Unsolved Problems on Noise – Lyon, June 2008

# Stick-slip motion of a sheared granular medium


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- Complex Systems Institute, CNR
- University of Rome
- University of Modena


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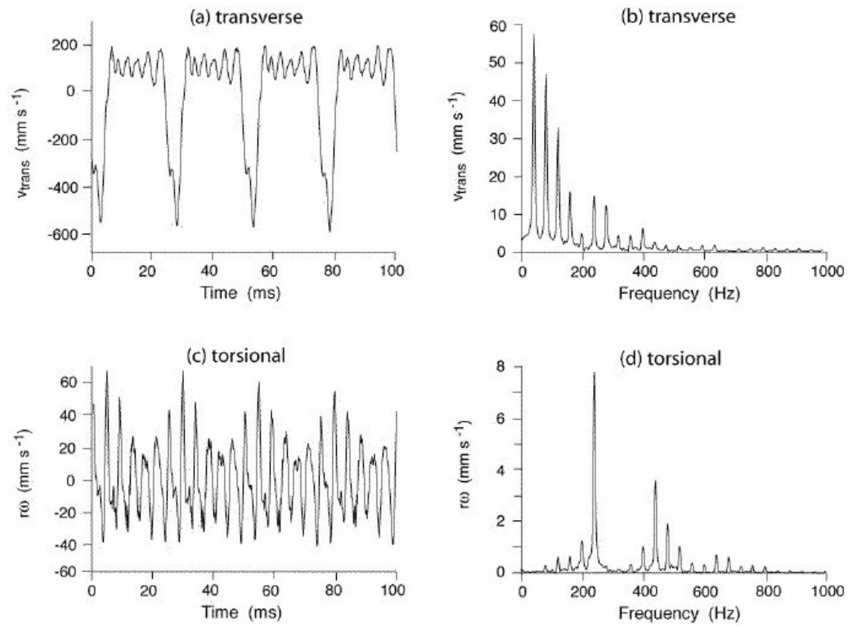
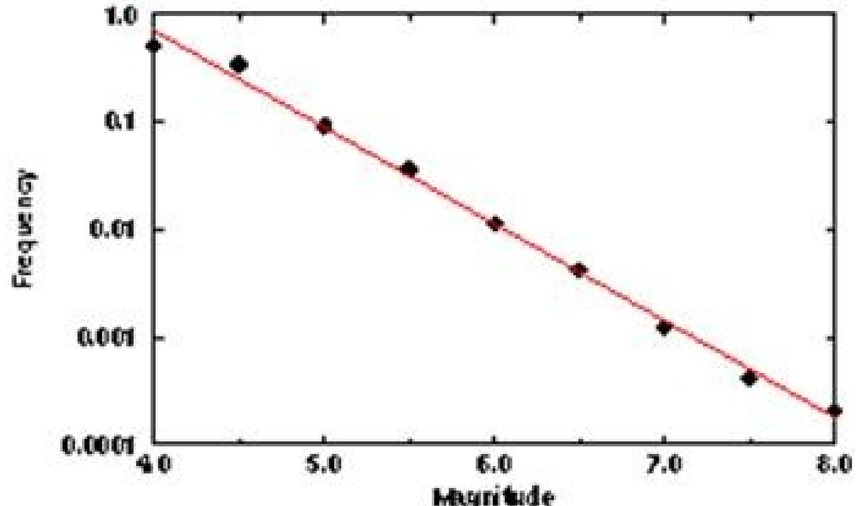
# Periodic stick-slip: elasticity and friction

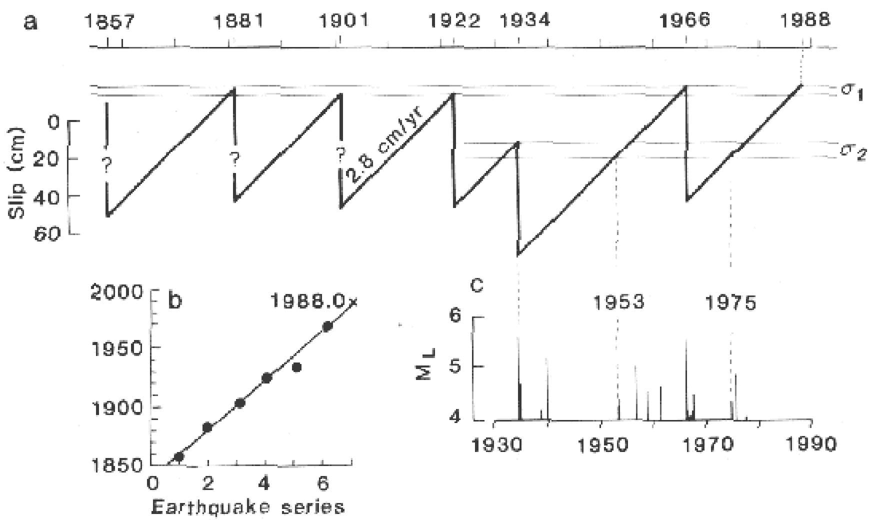



(http://www.unsw.edu.au/)

# *Irregular stick slip in tectonics - Gutenberg & Richter*

Earthquakes in the world: 1995

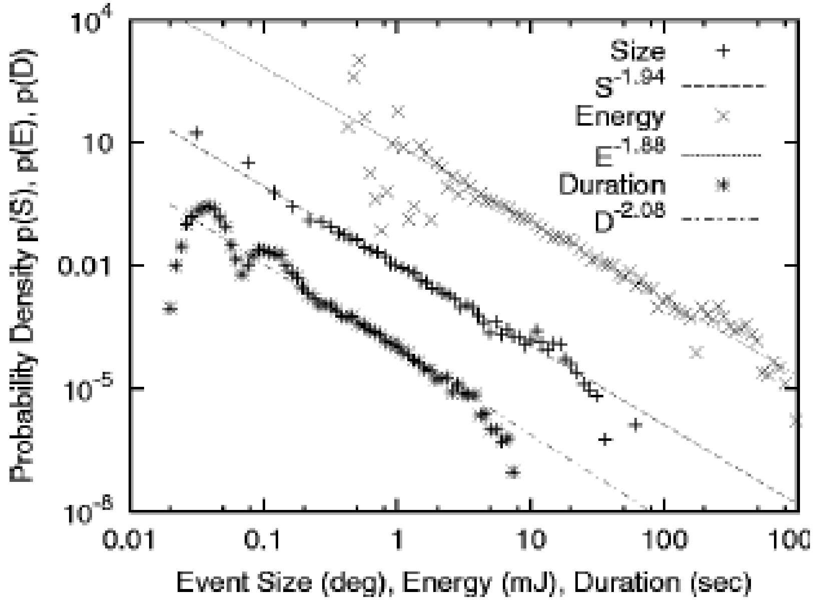


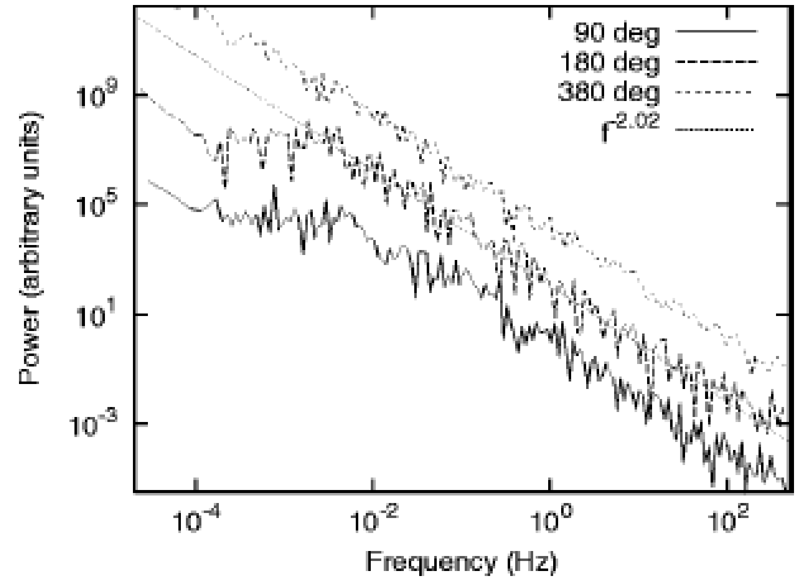




***log N(>M) = a - bM***

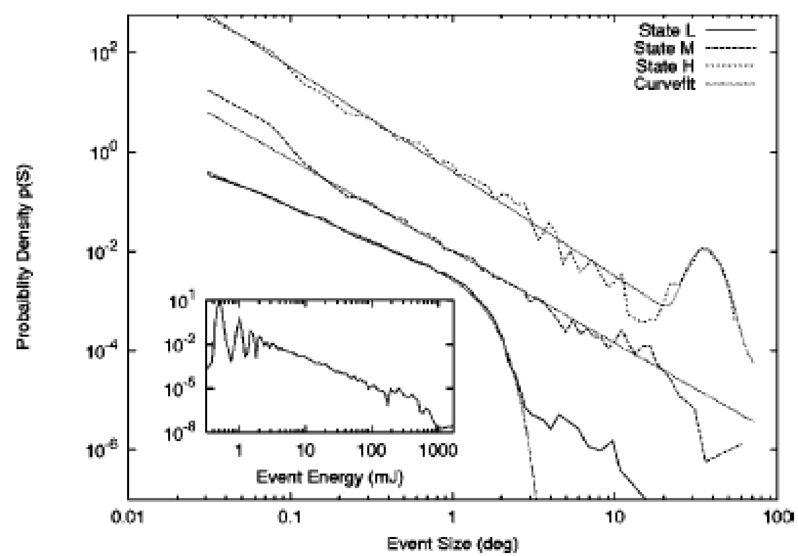
M = log A

# *Granular matter under shear stress*

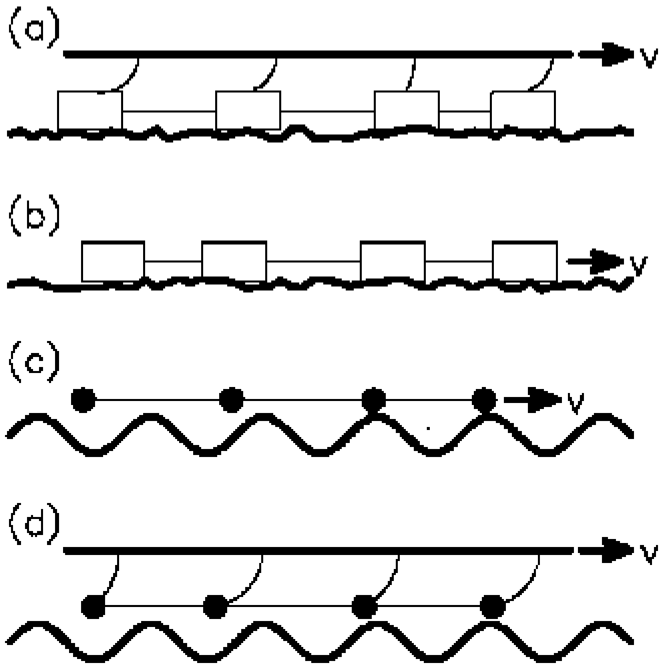



Dalton & Corcoran
- PRE63, 061312
- PRE65, 031310



# *Deterministic Models*



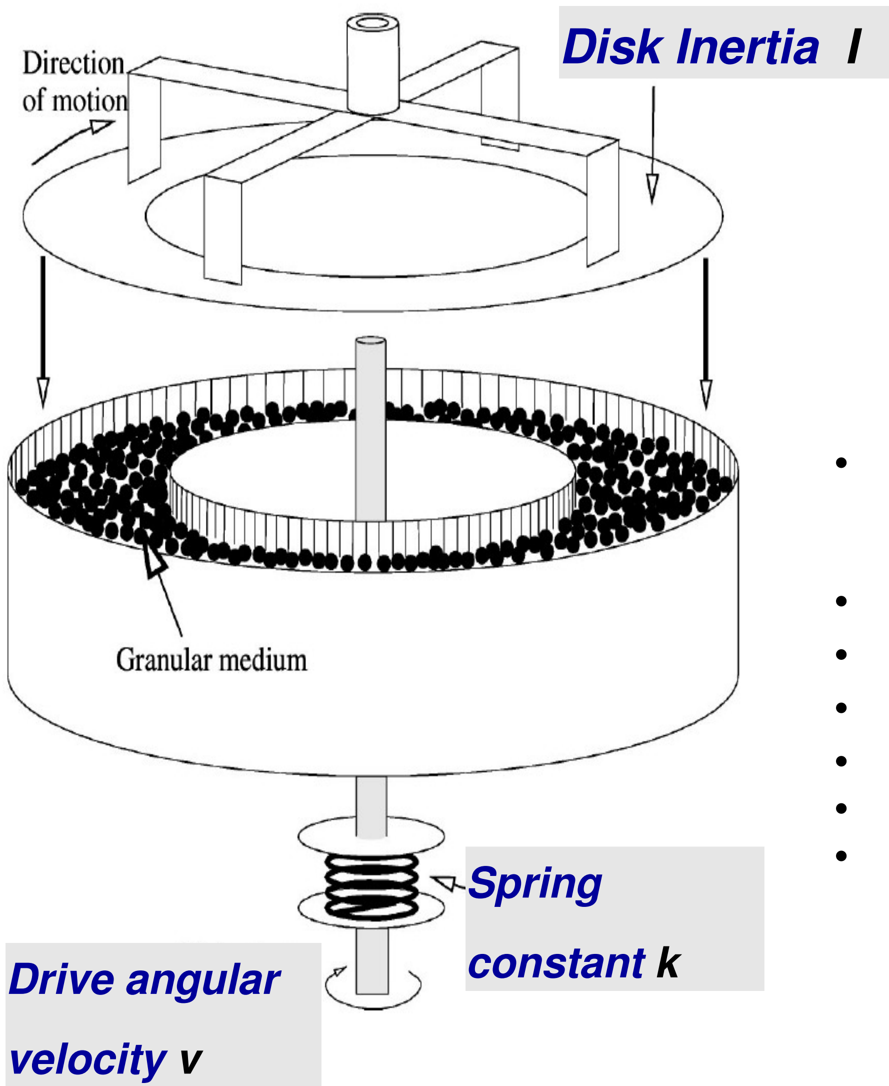

Experiment and nuerical,
Burridge, R. and Knopoff, L.
*Bull. Seis. Soc. Amer.* **57**, 341, 1967

T. Kontorova and Y. I Frenkel
Zh. Eksp. Teor. Fiz. **8** (1934)

$$m x_i''(t) = k_c[x_{i-1}(t) - 2x_i(t) + x_{i+1}(t)] - k_p[x_i(t) - ia - tv] - f\,F(f x_i'(t))$$

Large number of non-linear deterministic equations

# *Experiment: Grains sheared in a circular channel*




- Annular cell, sheared by an overhead plate driven by a motor via a torsion spring
- 2*mm* glass beads
- Spring: 0.1 < ***k*** < 1 *Nm/rad*
- Inertia: 0.015 < ***I*** < 0.02 *kg m*$^2$
- Motor & Gear: $10^{-5} < \omega <$ 50 rad/s
- Free dilation of the bed
- High angular resolution:
  35 μ *rad* at 10*kHz*

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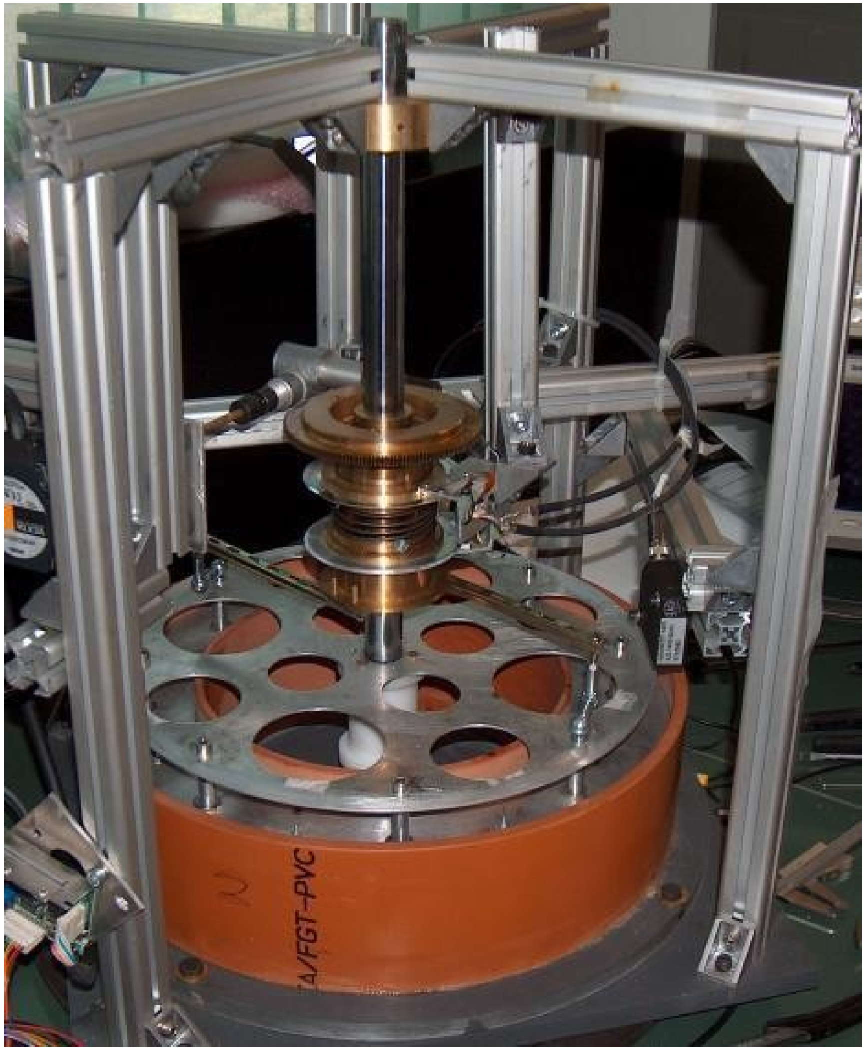

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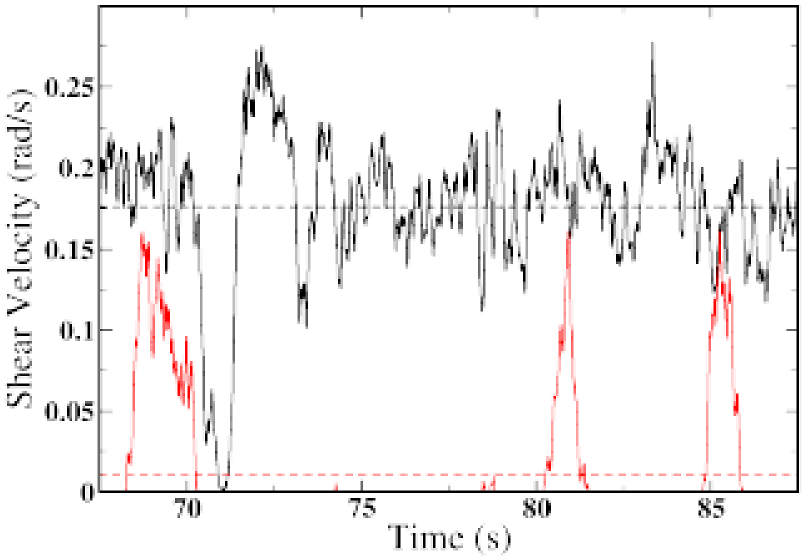
# *Plate velocity time series*

*Record the angular position of the top plate with respect to the motor in time, from which compute velocity, acceleration and torque*




At slow shear rate, solid-like behaviour, with intermittent and erratic motion

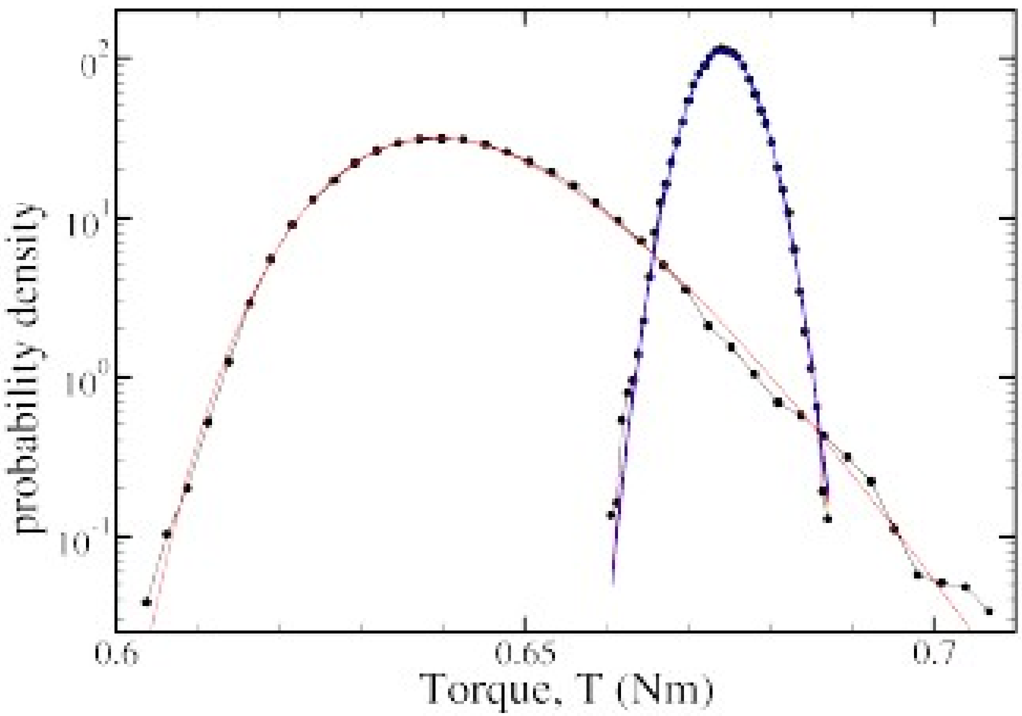
- stick-slip phase (crackling noise)

At increasing shear rate, solid-liquid transition to sliding phase at roughly $10^{-1}$ rad/s

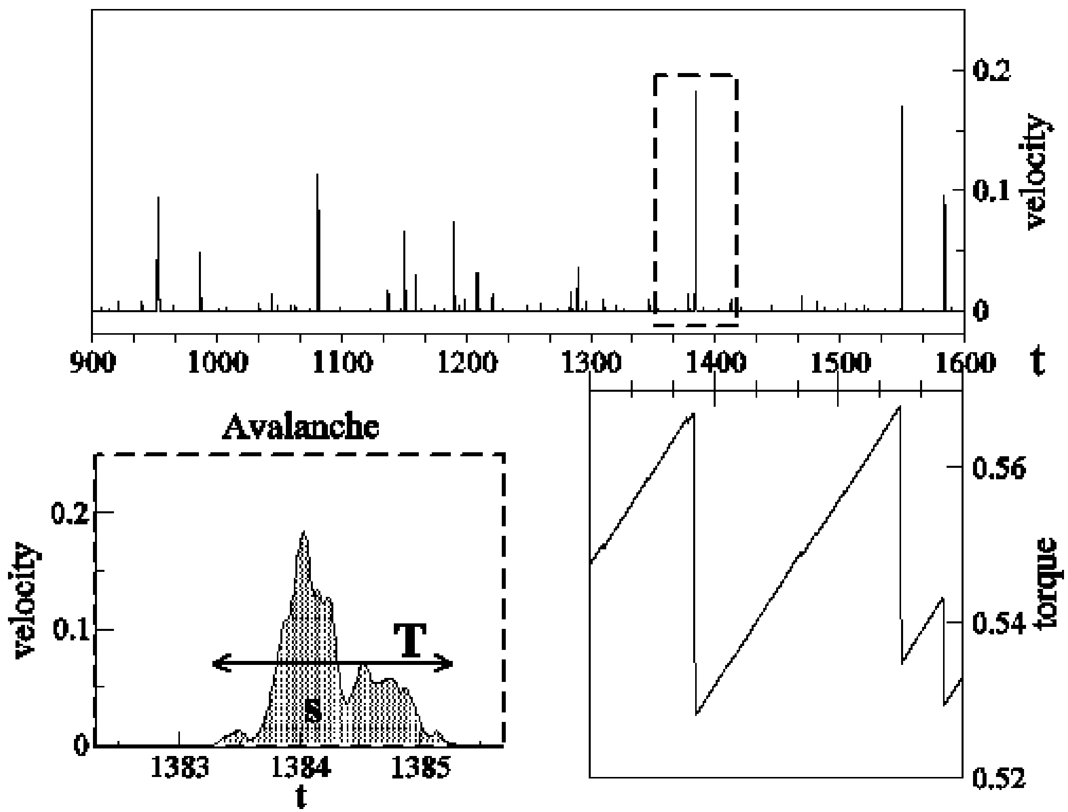
Solid phase: irregular slip events
Liquid phase: continuous sliding
- reflected in torque distribution:

Liquid phase: Gaussian fluctuations
- collision dominated

Solid phase: asymmetric torque
- correlated force chains



# *Objective: Description of the stochastic motion*




Slip motion characterization: statistical distribution of slip parameters: size ***s,*** duration ***T***, instantaneous velocity ***v(t)***

## *Stochastic motion equation*

Simple Langevin equation

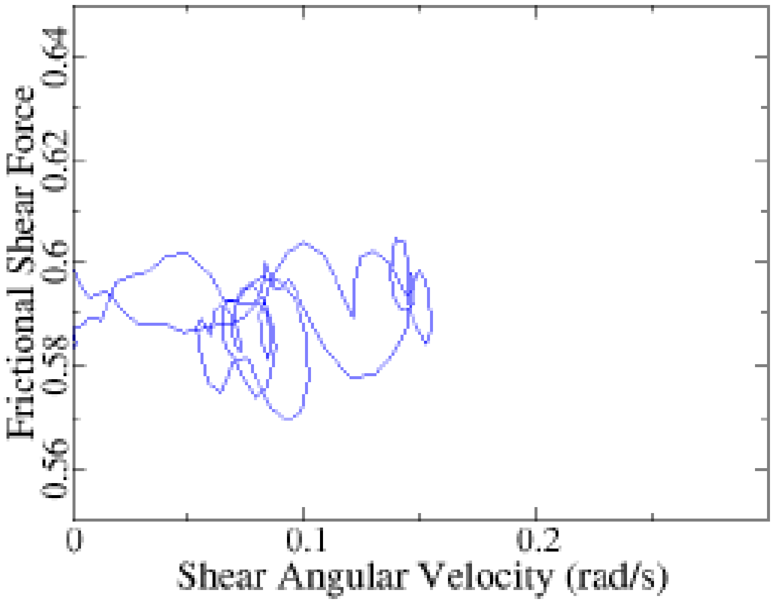
$$I\ddot{\theta} = \kappa(\omega_D t - \theta) + F(t, \theta, \dot{\theta}, \ldots)$$

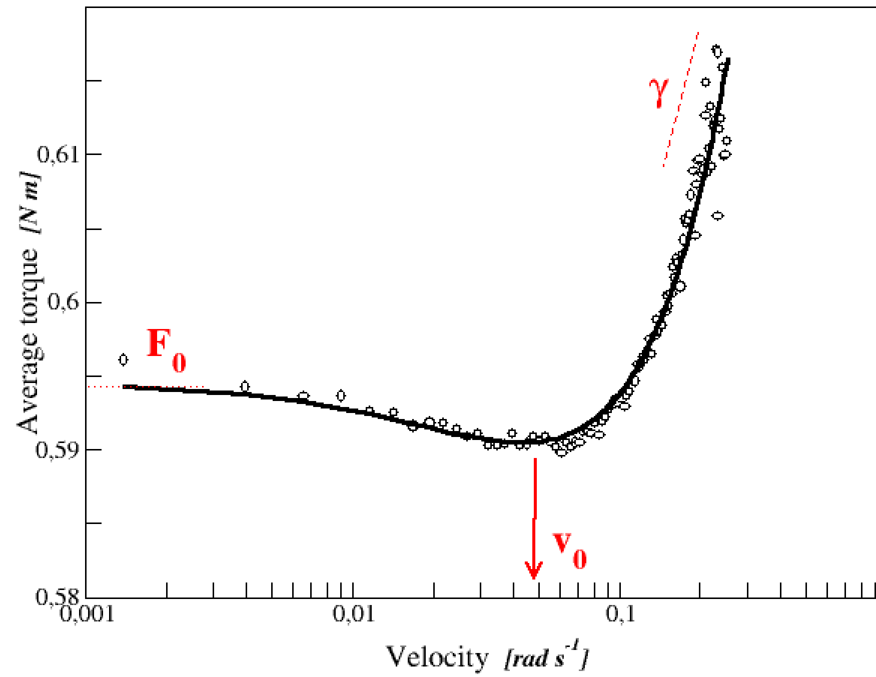
***I*** = inertia of the plate
***k*** = spring constant
***θ*** = angular position
$\boldsymbol{\omega_D}$ = driving (angular) velocity
***F*** = fluctuating reaction torque

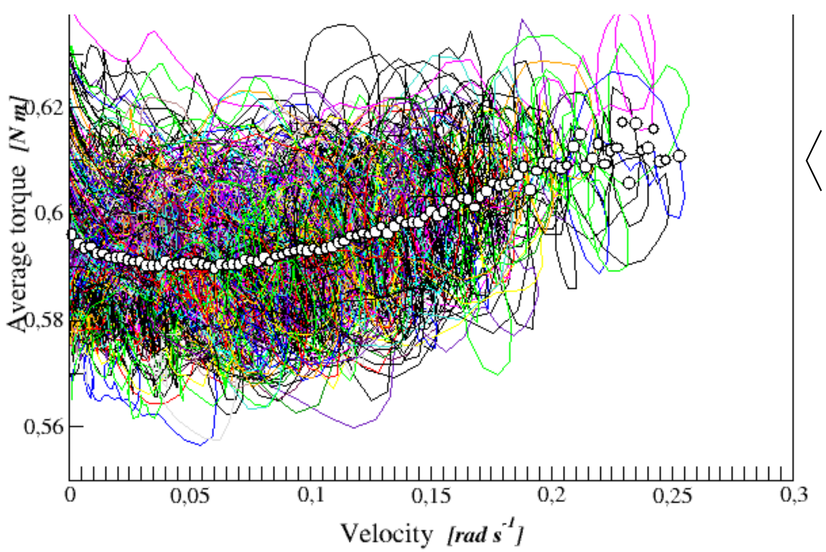
Assume: $$F(\theta, \dot{\theta}) = \langle F(\dot{\theta}) \rangle + F_\mu(\theta)$$

**<*F*>** = Average viscous force
$\boldsymbol{F_\mu}$ = fluctuating component

# *Average viscous force*




$$\langle F(\dot{\theta})\rangle = F_0 + \gamma(\dot{\theta} - 2\,\omega_0 \ln(1 + \dot{\theta}/\omega_0))$$

Assumed to depend only on the velocity:
- independent of spring and driving

- parameters from a fit of experimental data

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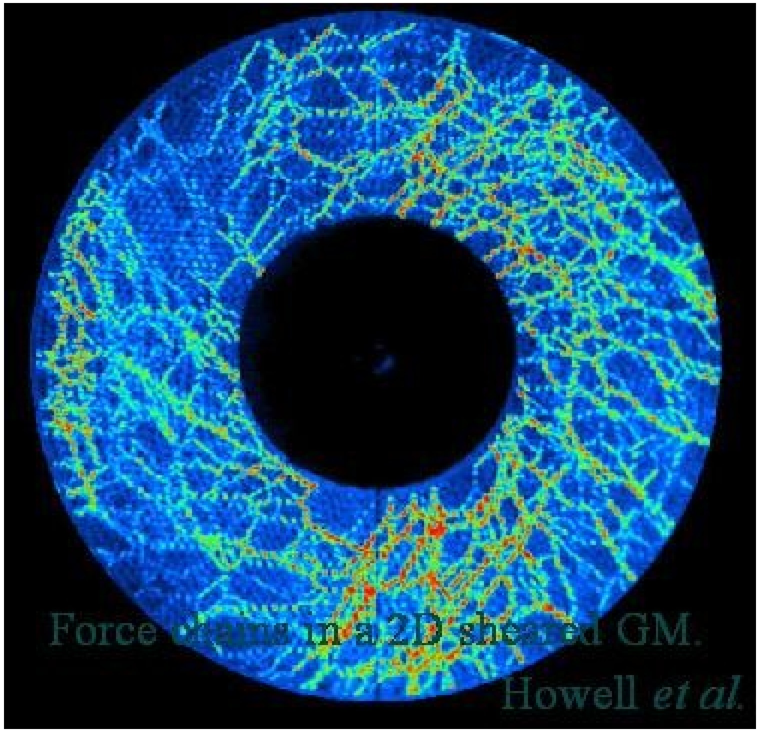
# *Fluctuating force component*

*Stress propagation takes place along a network of localized force chains.*
*The fluctuating force results from correlated changes in the chains during motion.*




Howell at http://www.phy.duke.edu/~bob/

$$F_\mu(\theta + d\theta) = F_\mu(\theta) + \delta F_\mu(\theta)$$

Brownian walk bounded by some potential
- Ornstein-Uhlenbeck process
- Wiener-Khintchine yields power-spectrum

$$\frac{dF_\mu}{d\theta} = \eta(\theta) - aF_\mu$$

$$S(k) = \langle \left| \int F_\mu(\theta) e^{-i\theta k} d\theta \right|^2 \rangle = \frac{2D}{a^2 + k^2}$$

$$\langle \eta(\theta)\eta(\theta') \rangle = D\,\delta(\theta - \theta')$$

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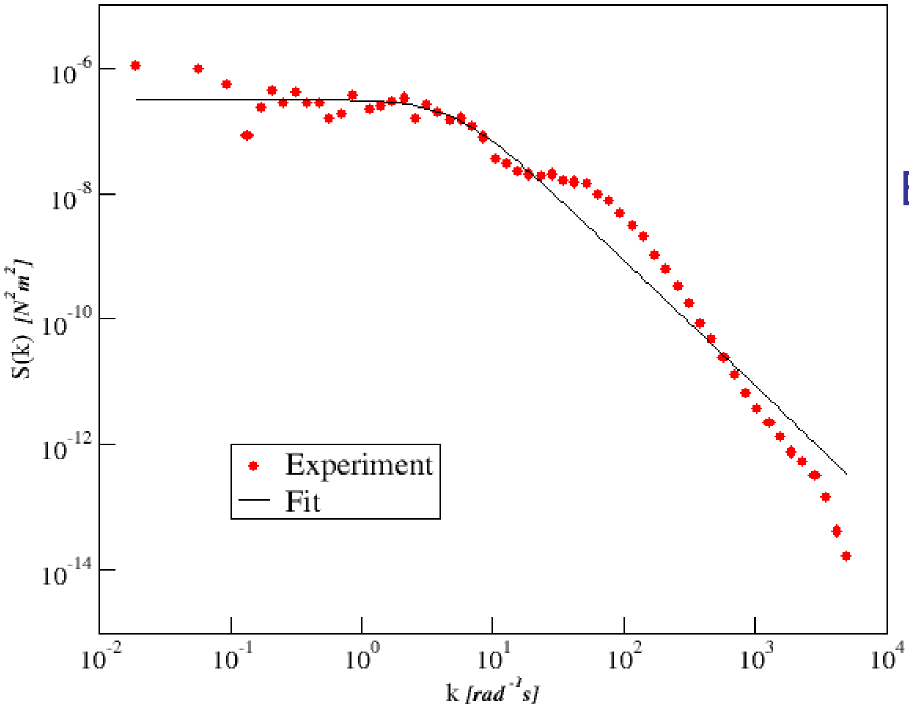


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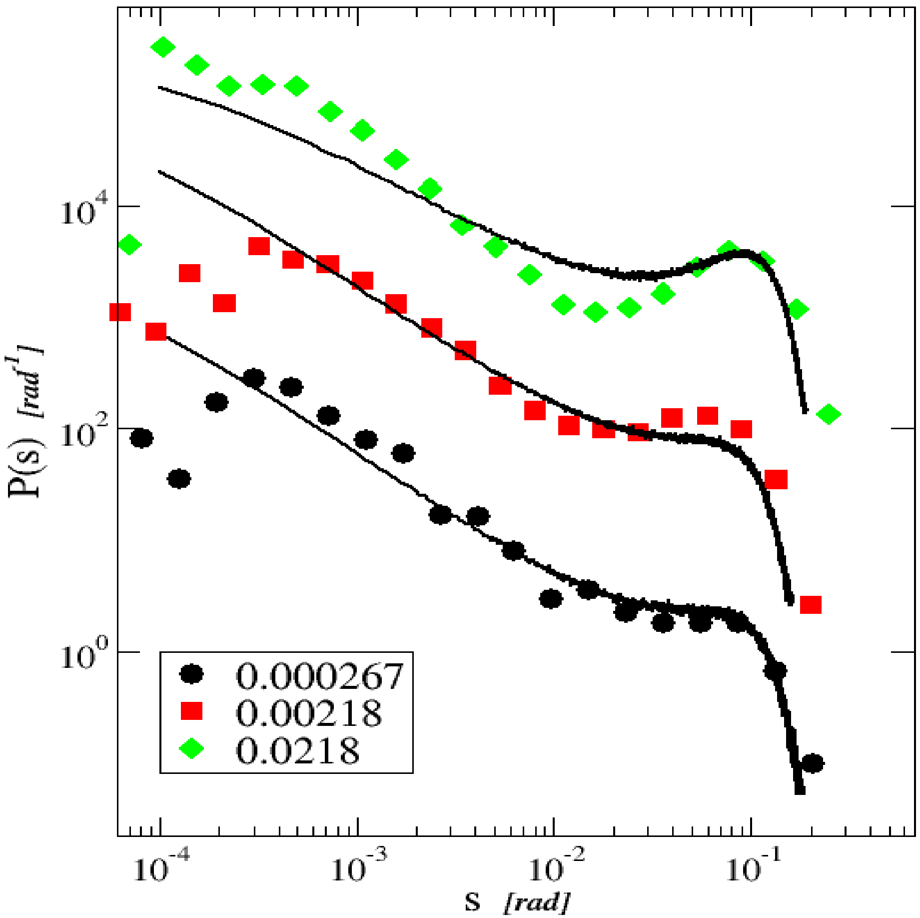
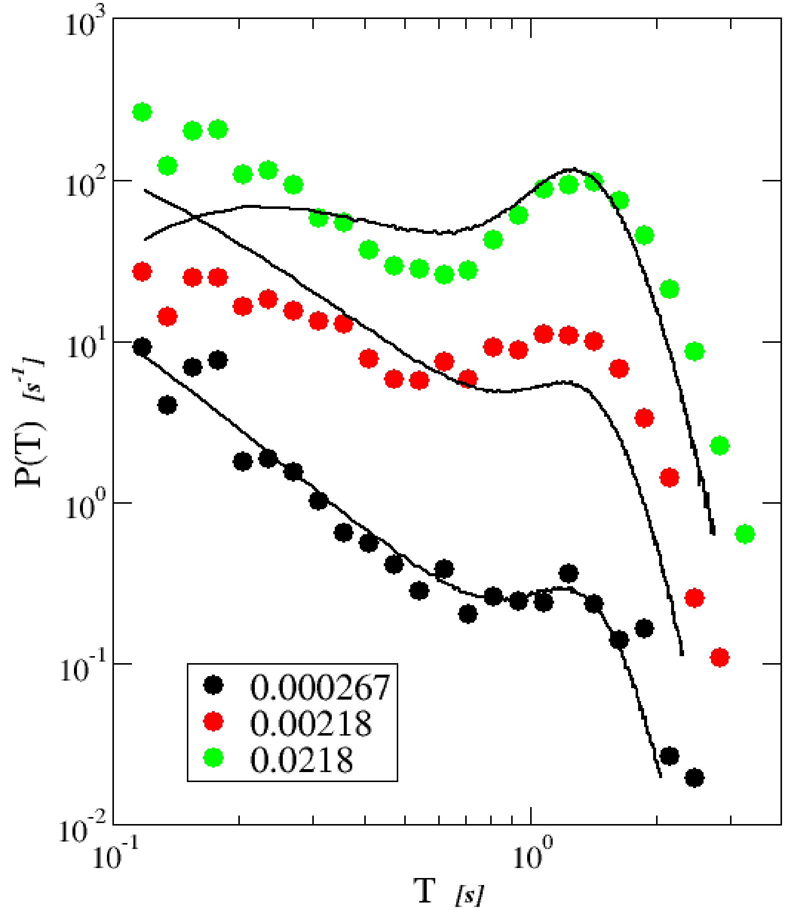
$$\frac{dF_\mu}{d\theta}=\eta(\theta)-aF_\mu \qquad \langle\eta(\theta)\eta(\theta')\rangle=D\delta(\theta-\theta')$$

Now all we need is a good *η(θ)* and we can numerically integrate and simulate the system!

# *Numerical integration: Results – Size and duration distributions*



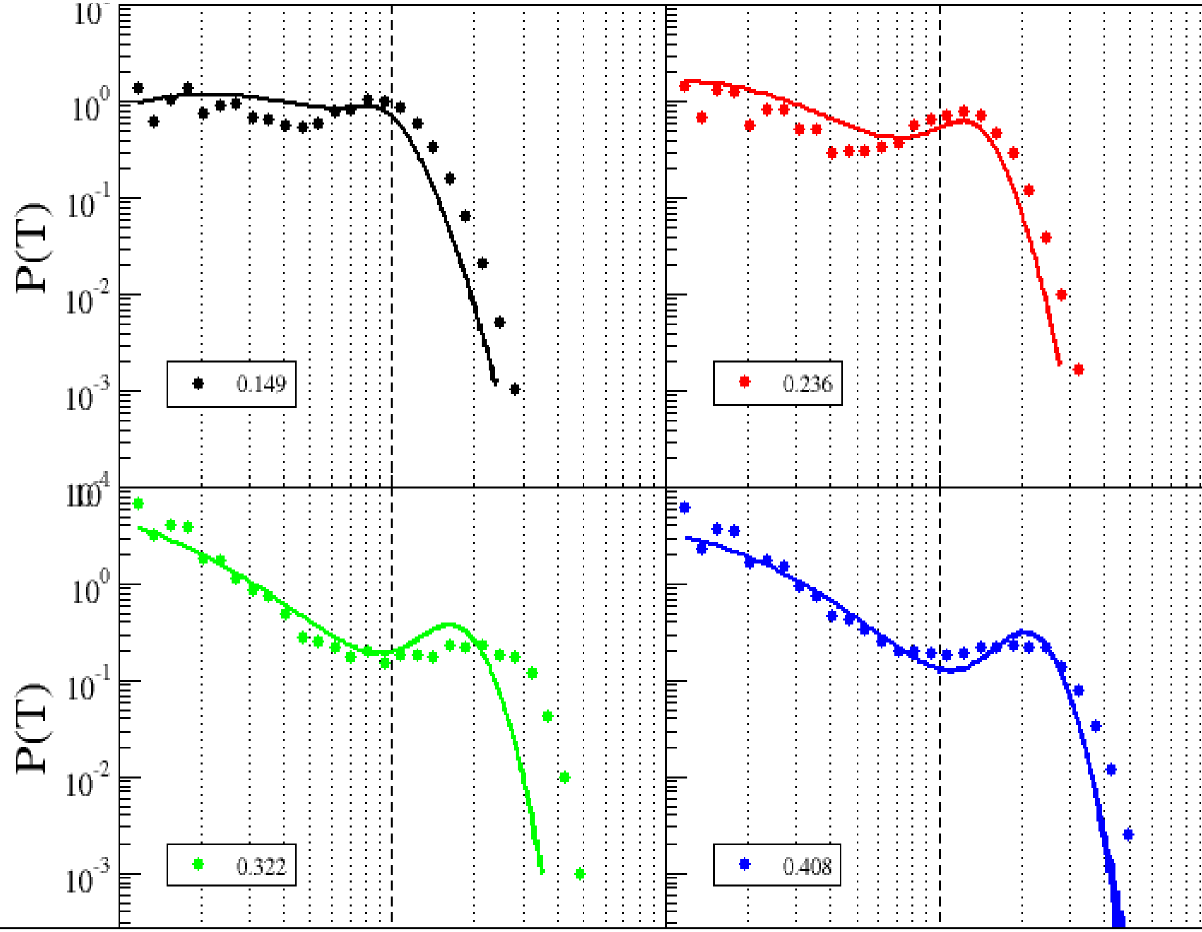

**No adjustable parameters**

(Typical values: *$a$ ~15 rad$^{-1}$, D~0.01 N$^2$/rad, $F_0$~0.55 Nm, $\gamma$~0.2 Nms/rad, $v_0$~0.03 rad/s*)

*Just using $I_{eff}$ ~ 2 I a better agreement is obtained.*

*(Also Nasuno et al. (1997) notice $M_{eff}$ ~ 1.7 M for slow drive periodic stick-slip).*

# *Numerical integration: Results – Several Values of T=√I/K*

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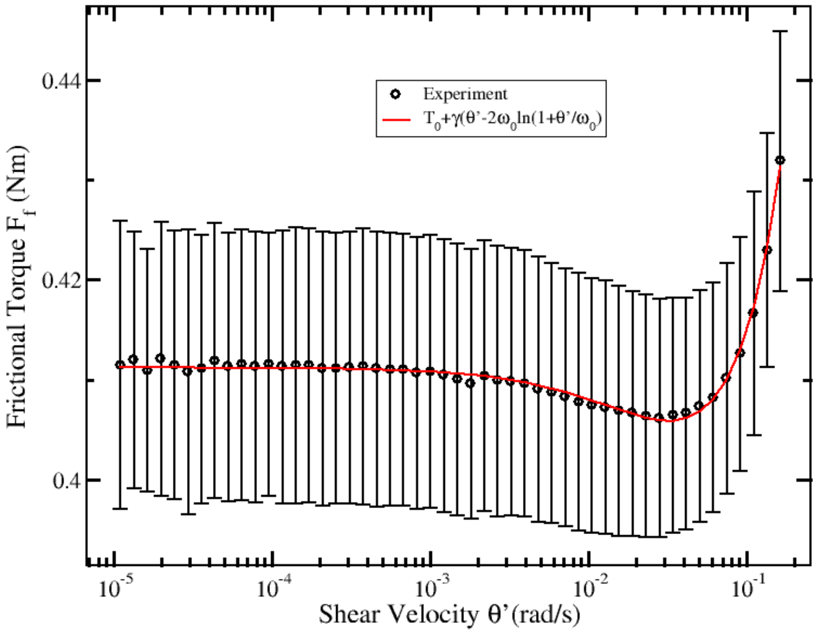
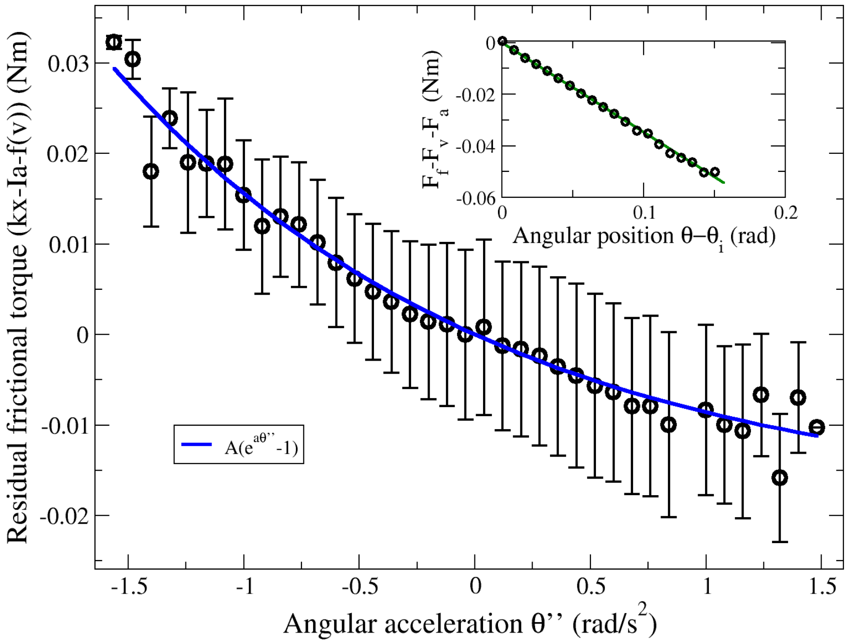
# *Numerical integration: Model shortcomings*

A residual dependence of *F(...)* on θ'' is observed
⇒ $F_r(\theta)$ not truly a random Brownian noise

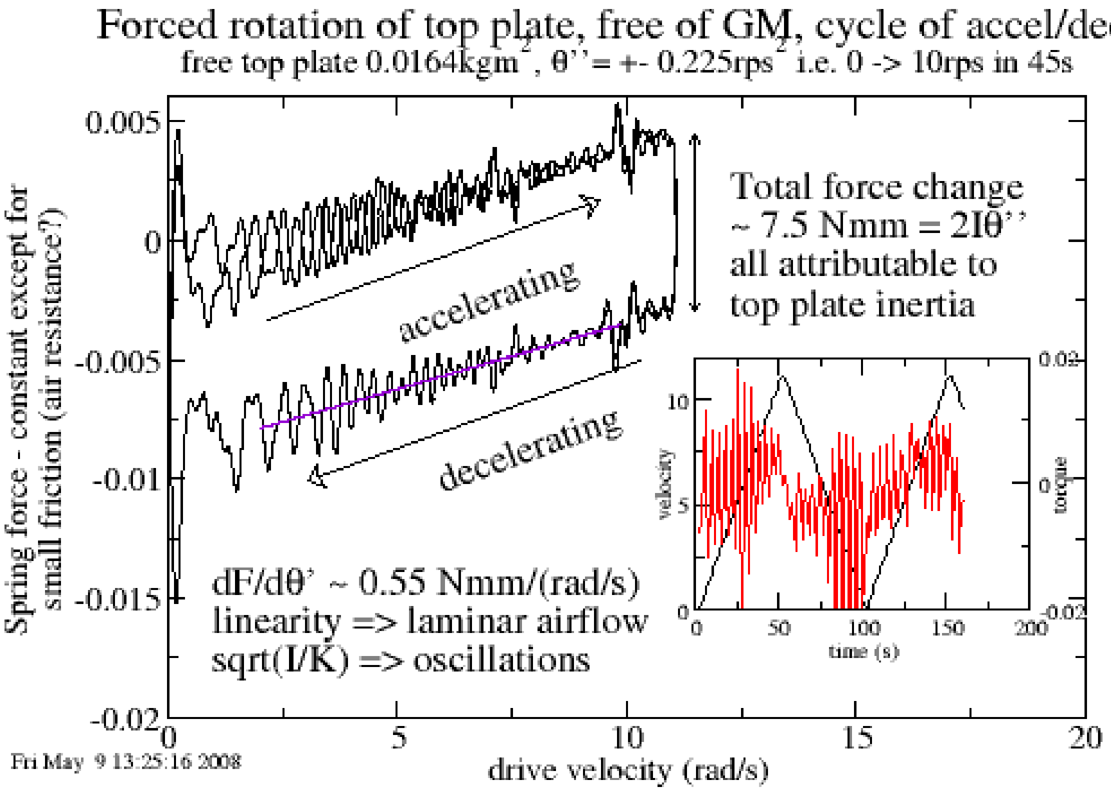


# Numerical integration: Model shortcomings – possible solution?

Idea: to isolate the acceleration dependent torque term,
run the system at various velocities and accelerations!
- calculate the inertial term as a function of velocity,
  and acceleration if necessary

Example: Free driven rotation of the top plate



# *Numerical integration: Model shortcomings – possible solution?*

Idea: to isolate the acceleration dependent torque term,
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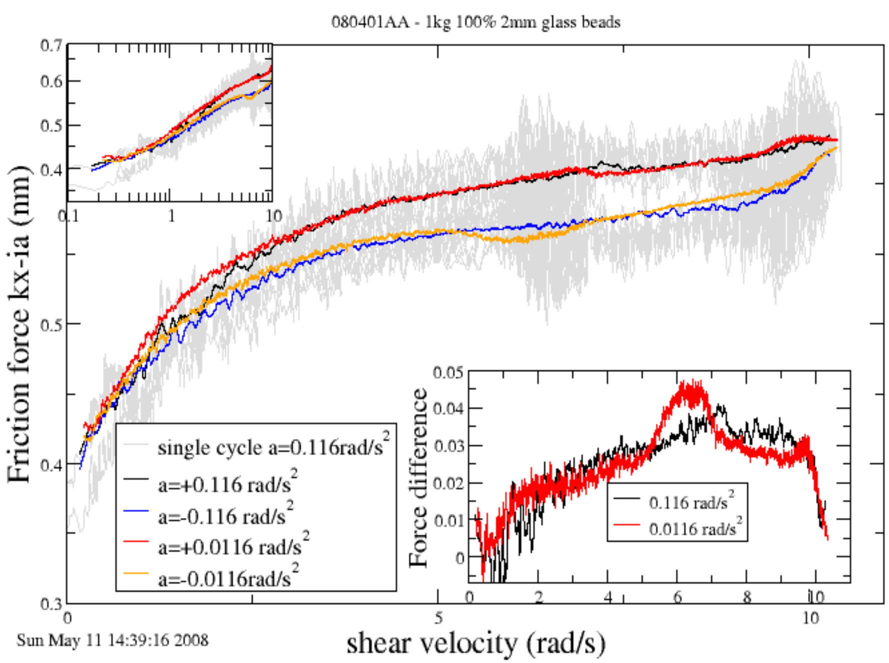
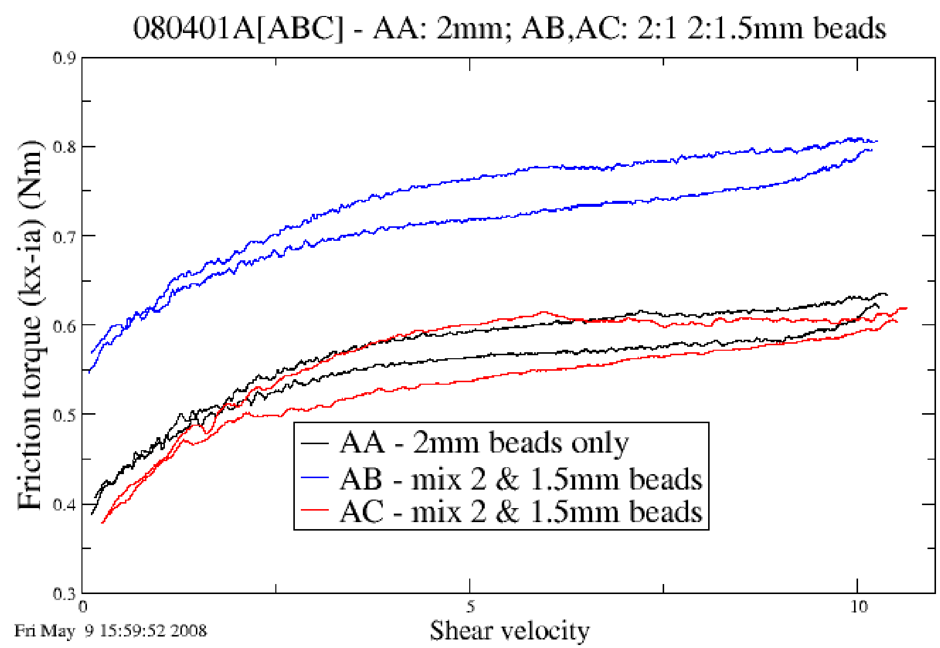
Problem: spring permits sqrt(I/K) oscillations
- too much variation



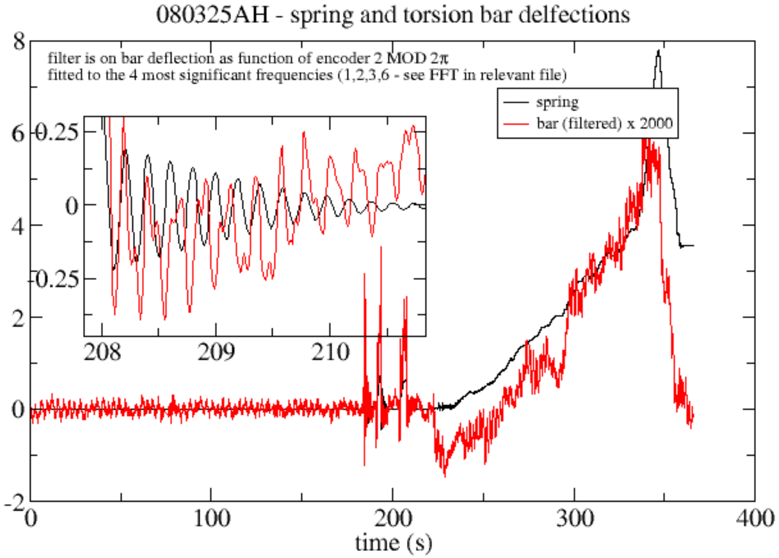



# *Numerical integration: Model shortcomings – possible solution?*

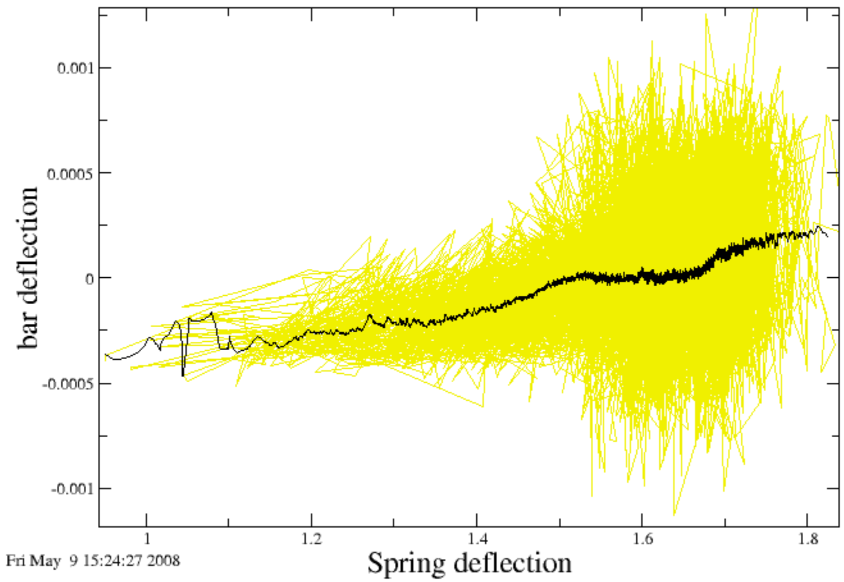
Problem: spring permits sqrt(I/K) oscillations
- too much variation

Solution: use the axle as a torsion bar?
-- for average behaviour, *maybe*, but for us, *<u>no biscuit</u>*

No GM – oscillation and 'static' forcing




GM present, 0-10-0 rad/s




Worry: which is the torque applied to the top plate then,
that measured by the bar or that by the spring?
- need torsion sensor!

# *Formal comparison with magnetic domain wall motion: Barkhausen Noise*

Correlation between magnetic domain wall motion and force chains motion

$$\Gamma \frac{dx}{dt} = H_a + H_{dem} + H_{pin}$$

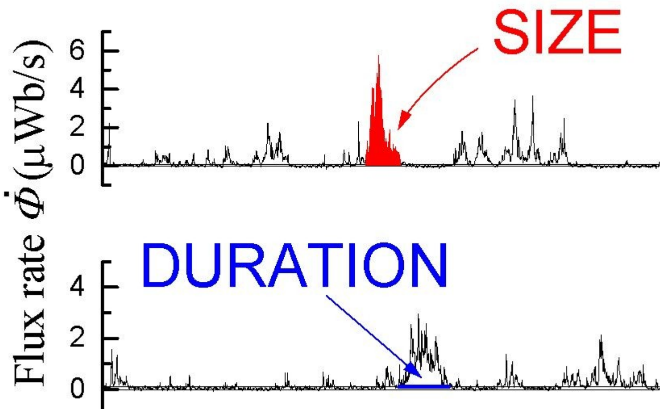
*x* = magnetization

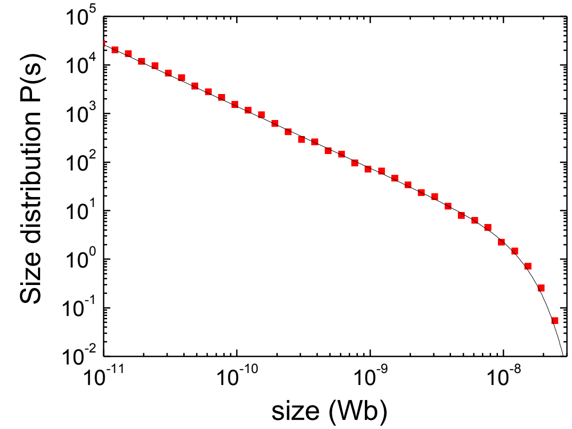
| Damping | Applied field | Demagnetizing field | Pinning |
|---|---|---|---|
| $\bar{F}$ | $Vt$ | $-kx$ | w |
| | | | $\langle (w(x) - w(x'))^2 \rangle = D\|x - x'\|$ |
| | | | fluctuating force |

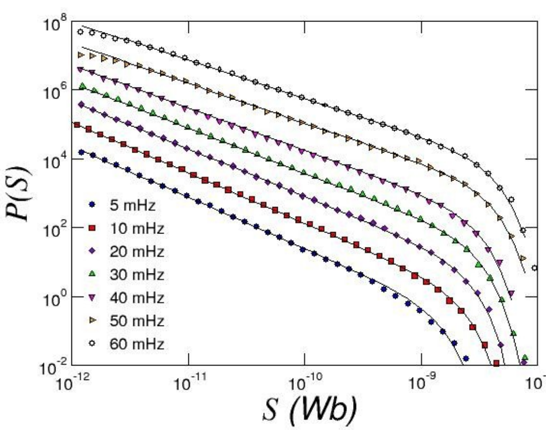
Alessandro *et al.*, *Appl. Phys.* **68**, 2901–2908 (1990) - *ABBM model*

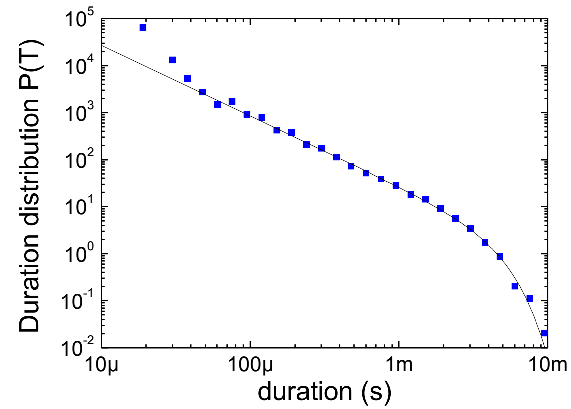
# *Barkhausen Noise*

No inertial peak




S. Zapperi, P. Cizeau, G. Durin and H.E. Stanley,
*Phys. Rev. B* **58**, 6353 (1998).

# *Summary and perspectives*

A large class of dynamical instabilities can be quantitatively described by similar stochastic processes of the Brownian kind: similar phenomenology is observed in different systems, i.e. friction on solids, layers of crossing polymers, and completely different phenomena like *Barkhausen noise in ferromagnets.*

Get one level down and derive stochastic forces and the friction law from ab-initio or effective description of the systems (e.g. force chains)

Write down the associated Fokker Planck equation

Describe fluctuations of physical quantities in different heterogeneous systems by general stochastic equations

Understand the onset of asymmetric and stable probability distributions in correlated systems

Torque sensor arriving – direct sensing of torque with no torsion spring will reduce stick-slip and allow better determination of viscous friction law

F. Dalton et al. *:*
***"Shear stress fluctuations in the liquid-solid granular transition",***
PRL **95,** 138001 (2005)

A. Baldassarri et al.:
***"Brownian forces in sheared granular matter",***
PRL **96,** 118002 (2006)

"*FIRB*", Italian funded project:
*" Rheological properties of granular media"*

*TRIGS*

UE funded project on triggering, instabilities and crackling noise: granulars, fractures, friction, magnets, vortices, earthquakes,….