Dynamics and Third Cumulant of Quantum Noise

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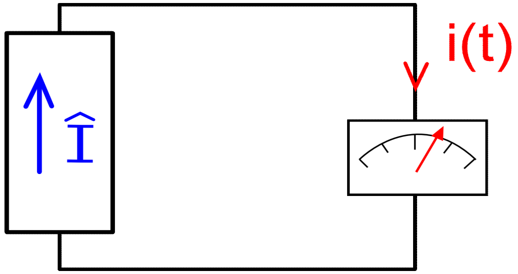
How to measure a Quantum current Î ?

Coupling between Î and i(t) measured by a classical detector



Quantum
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Classical detector
(linear amplifier)

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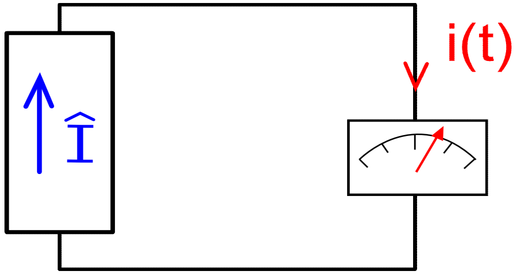

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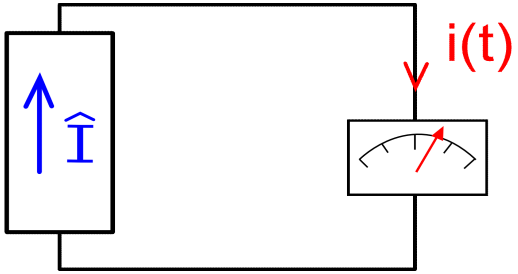

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but … fluctuations at a given frequency ………… $\Delta i^2_{meas.}(\omega)$

$$\langle \hat{I}(-\omega) \times \hat{I}(\omega) \rangle - \langle \hat{I} \rangle^2$$ or $$\langle \hat{I}(\omega) \times \hat{I}(-\omega) \rangle - \langle \hat{I} \rangle^2$$?

do not commute

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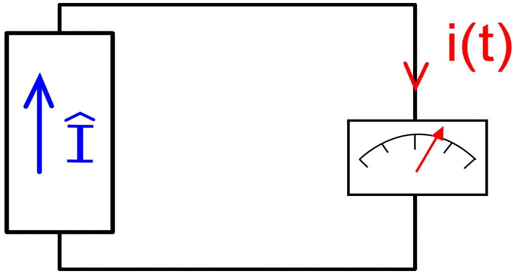

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L.D. Landau and E.M. Lifshitz, *Statistical Physics*

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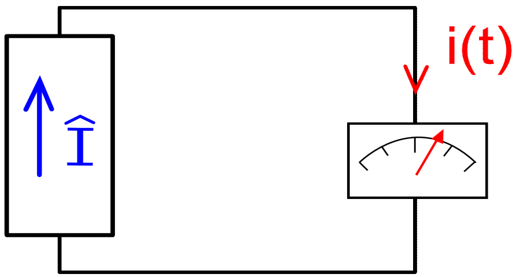

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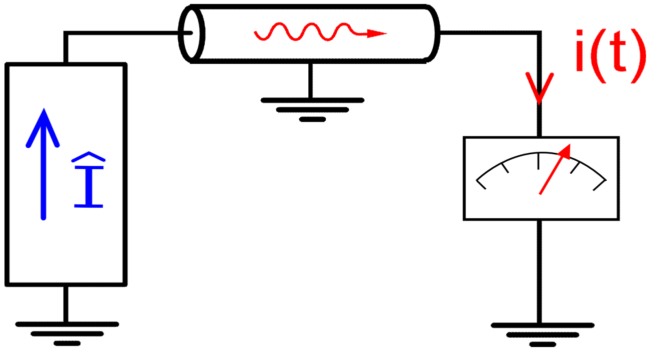
Is it perfectly clear ?

Measurement of Quantum Noise

$\Delta i^2_{meas.}(\omega)$ can be seen as a photo-detection problem (Nyquist argument)



energy conservation

$$R\,\Delta i^2_{meas.}(\omega) = N_{photon}(\omega) \times \hbar\omega \times \delta\omega$$

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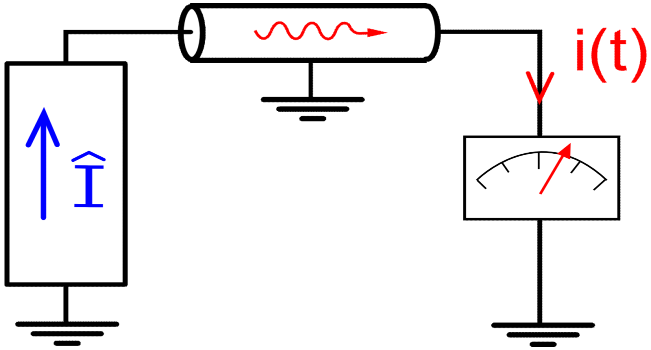

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absorption term

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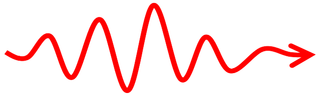
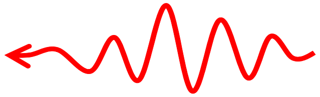


emission term

$$\left\langle \delta\hat{I}(-\omega) \times \delta\hat{I}(\omega) \right\rangle \propto N(\omega) + 1$$



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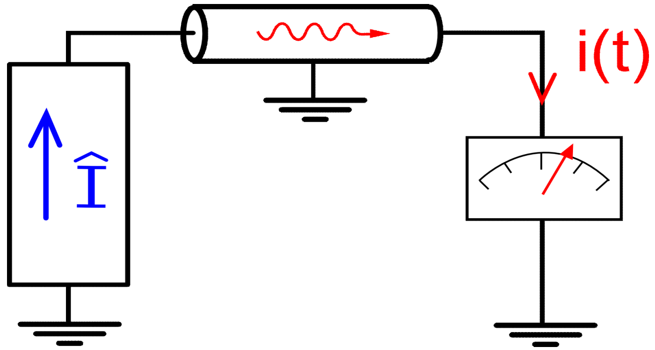

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… it has been proven for a quantum detector (SIS junction) that

Theory by L. Levitov, G. Lesovik
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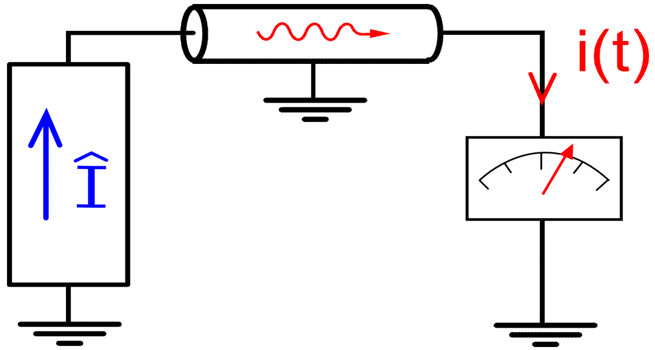

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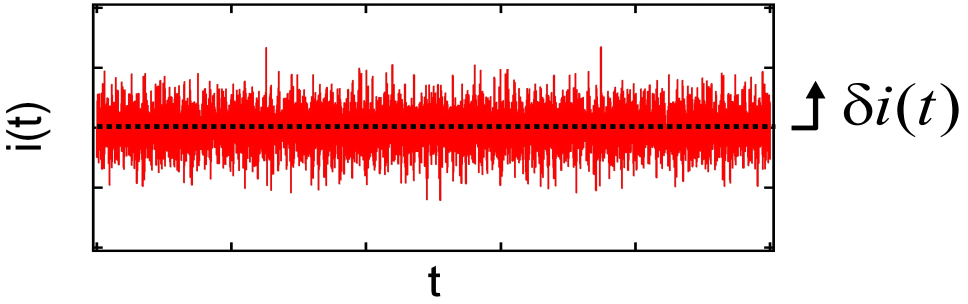
But it is not clear for a linear amplifier

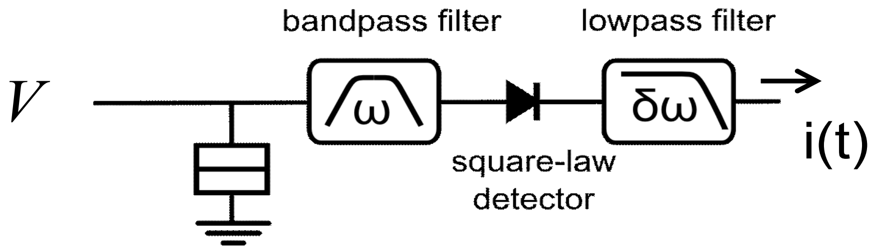
$$\Delta i^2_{meas.}(\omega) \propto \hbar\omega \left(N_{photon}(\omega) + \frac{1}{2} \right)$$

measurement of Zero Point Fluctuations

U. Gavish, PRB 62 R10 637 (2000)

Quantum Noise in a tunnel junction

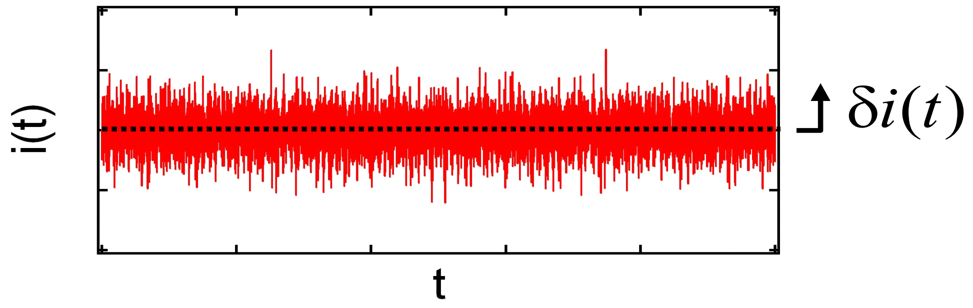


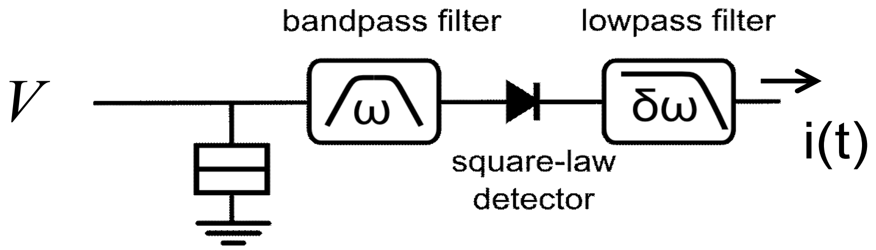


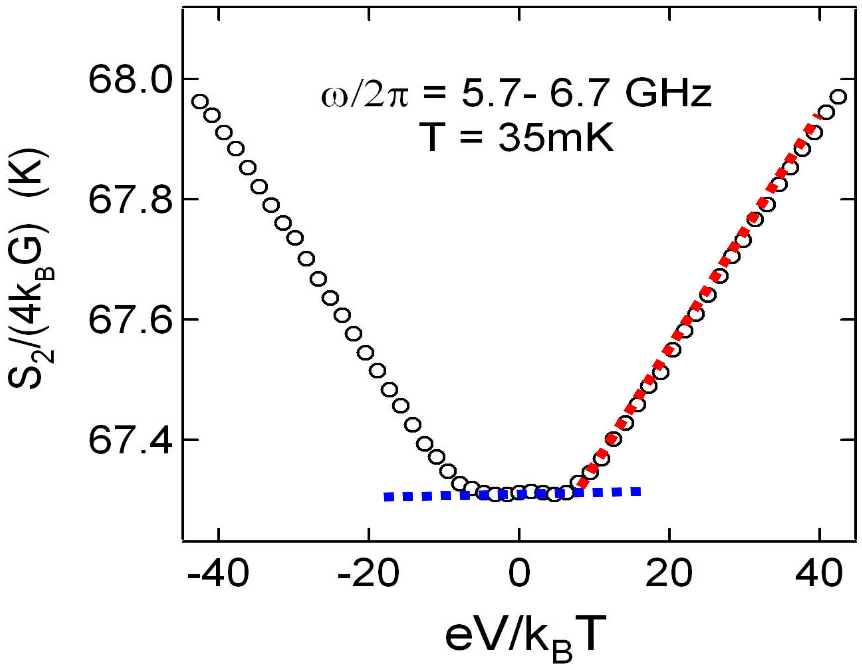
Noise spectral density

$$S_2(V,\omega) = \langle \delta i(\omega)\, \delta i(-\omega) \rangle$$

Quantum Noise in a tunnel junction







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Equilibrium noise (zero point fluctuations)

$$S_2^{(éq)}(eV << \hbar\omega) = 2G\hbar\omega$$

Shot noise (discreteness of charge)

$$S_2(eV >> \hbar\omega) = 2e\, I$$

Sample: L. Spietz, Yale Universit

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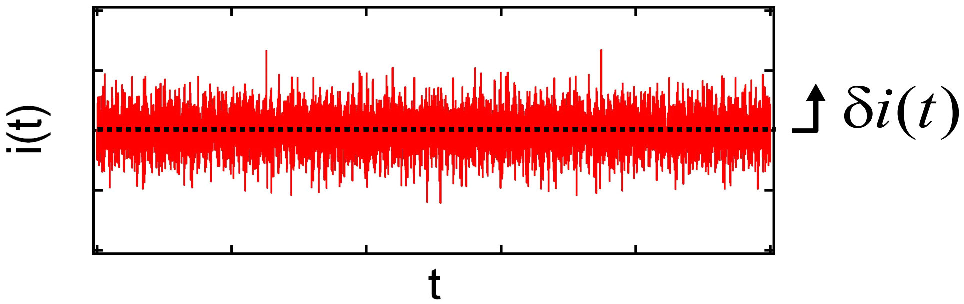

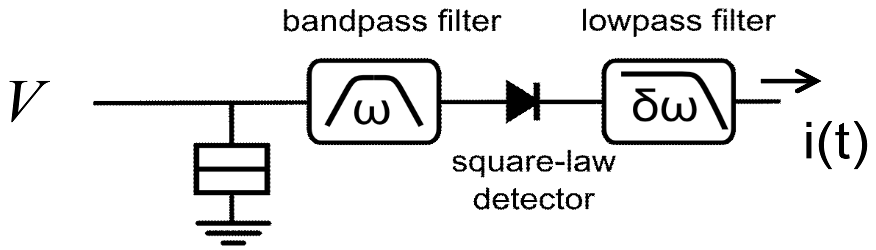

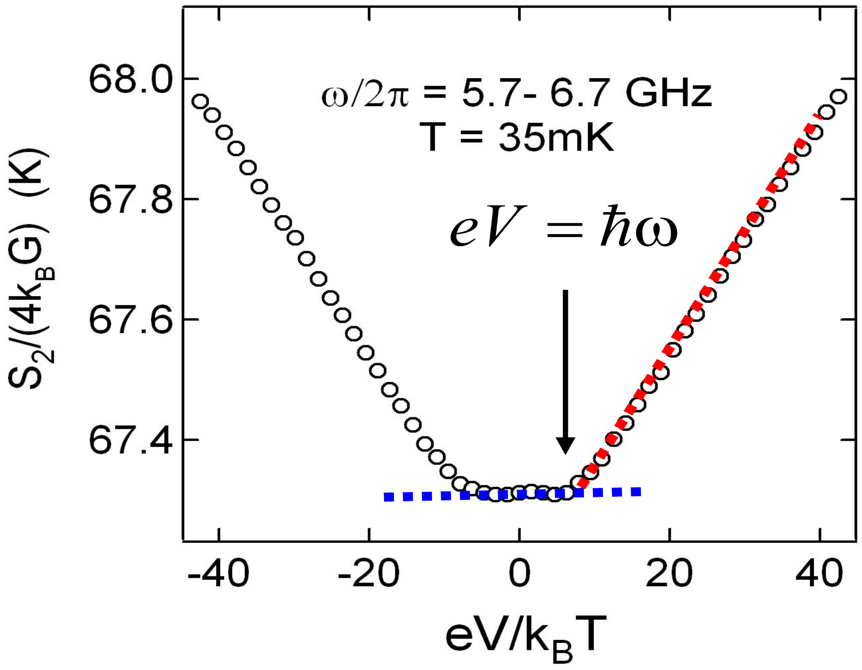

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Quantum regime $\hbar\omega >> k_BT, eV$

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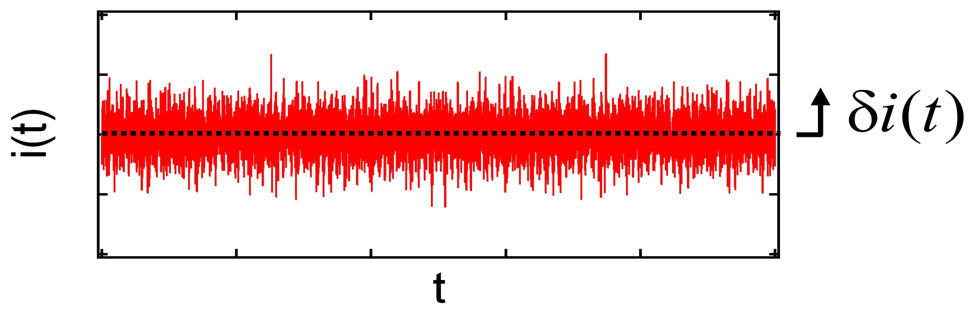

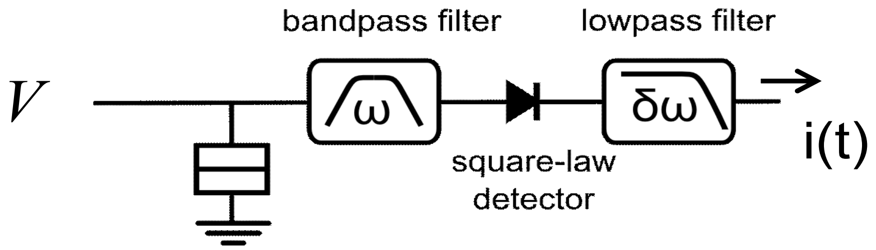

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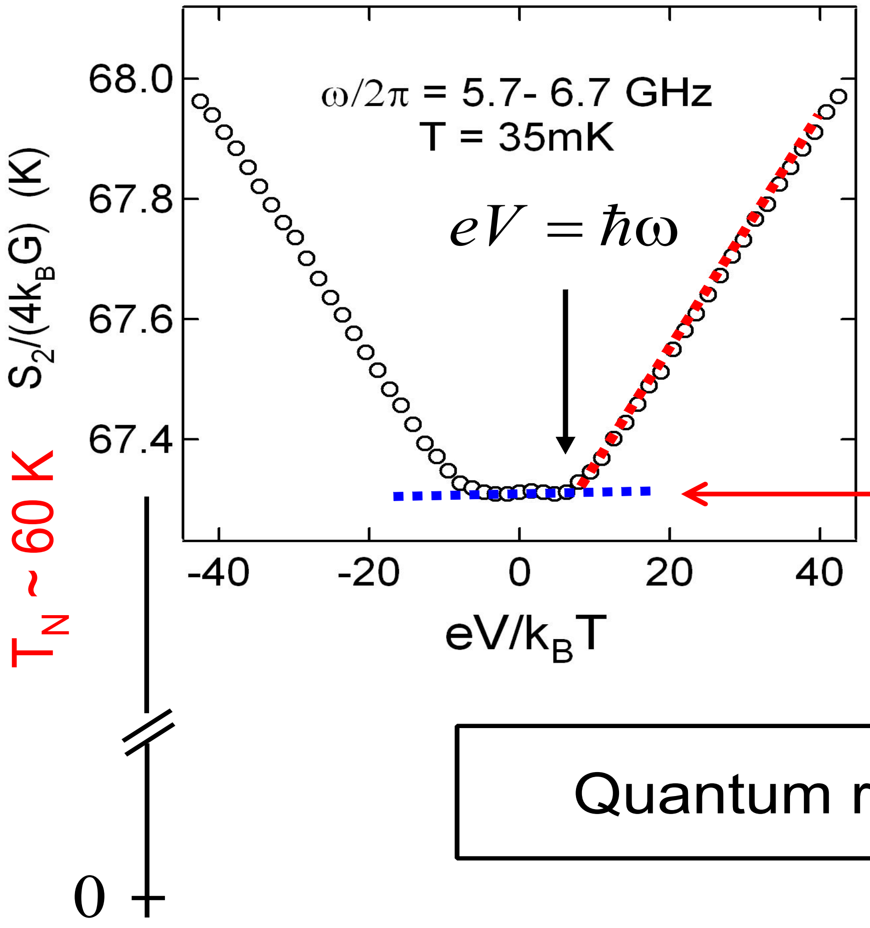

?

0
or
$\hbar\omega/2$

Equilibrium noise (zero point fluctuations)

$$S_2^{(éq)}(eV \ll \hbar\omega)=2G\hbar\omega$$

Shot noise (discreteness of charge)

$$S_2(eV \gg \hbar\omega)=2e\,I$$

Quantum regime $\hbar\omega \gg k_BT, eV$

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Experimental Answer

Measurement of Third Cumulant in a Tunnel Junction

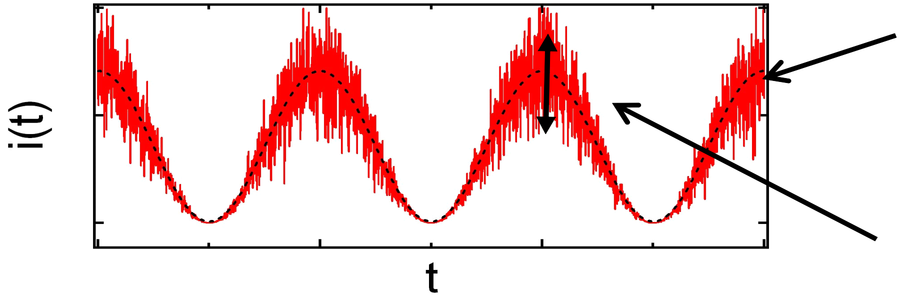
I. Definition and measurement of Quantum Noise Dynamics

1. Definition of the quantity which describes properly noise dynamics
2. Measurement of Quantum noise dynamics in a tunnel junction

II. Preliminary measurement of Third Cumulant

1. Subtraction of environmental terms
2. Estimation of Third Cumulant of current fluctuations

I. Noise Dynamics



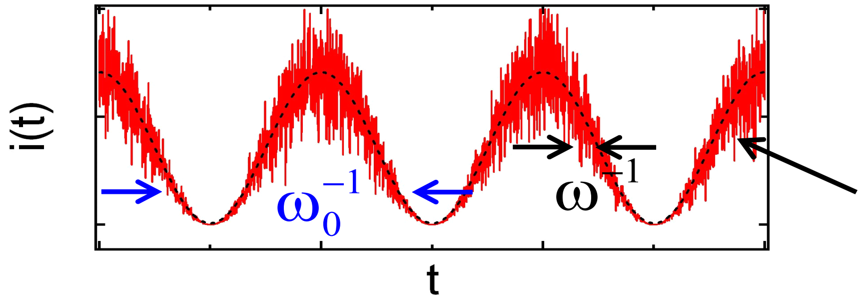
$$\left\langle I \right\rangle = G(\omega_0)\,\delta V$$

$$\left\langle \Delta i^2 \right\rangle = X(\omega,\omega_0) = \chi(\omega_0)\,\delta V$$

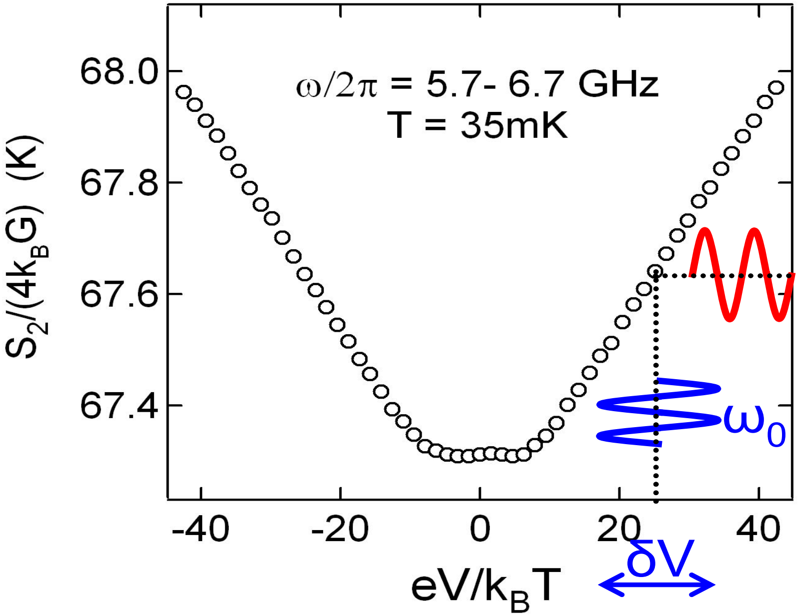
How fast does Quantum Noise respond ?

1. Definition of the quantity which describes properly noise dynamics
2. Measurement of Quantum noise dynamics in a tunnel junction

Adiabatic Noise Dynamics $\omega_0 \ll \omega$



$\langle i \rangle = G(\omega_0)\,\delta V$



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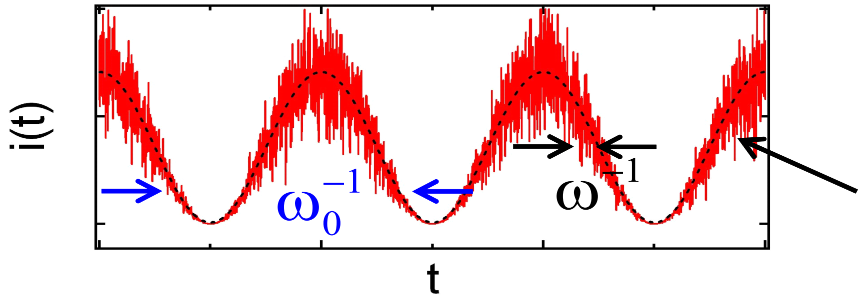

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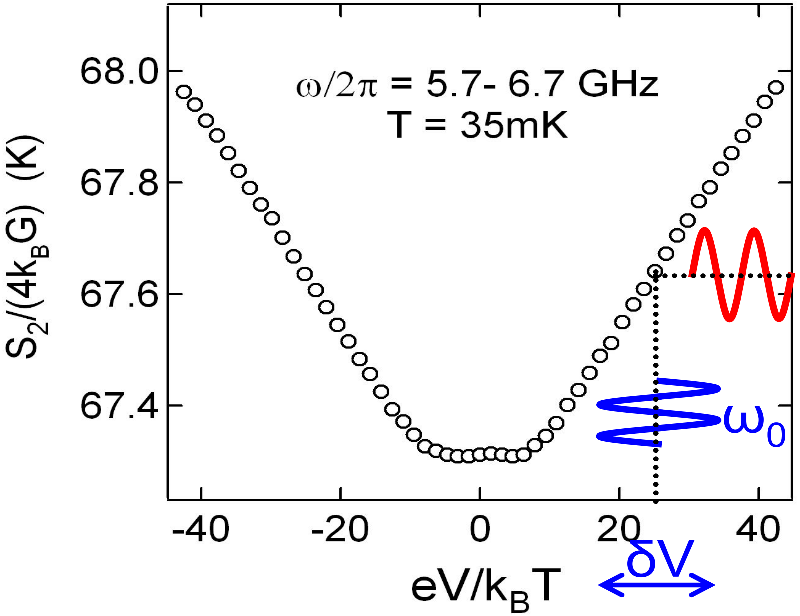
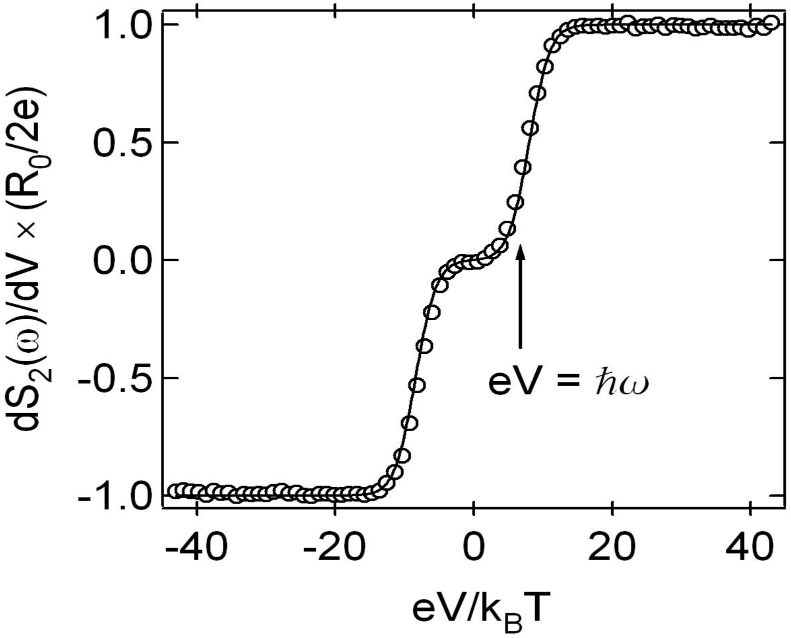
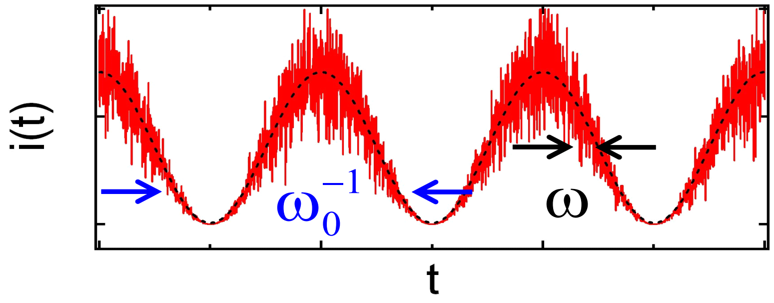




Dynamical response function $\propto \dfrac{dS_2(\omega)}{dV}$

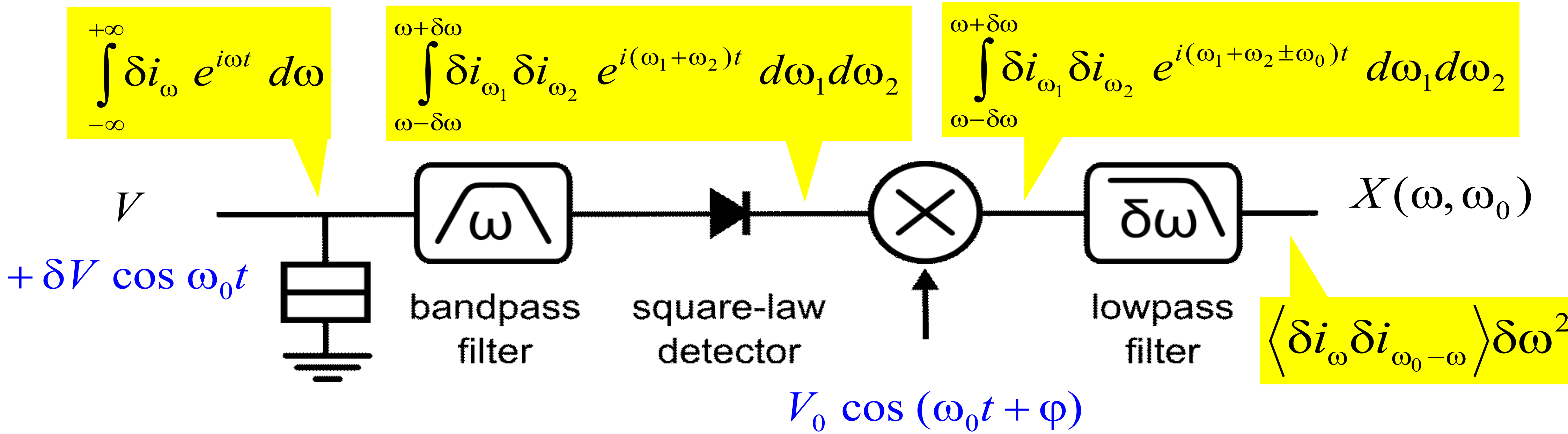
Quantum Noise dynamics



Homodyne technique to measure the in-phase and out of phase response

$$\boxed{X(\omega,\omega_0)=\langle \delta i(\omega)\,\delta i(\omega_0-\omega)\rangle}$$

mean square beating at ω_0



$$\int_{-\infty}^{+\infty} \delta i_\omega\, e^{i\omega t}\, d\omega$$

$$\int_{\omega-\delta\omega}^{\omega+\delta\omega} \delta i_{\omega_1} \delta i_{\omega_2}\, e^{i(\omega_1+\omega_2)t}\, d\omega_1 d\omega_2$$

$$\int_{\omega-\delta\omega}^{\omega+\delta\omega} \delta i_{\omega_1} \delta i_{\omega_2}\, e^{i(\omega_1+\omega_2\pm\omega_0)t}\, d\omega_1 d\omega_2$$

V

$+\delta V \cos \omega_0 t$

bandpass filter

square-law detector

lowpass filter

$X(\omega,\omega_0)$

$\langle \delta i_\omega \delta i_{\omega_0-\omega} \rangle \delta\omega^2$

$V_0 \cos(\omega_0 t+\varphi)$

Case of the tunnel junction

usual symmetrization

$$X(\omega,\omega_0)=\frac{1}{2}\left\{\left\langle \hat{i}(\omega)\,\hat{i}(\omega_0-\omega)\right\rangle+\left\langle \hat{i}(\omega_0-\omega)\,\hat{i}(\omega)\right\rangle\right\}$$

Case of the tunnel junction

usual symmetrization

$$X(\omega,\omega_0)=\frac{1}{2}\left\{\left\langle\hat{i}(\omega)\,\hat{i}(\omega_0-\omega)\right\rangle+\left\langle\hat{i}(\omega_0-\omega)\,\hat{i}(\omega)\right\rangle\right\}$$

in the Landauer – Büttiker formalism for a tunnel junction:

$$X(\omega,\omega_0)=\frac{1}{2}\sum_n J_n(z)\,J_{n+1}(z)\left(S_2(eV+\hbar\omega+n\hbar\omega_0)\right.$$
$$\left.-S_2(eV-\hbar\omega-n\hbar\omega_0)\right)$$

with $J_n =$ Bessel function of 1st kind and $z=e\,\delta V/\hbar\omega_0$

J. Gabelli and B. Reulet, arXiv 0801.1432

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with $J_n =$ Bessel function of 1st kind and $z=e\,\delta V/\hbar\omega_0$

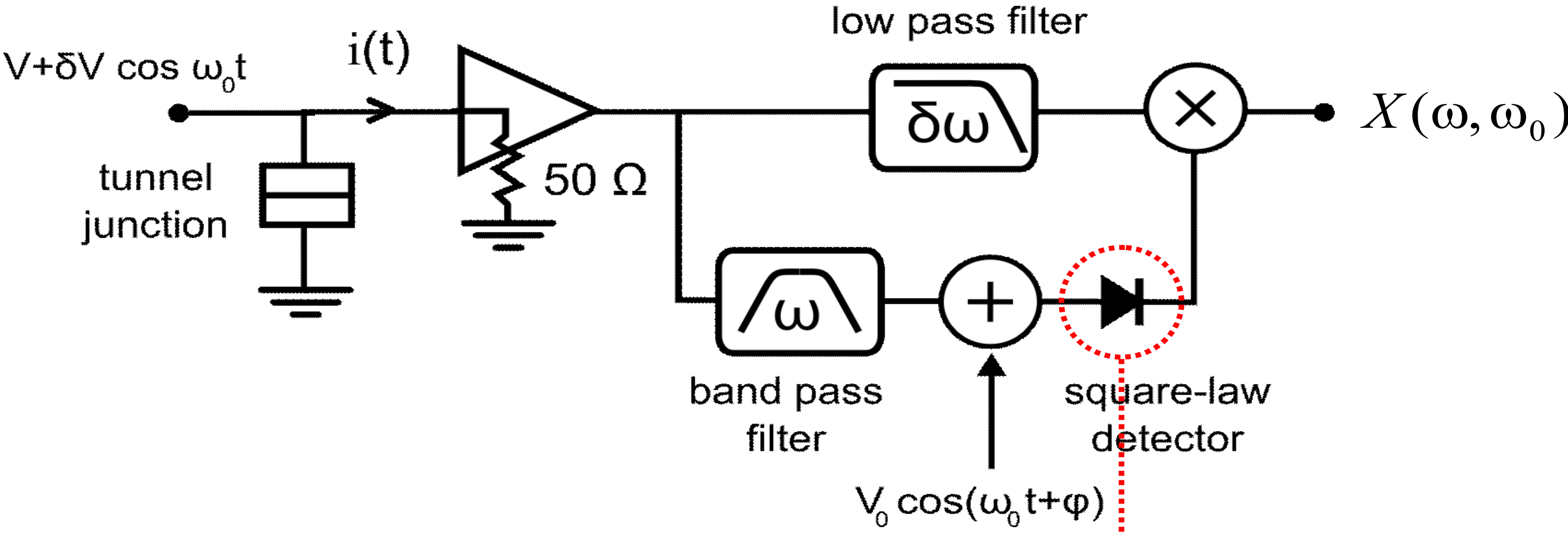
J. Gabelli and B. Reulet, arXiv 0801.1432

$X(\omega,\omega_0)$ is clearly different from photo-assisted noise

$$S_2^{ph}(\omega,\omega_0)=\sum_n J_n^2(z)\,\big(S_2(eV+\hbar\omega+n\hbar\omega_0)+S_2(eV-\hbar\omega+n\hbar\omega_0)\big)$$

Experimental Setup

Quantum regime $\hbar\omega \sim \hbar\omega_0 >> k_B T, eV$



for technical reason $\omega \sim \omega_0$

$$\delta\omega / 2\pi \approx 100\ MHz << \omega / 2\pi \approx 6\ GHz$$

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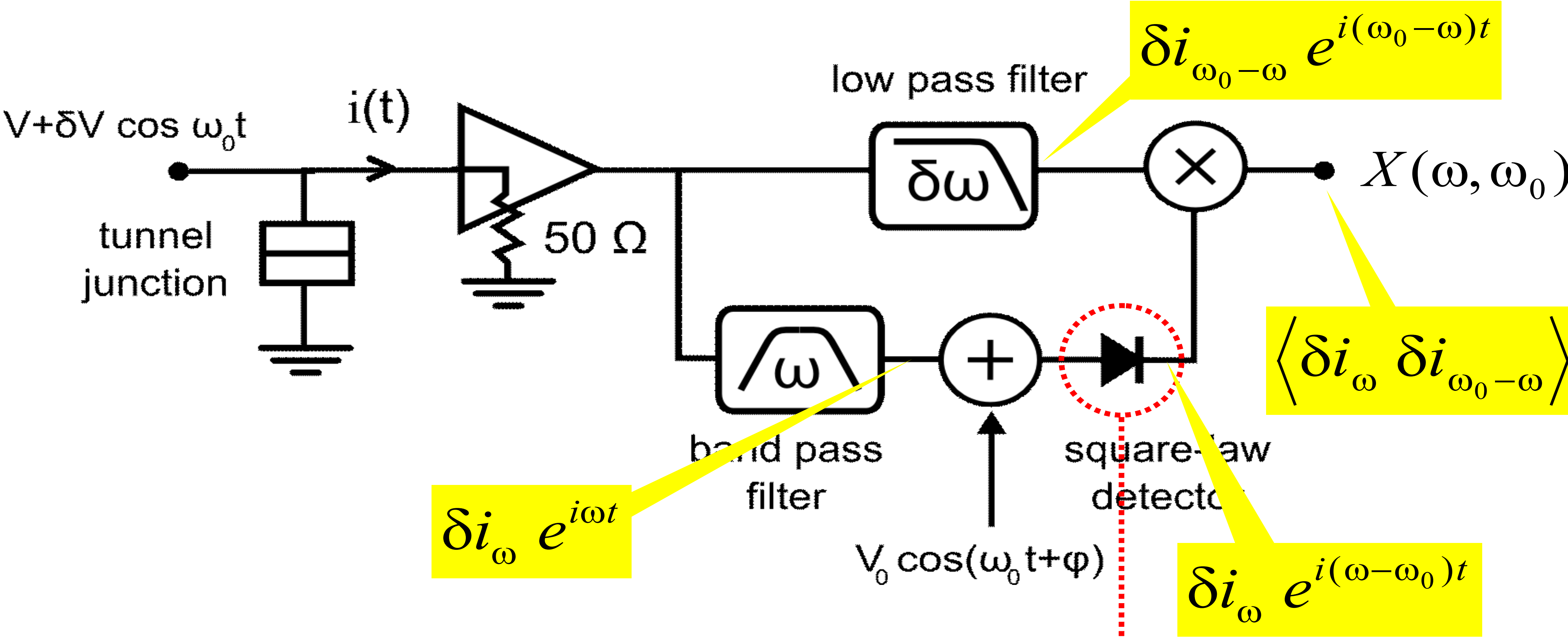

$$\delta i_{\omega_0-\omega}\; e^{i(\omega_0-\omega)t}$$

$$\left\langle \delta i_\omega\, \delta i_{\omega_0-\omega} \right\rangle$$

$$\delta i_\omega\; e^{i\omega t}$$

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Calibration – No fitting parameter

Quantum regime $\hbar\omega \sim \hbar\omega_0 >> k_B T, eV$

Noise

$$S_2(\omega) \rightarrow \begin{cases} \text{conductance } G \\ \text{temperature } T \\ \text{gain } g \end{cases}$$

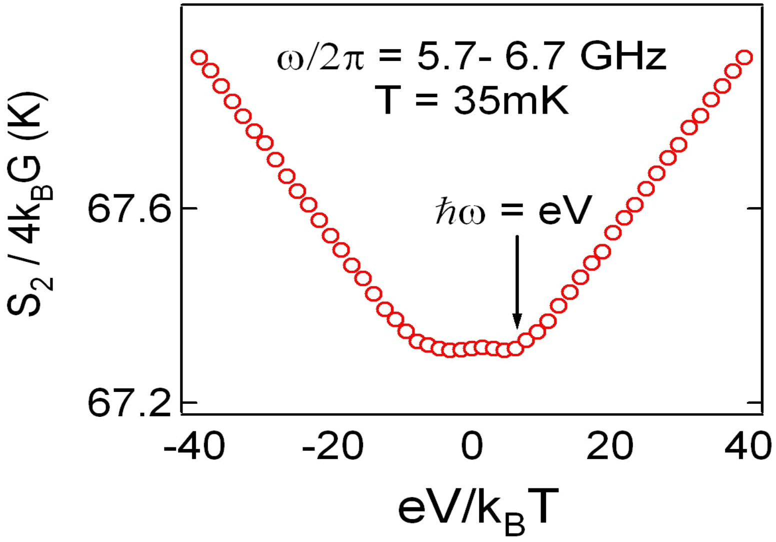
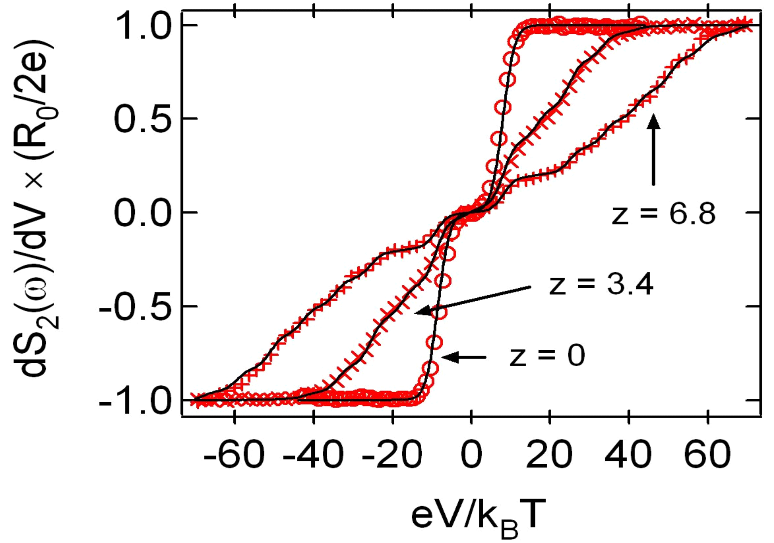


Photo-assisted noise

$S_2^{ph}(\omega) \rightarrow$ excitation coupling

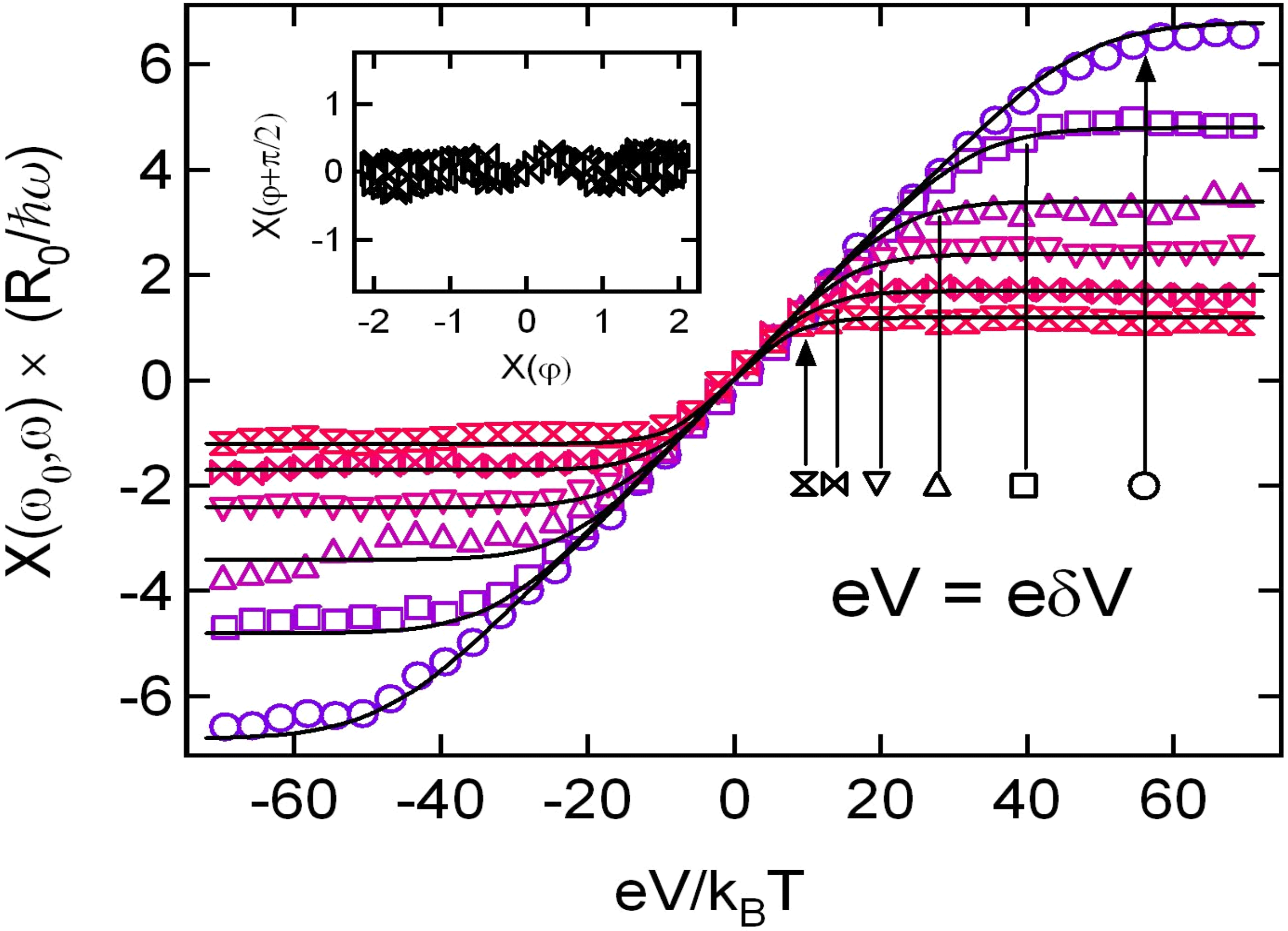
$$z = \frac{e\,\delta V}{\hbar\omega_0}$$



R. Schoelkopf et al PRL 78 3370 (1997), PRL 80 2437 (1998) *J. Gabelli and B. Reulet, PRL 100, 026601 (200*

Experimental results

Quantum regime $\hbar\omega \sim \hbar\omega_0 >> k_B T, eV$



Noise responds in phase in quantitative agreement with theory

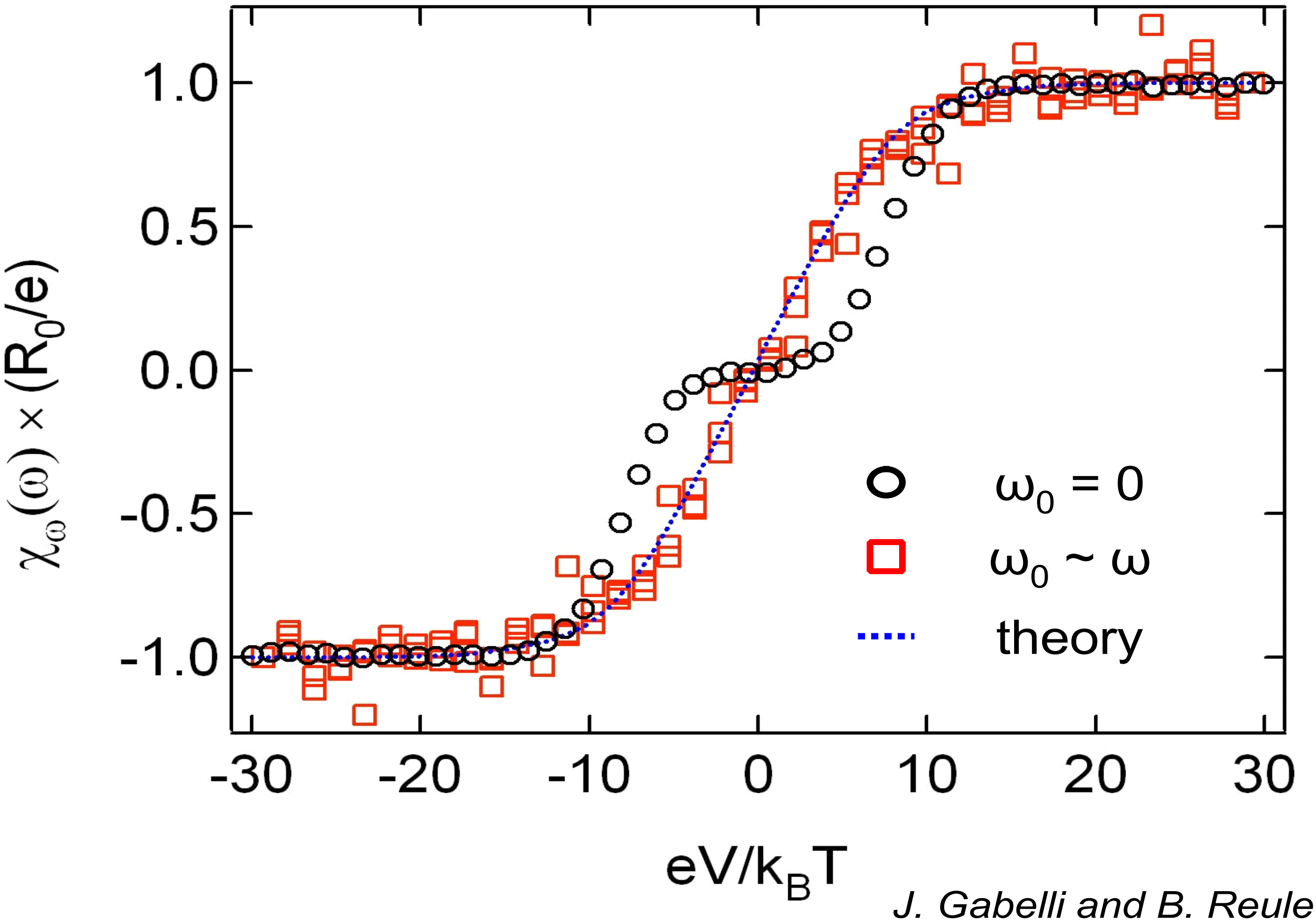
No fitting parameter

Sample: L. Spietz, Yale University

Quantum noise dynamics

Definition of noise susceptibility $$\chi_\omega(\omega) = \lim_{\delta V \to 0} \frac{X(\omega,\omega)}{\delta V} \neq \frac{dS_2(\omega)}{dV}$$

Noise responds in phase but non adiabatically



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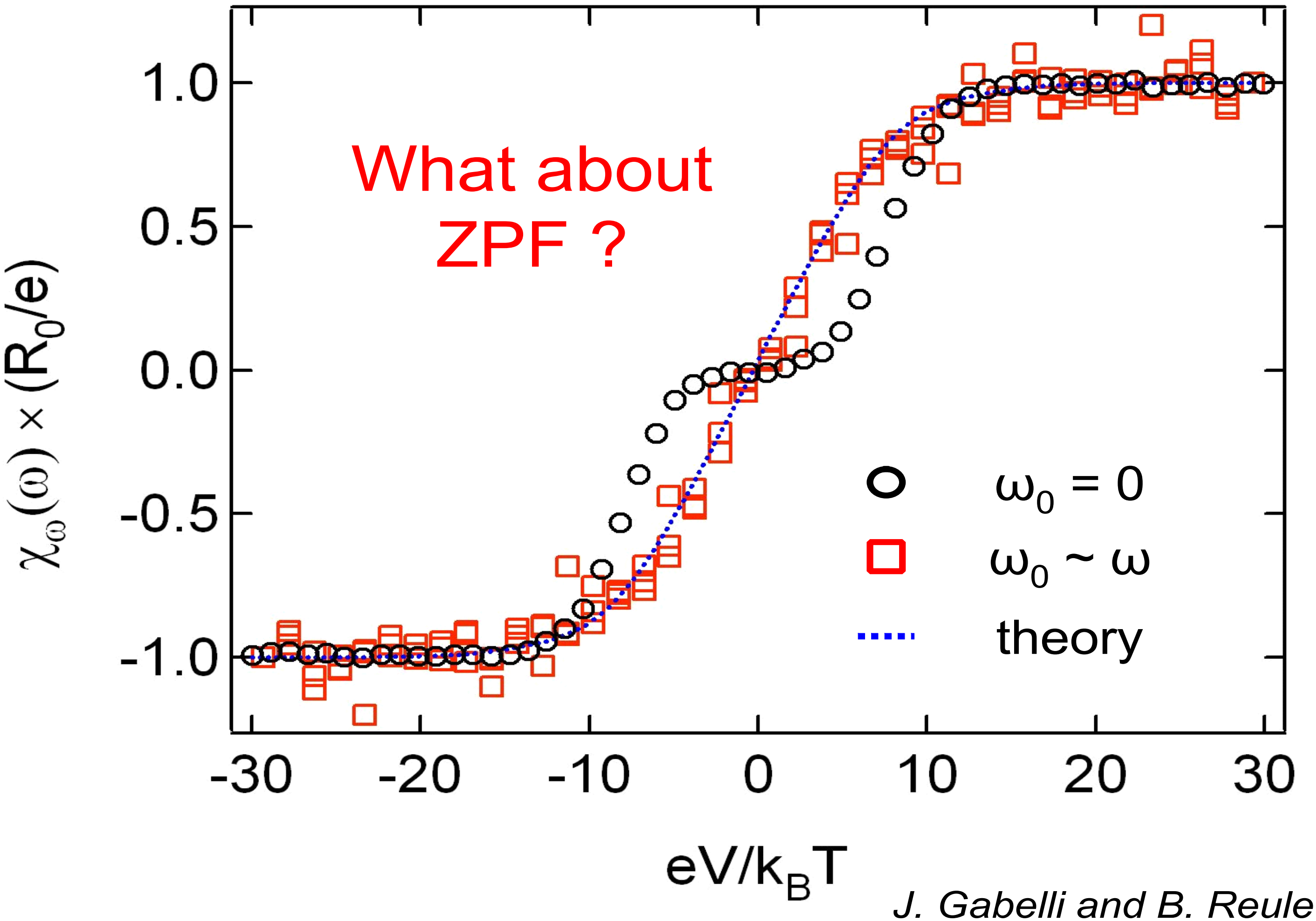

What about ZPF ?

J. Gabelli and B. Reulet, PRL 100, 026601 (2008

Noise dynamics and environmental effects

High order Cumulant = Measurement of P(i), probability distributio

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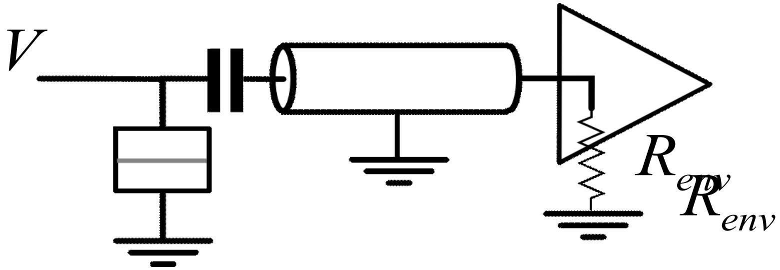
Probalility distribution strongly affected by environment (*B. Reulet et al., PRL* 91 pp. 196601 (2003))

$$P(i,V-R_{env}\ i) \cong P_V(i,V)-R_{env}\ i\,\frac{\partial}{\partial V}P_V(i,V)$$

not perfectly voltage bias



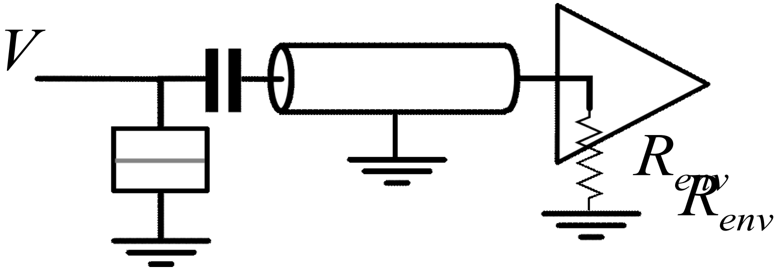
Noise dynamics and environmental effects

High order Cumulant = Measurement of P(i), probability distributio

Probalility distribution strongly affected by environment (*B. Reulet et al., PRL* 91 pp. 196601 (2003))

$$P(i, V - R_{env}\ i) \cong P_V(i, V) - R_{env}\ i \frac{\partial}{\partial V} P_V(i, V)$$

not perfectly voltage bias



Effect on the moments: $\left\langle i^n \right\rangle = \left\langle i^n \right\rangle_V - R_{env} \boxed{\frac{\partial}{\partial V} \left\langle i^{n+1} \right\rangle_V}$

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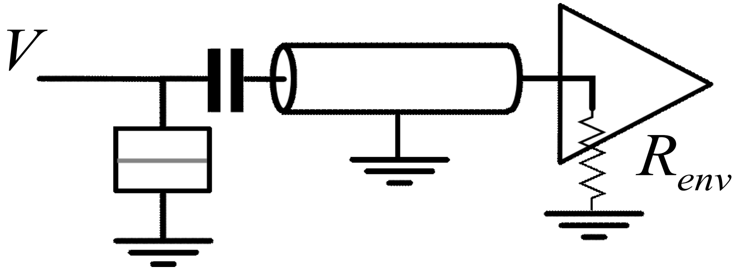

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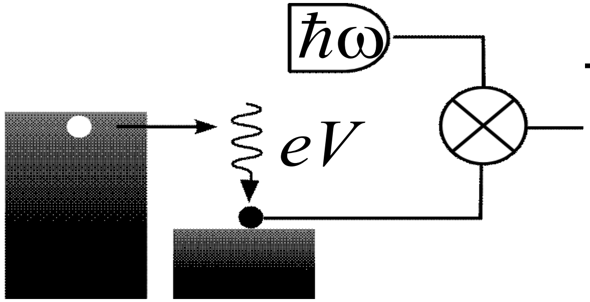
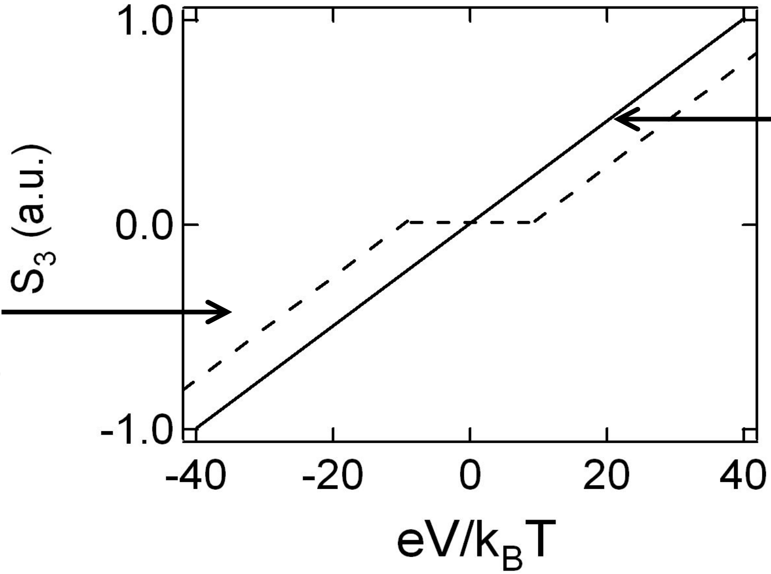
n = 1: Dynamical Coulomb Blockade Effect: $\frac{\partial}{\partial V}\left\langle i^2 \right\rangle_V \leftrightarrow \chi_\omega(\omega)$

n = 3: Third Cumulant measurement: $\frac{\partial}{\partial V}\left\langle i^4 \right\rangle_V \leftrightarrow S_2(\omega_1) \times \chi_{\omega_1}(\omega_2)$

II. Third Cumulant

intuitive result in terms
of photo-detection





theoretical prediction:
3rd cumulant of Poissonian
distribution

$$S_3(\omega,0) = e^2 I$$

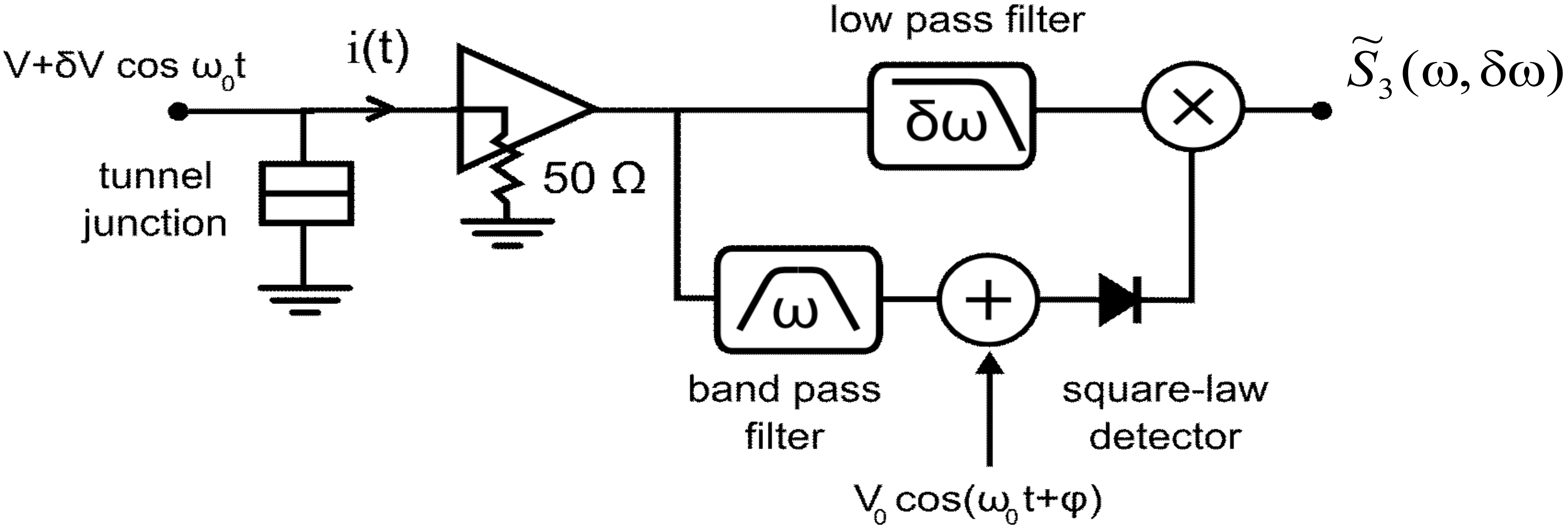
Zaikin et al. (PRB 2003)
Hekking et al. (PRB 2006)

What does a linear amplifier exactly measure ?

1. Experimental setup
2. Estimation of Third Cumulant of current fluctuations

Experimental difficulties

… we can use the setup for $\chi_\omega(\omega)$



$$\delta\omega / 2\pi \approx 100\ MHz << \omega / 2\pi \approx 6\ GHz$$

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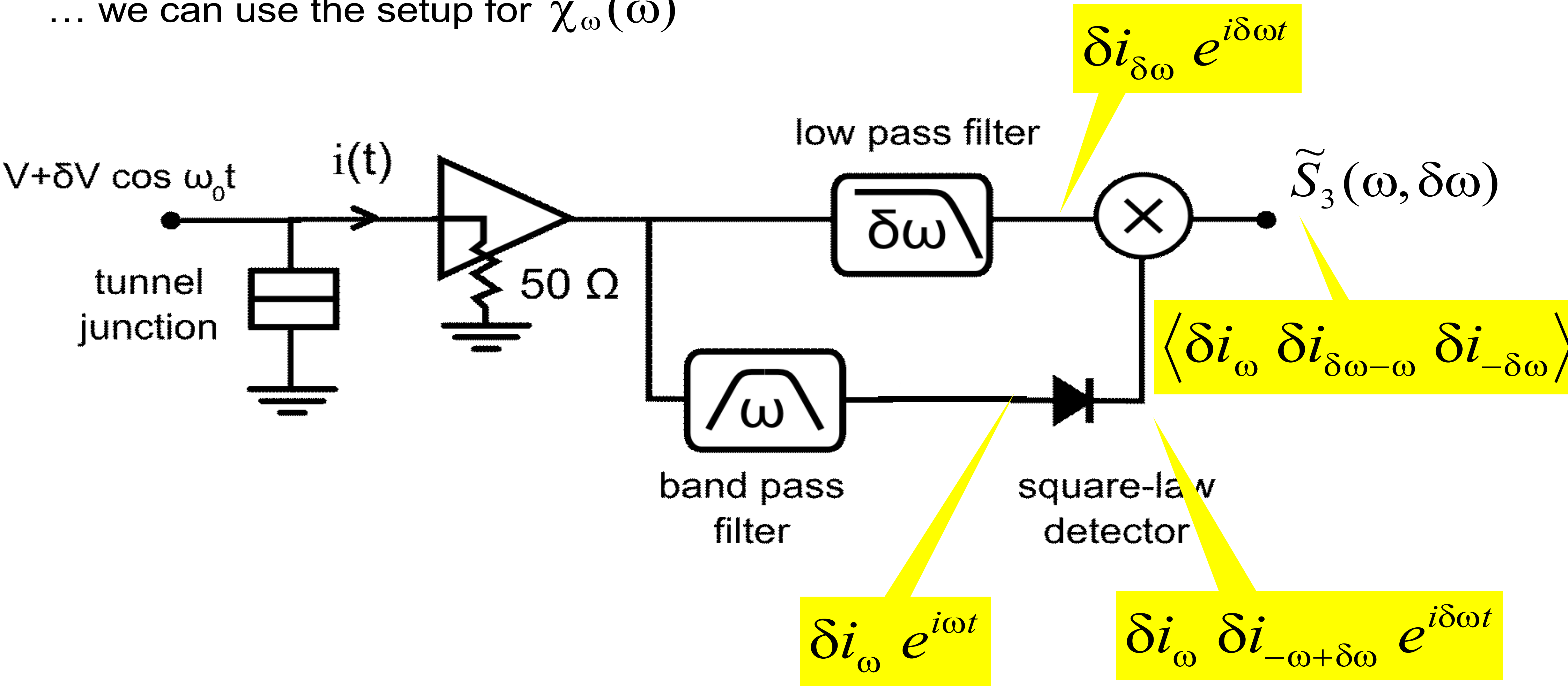

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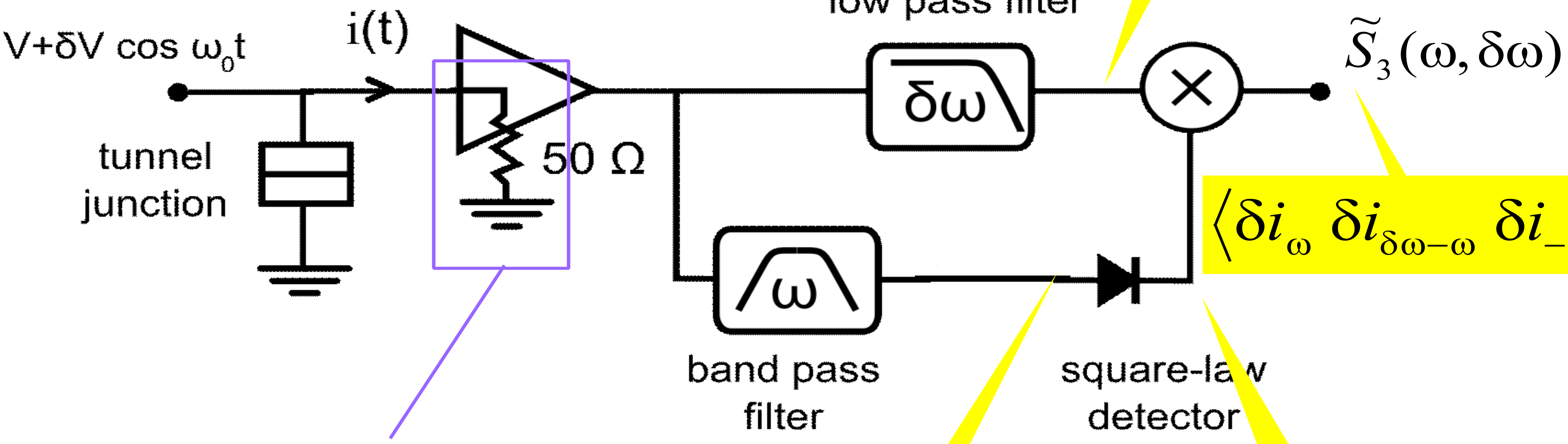


$\delta i_{\delta\omega}\ e^{i\delta\omega t}$

$\langle \delta i_\omega\ \delta i_{\delta\omega-\omega}\ \delta i_{-\delta\omega} \rangle$

$\delta i_\omega\ e^{i\omega t}$

$\delta i_\omega\ \delta i_{-\omega+\delta\omega}\ e^{i\delta\omega t}$

Environmental contribution to S_3

$$-R_{env.}\ S_2 \times \chi$$

$$\delta\omega / 2\pi \approx 100\ MHz << \omega / 2\pi \approx 6\ GHz$$

Feedback and noise susceptibility

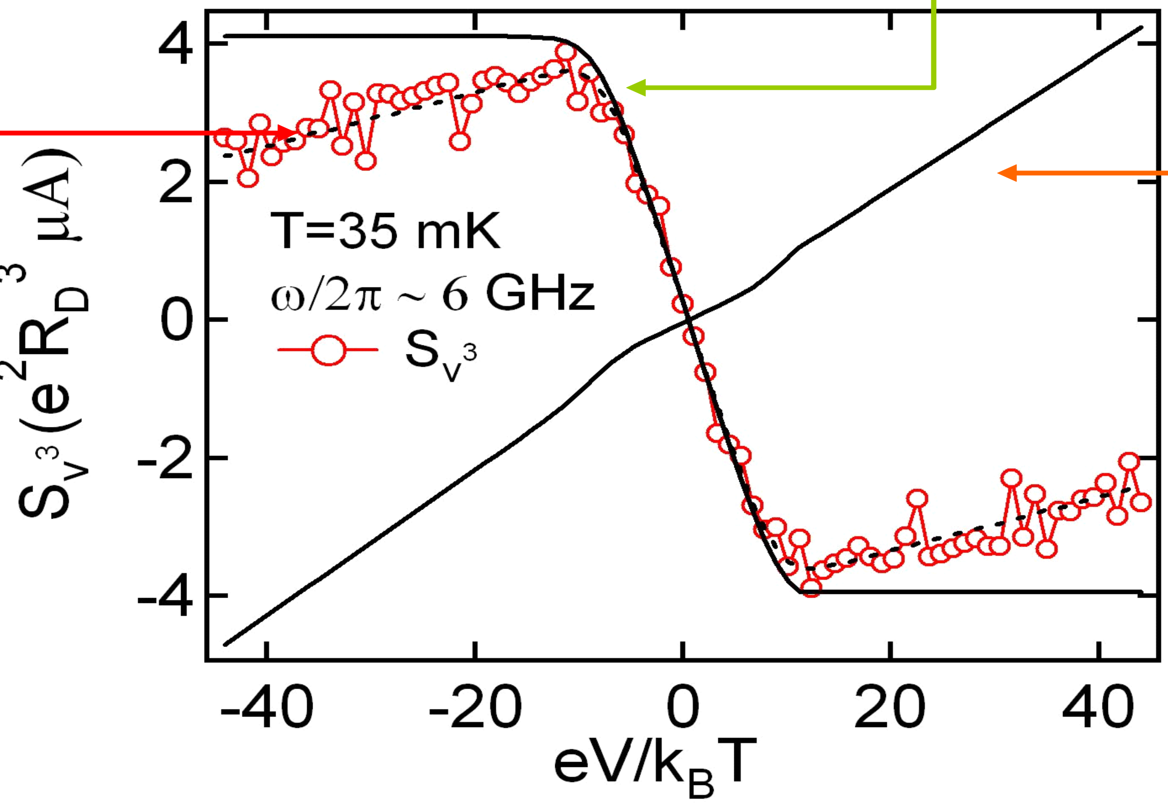
$$S_{V^3}(\omega,0) = R_{env.}^{\;3}\left(-S_3(\omega,0)+S_{2,env}(0)\times\chi_0(\omega)+S_2(\omega)\times\chi_\omega(\omega)\right.$$

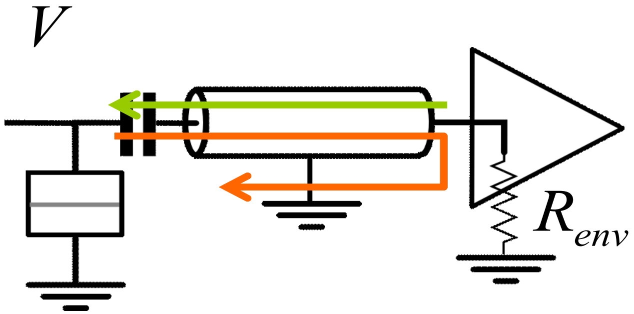
what we are looking for…

fluctuations due to noise of sample itsel

what we measure …

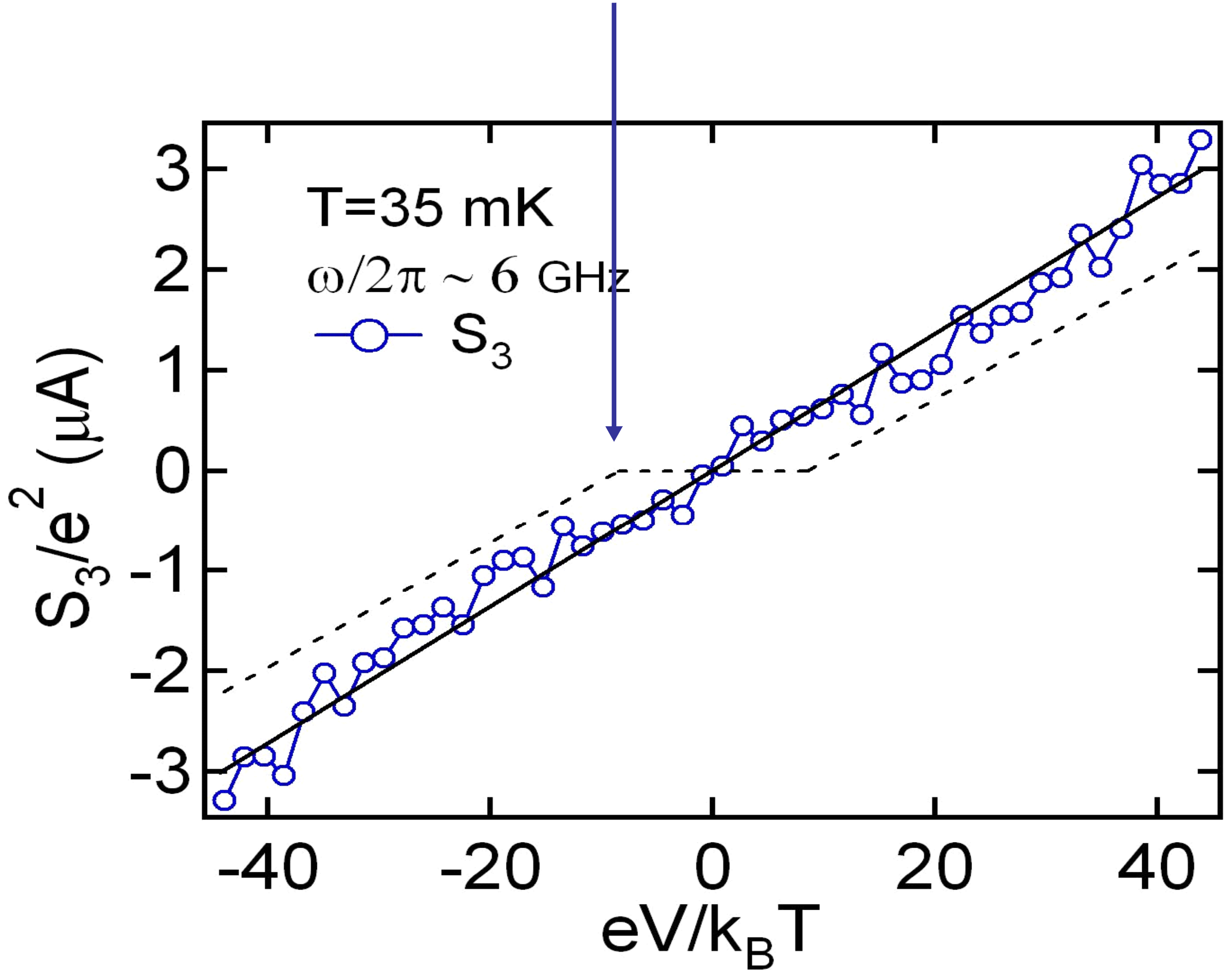
fluctuations due to noise of environment





S_3 measurement

$$S_{V^3} = Z_{//}^{\,3}\left(-\,S_3 \quad + \quad S_{2,\mathrm{env}}\times\chi \quad + \quad S_2 \times \chi\right)$$

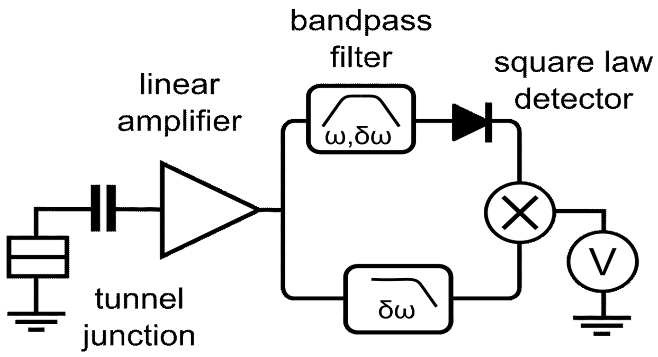


Fitting parameters in good agreement with expected values

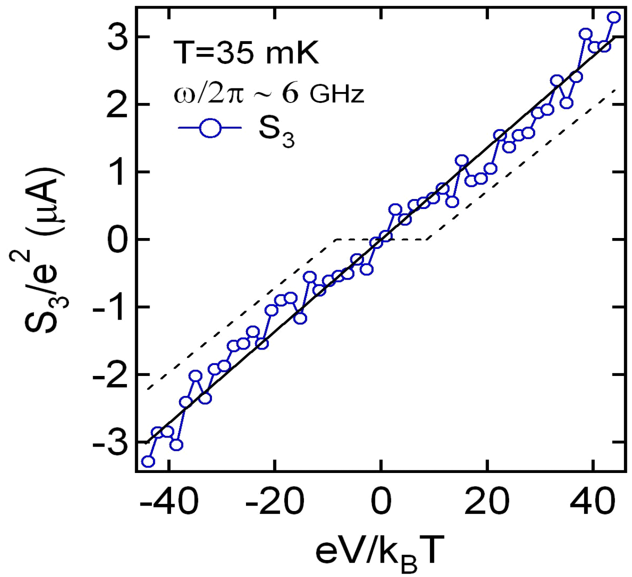
$$Z_{\mathrm{env}} = 42\,\Omega$$

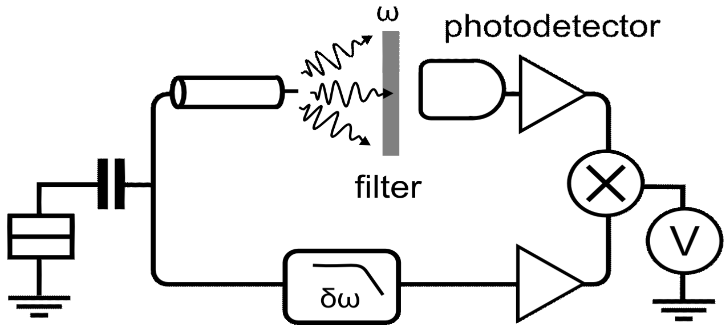
$$T^{*}_{\mathrm{env}}(\omega,0) = 3{,}6\ K$$

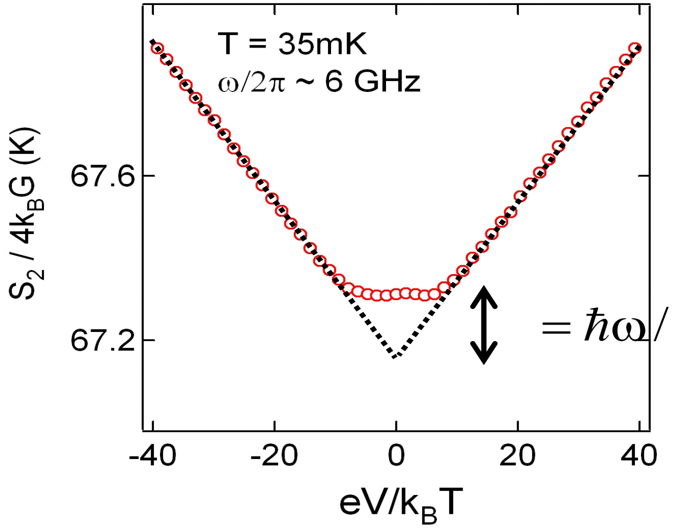
What do we measure with a linear amplifier ?



$\neq$







$= \hbar\omega / 2 \neq 0$

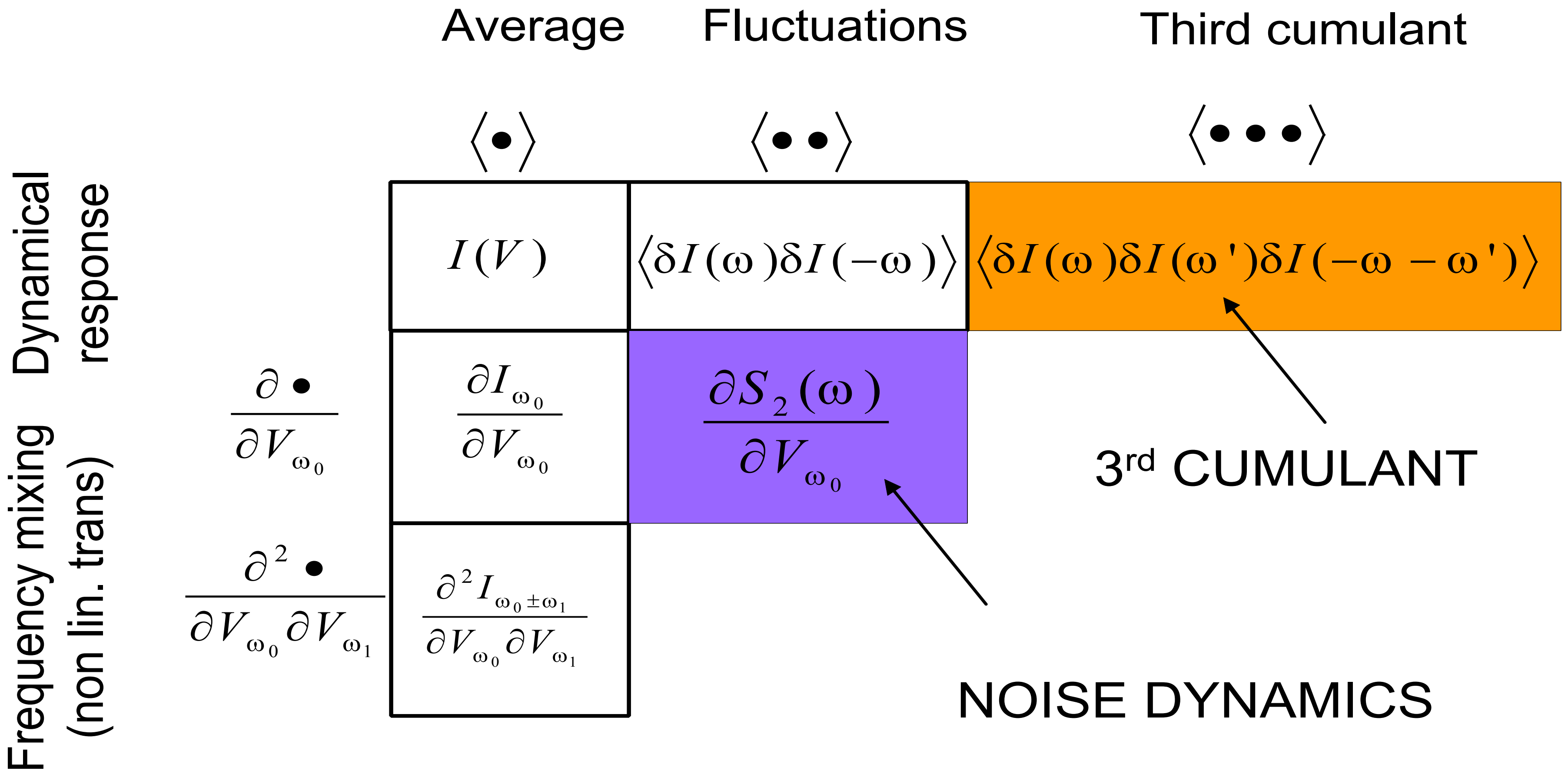
a 1st step to show that linear amplifier

Measures Zero Point Fluctuations ?

Conclusion

I. Quantum noise does not respond adiabatically for $\hbar\omega > eV$

II. $S_3(\omega,0) = e^2 I$ remains equal to classical value of poissonian process



		Average $\langle \bullet \rangle$	Fluctuations $\langle \bullet\bullet \rangle$	Third cumulant $\langle \bullet\bullet\bullet \rangle$
Dynamical response		$I(V)$	$\langle \delta I(\omega)\delta I(-\omega) \rangle$	$\langle \delta I(\omega)\delta I(\omega')\delta I(-\omega-\omega') \rangle$
Frequency mixing (non lin. trans)	$\frac{\partial \bullet}{\partial V_{\omega_0}}$	$\frac{\partial I_{\omega_0}}{\partial V_{\omega_0}}$	$\frac{\partial S_2(\omega)}{\partial V_{\omega_0}}$	
	$\frac{\partial^2 \bullet}{\partial V_{\omega_0}\partial V_{\omega_1}}$	$\frac{\partial^2 I_{\omega_0 \pm \omega_1}}{\partial V_{\omega_0}\partial V_{\omega_1}}$		

3rd CUMULANT

NOISE DYNAMICS