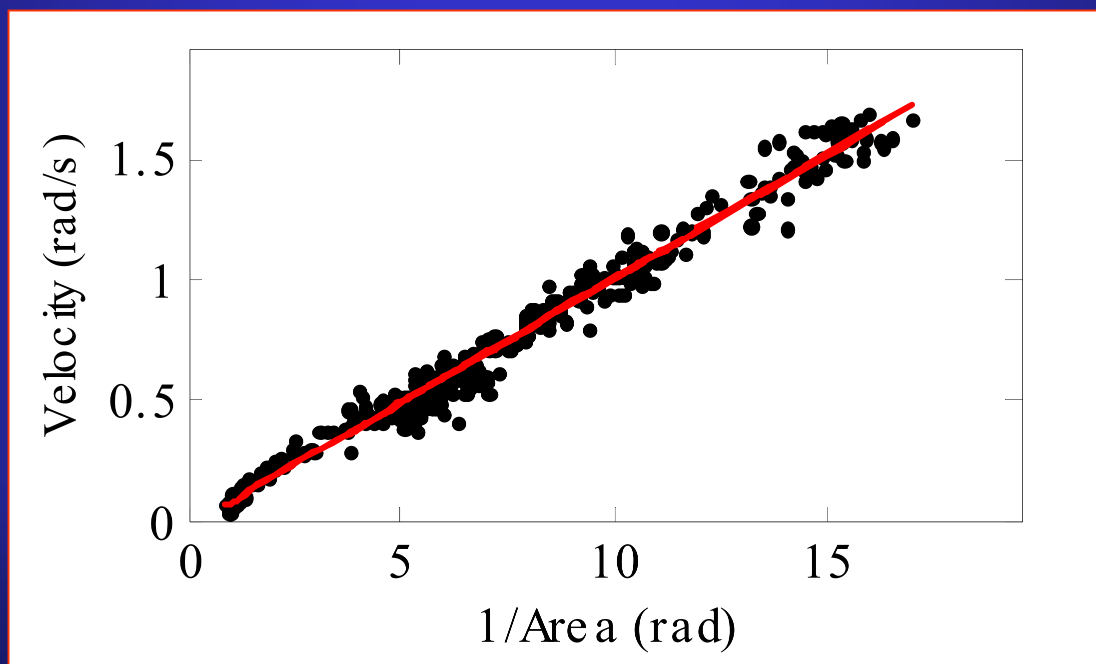
downhill: deterministic, fluctuation-free path from saddle S to attractor A_2

Constant probability current along most probable switching path

Gaussian cross section







$A_{\text{cross section}}$ x velocity = switching rate

Switching Paths for systems with time reversal symmetry

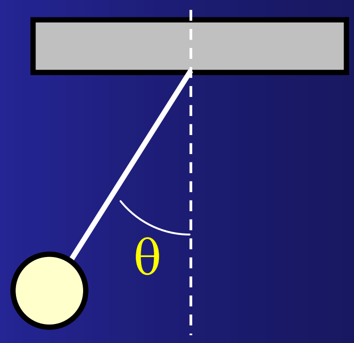
Time reversal symmetry

most probable "uphill" fluctuational path = time reversed fluctuation-free path

Example of time reversal:
linear oscillator

$$\frac{d^2\theta}{dt^2} + 2\Gamma\frac{d\theta}{dt} + \omega_0^2\theta = 0$$

t → -t
$$\frac{d^2\theta}{dt^2} + 2\Gamma\frac{d\theta}{-dt} + \omega_0^2\theta = 0$$



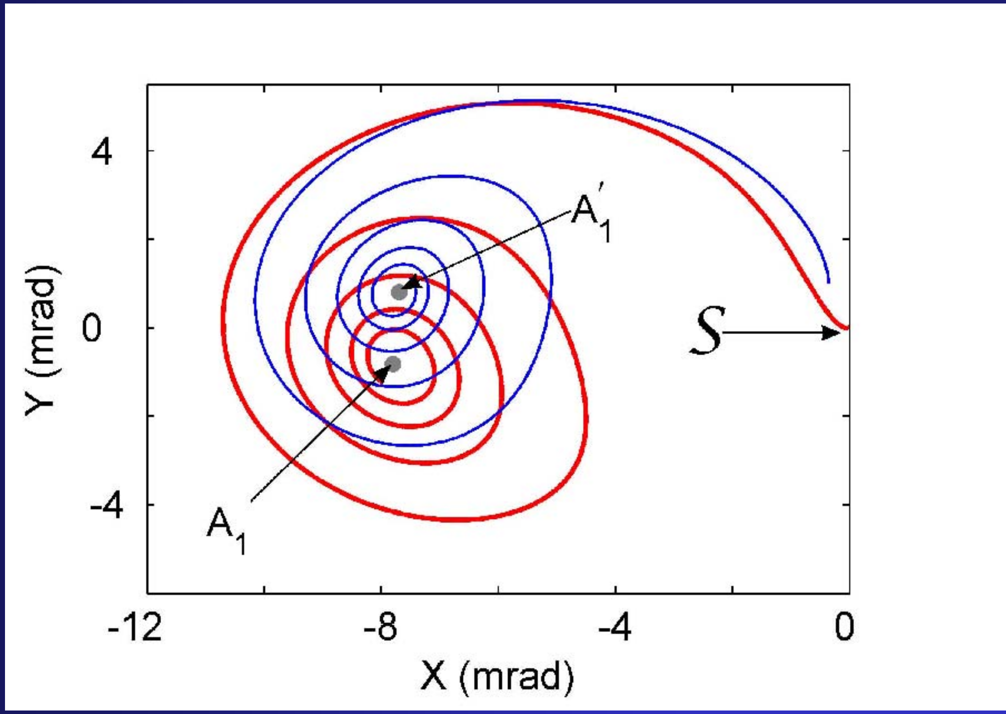
➔ Γ → -Γ

θ = dθ/dt = 0 (stable) → θ = dθ/dt = 0 (unstable)

In overdamped particle in bistable potential,

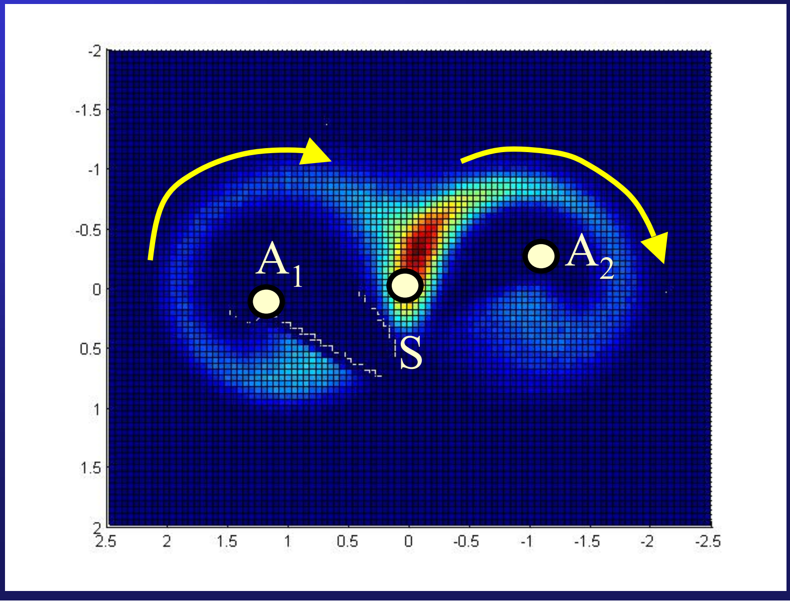
"uphill" fluctuational path
time reversed fluctuation-free path
} identical

Nonequilibrium oscillator: lack of time reversal symmetry



Red: "uphill" fluctuational path

Blue: time reversed fluctuational-free path



Differences from overdamped particle in bistable potential:

- " Uphill " fluctuational path ≠ time reversed fluctuation-free path
- Position of attractor shifted upon time reversal

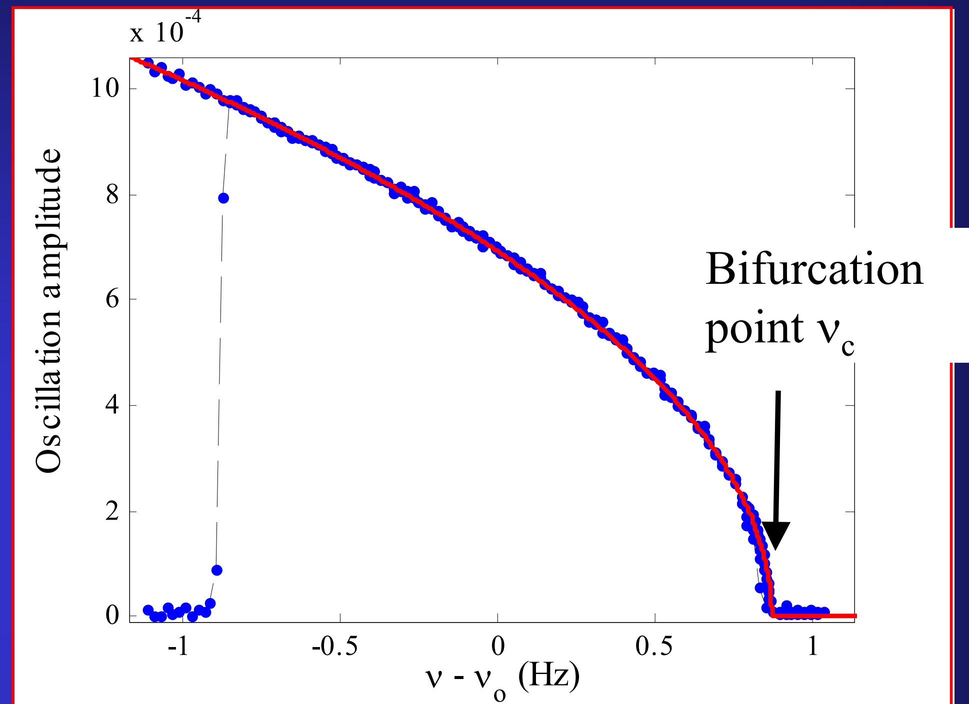
Attractor is an oscillating state, $d\theta/dt = 0$

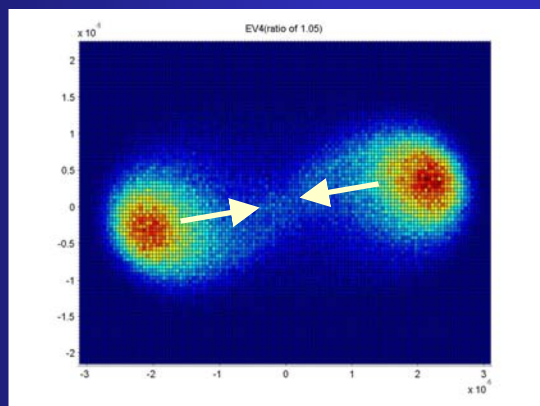
Critical exponent in activation energy for parametric resonance

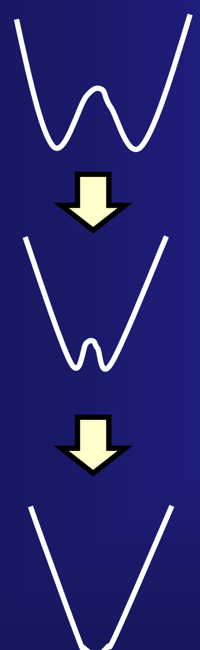
$$\Gamma \propto \exp\left(-R/I_N\right)$$

$$R \propto (\upsilon - \upsilon_c)^{\alpha}$$

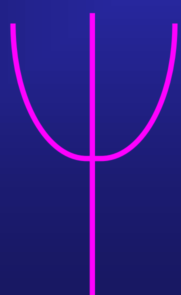
Critical exponent α

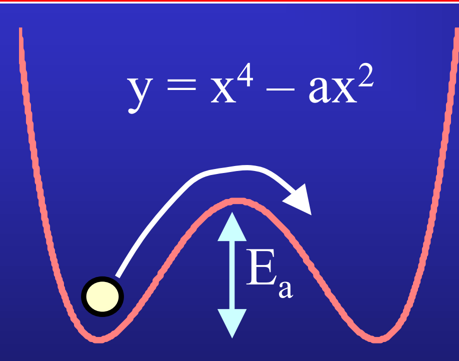






Pitchfork bifurcation





Overdamped motion in 1D

$$E_a \propto a^2$$

close to ν_c , universal scaling predicted with critical exponent α = 2 (Dykman *et al*. ’98)

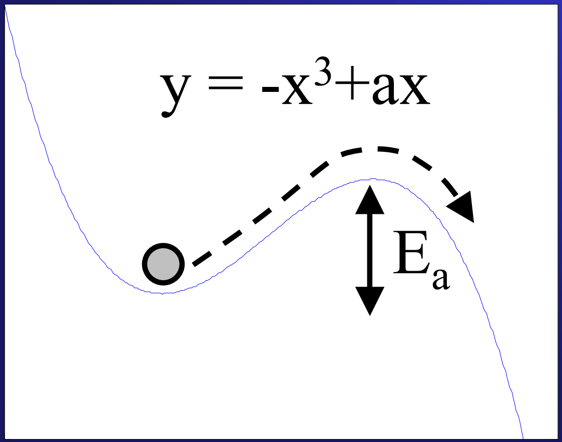
Activation barrier scaling for saddle node bifurcation

Critical exponent α

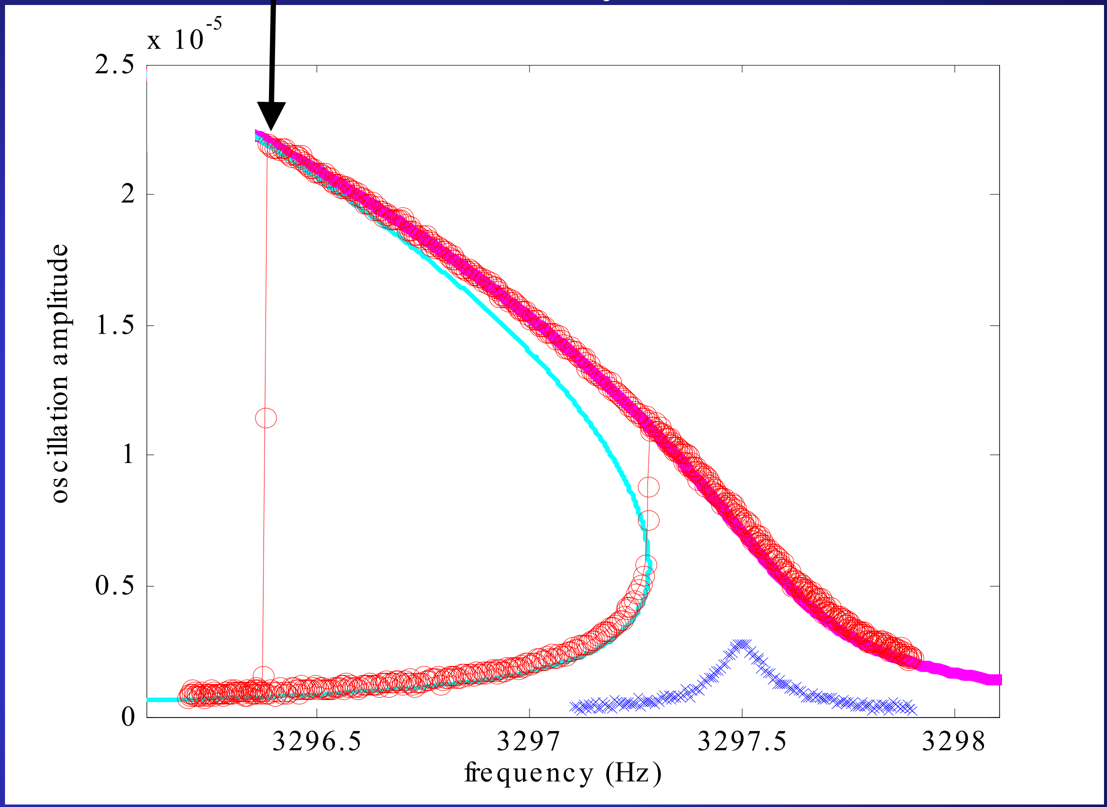
$$\Gamma \propto \exp\left(-c(\upsilon-\upsilon_c)^{\alpha} \Big/ I_N\right)$$

Near bifucation points:

Universal scaling with critical exponent α = 3/2 (Dykman & Krivoglaz '79)



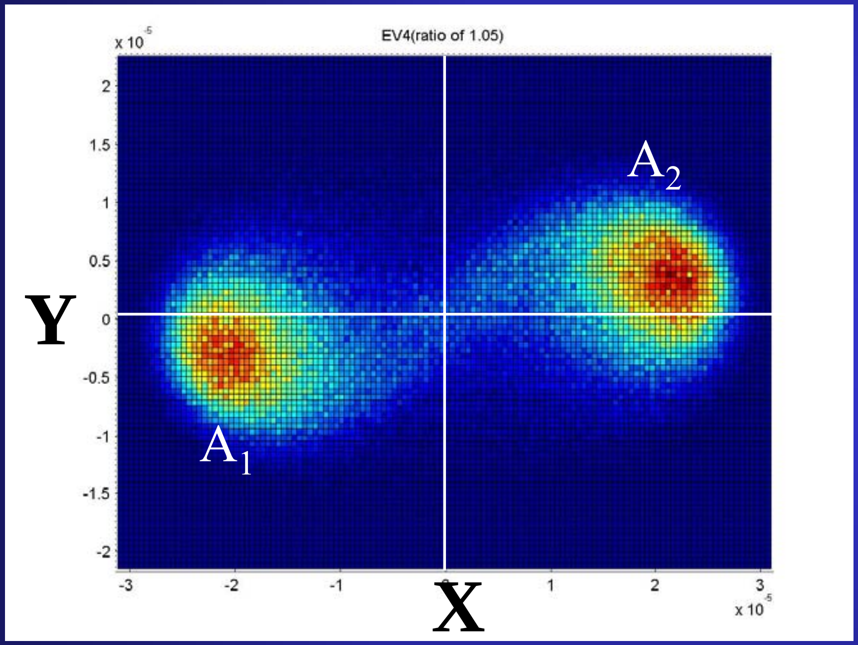
Bifurcation point ν_c

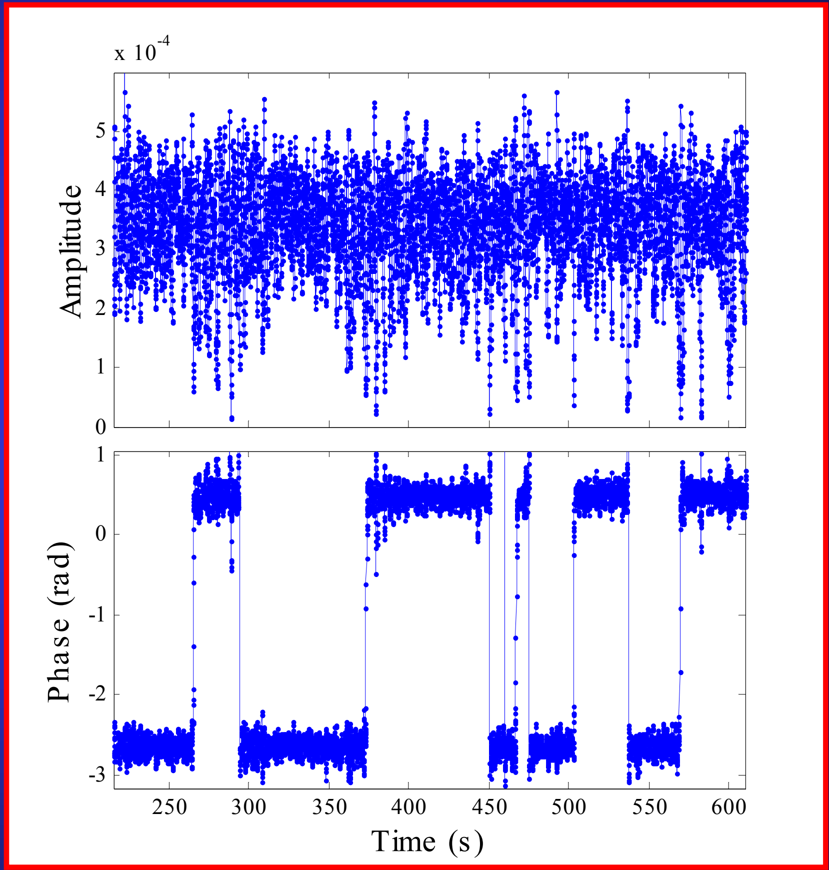


3/2 scaling expected to occur for all spinodal bifurcations:

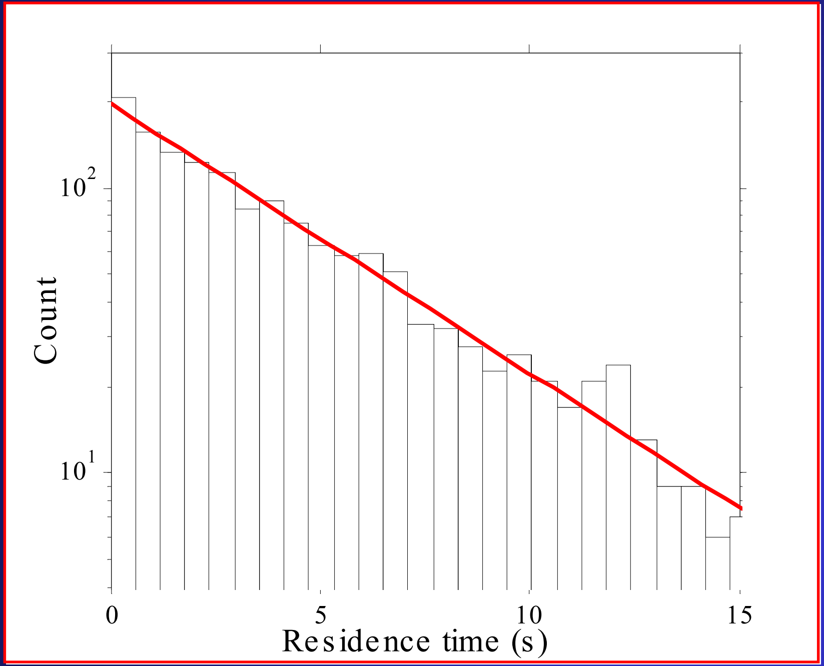
- RF driven Josephson junctions
- Nanomagnets driven by polarized currents
- Double barrier resonant tunneling structures.

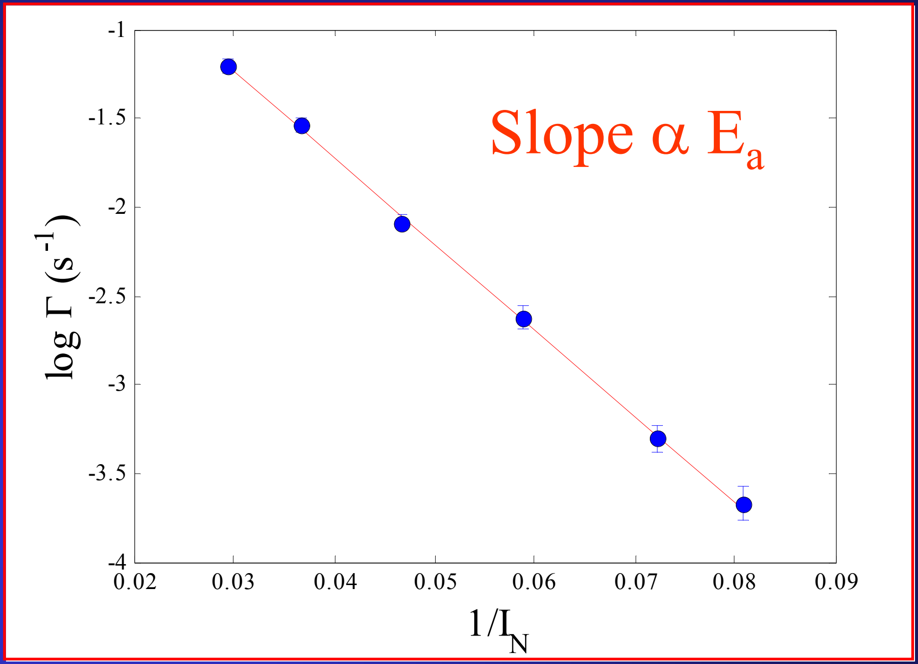
Fluctuation induced switching in parametric oscillator





Activated phase slip





Poisson's statistics:

exponentially decay histogram

➔ Randomness in switching

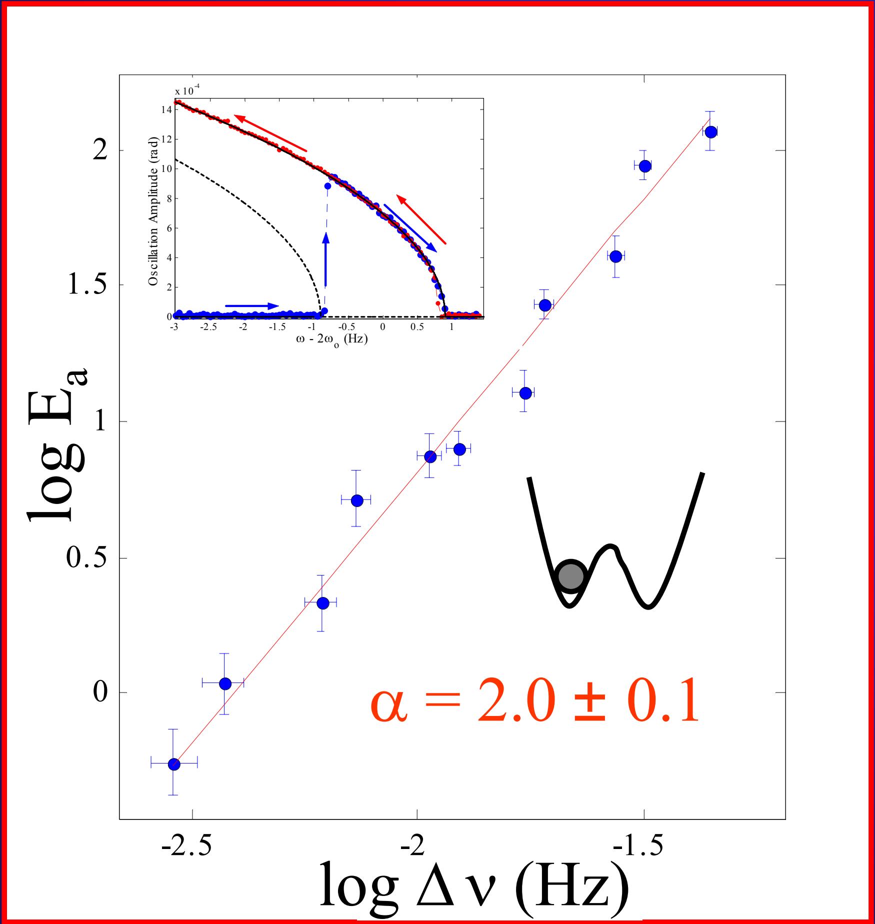
Activated behavior

$$\Gamma \propto \exp\left(-E_a / I_N \right)$$

$$\log(\Gamma) = -E_a / I_N + C$$

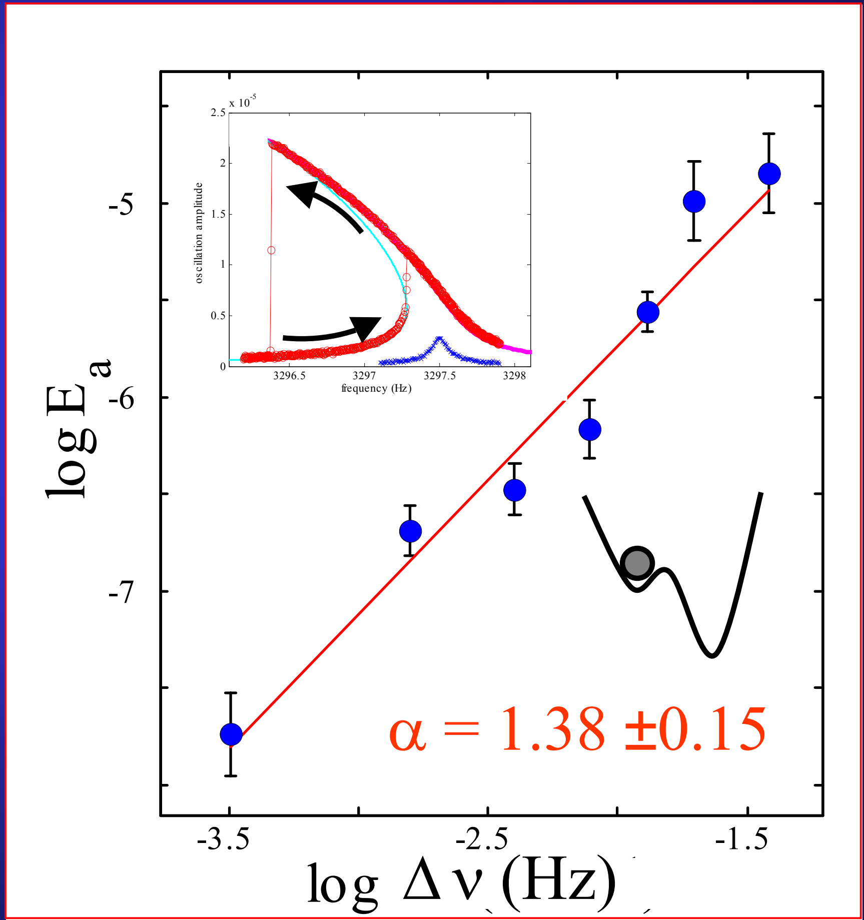
Two different scaling relations for activated switching in micromechanical torsional oscillator

Pitchfork bifurcation:
parametrically driven oscillator



Chan & Stambaugh, PRL 99, 060601 (2007).

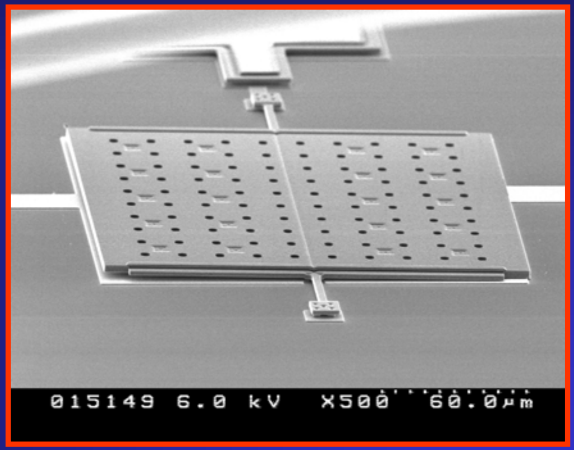
Saddle node bifurcation:
resonantly driven Duffings oscillator

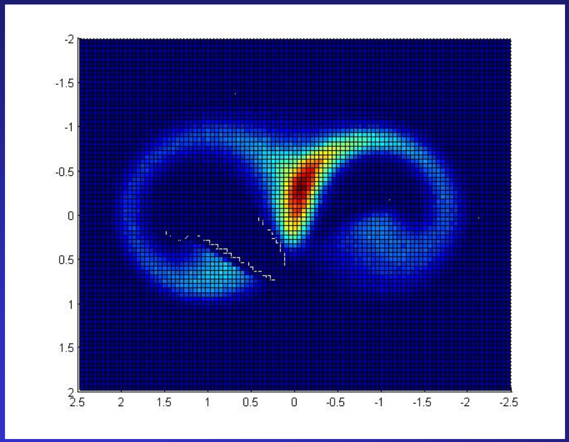


Stambaugh & Chan, PRB 73, 172302 (2006).

Consistent with universal scaling behavior (Dykman et al.)

Summary





- activated switching between coexisting states in a micromechanical torsional oscillator driven into parametric resonance.
- scaling of the activation barrier close to bifurcation points.
- measurement of the most probable switching path in 2 dimensions

  Switching Probability Distribution

  constant probability current along the path.

  experimental evidence for lack of time reversal symmetry

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