
Best regards from Norway!

Many-Electron Theory of Low-Frequency Noise in Hopping Insulators

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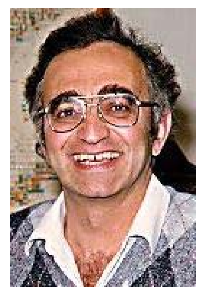

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Phys. Rev. B (2006); Phys. Rev. Lett. (2007), J. Phys. C (in press)

Noise spectrum:

$$S_I(\omega) \equiv 2\int_0^\infty d\tau\, e^{i\omega\tau}\left[\langle I(t+\tau)I(t)\rangle - \bar{I}^2\right]$$

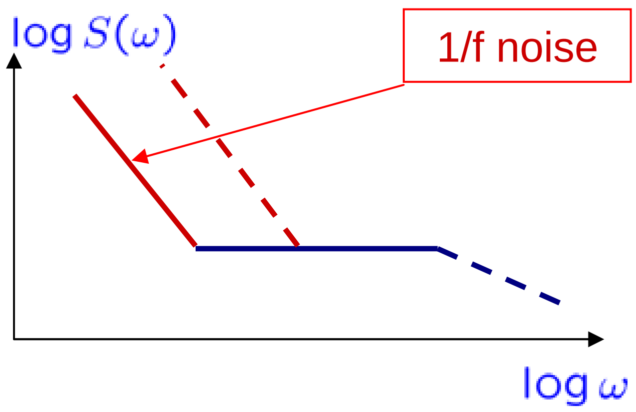
Near equilibrium - thermal noise:

$$S_I(\omega) \equiv 2\hbar\omega \coth\left(\frac{\hbar\omega}{2kT}\right)\Re G(\omega)$$

Fluctuation-dissipation theorem

Near the equilibrium the noise is voltage-independent.

It provides the same information as AC conductance (however, without AC pumping)



Properties:

$S_I(\omega) \propto I^2 \Longrightarrow$ noise of conductance G

Spectrum is rather universal – there is no saturation even at very low frequencies (down to 10-7 Hz)

History

Electronic emission in thermionic tube - Johnson (1925)

The name "flicker noise" belongs to Schottky (1926)

Carbon microphones – Christensen&Pearson (1936)

Various semiconductor devices – 1940-1950

Phenomenology

The Hooge relation:

$$S_G(\omega) \equiv \frac{\langle \delta G^2 \rangle_\omega}{G^2} = \frac{\alpha(T,\omega)}{\mathcal{N}\omega}$$

$\alpha(T,\omega)$ - dimensionless (Hooge) parameter

$\mathcal{N}$ - originally, number of charge carriers

- The Hooge parameter very weakly depends on frequency.
- Both order of magnitude and temperature dependence are non-universal. In metals α usually increases with temperature.

The concept of non-exponential kinetics

Assuming $\langle \delta I(t)\delta I(0)\rangle = \overline{(\delta I)^2} e^{-|t|/\tau}$ one obtains:

$$S_I(\omega) = 2\int_0^\infty \langle \delta I(t)\delta I(0)\rangle \cos(\omega t)\, dt = \overline{(\delta I)^2}\, \boxed{\frac{2\tau}{1+\omega^2\tau^2}}$$

Lorentzian

In more general case, $\delta I(t)$ is a superposition of many processes characterized by different relaxation times.

One can introduce a weight function, $p_I(\tau)$, including the number of the systems with given relaxation time

$$S_I(\omega) = \int_0^\infty d\tau\, p_I(\tau) \frac{2\tau}{1+\omega^2\tau^2}$$

$$p_I(\tau) \propto 1/\tau, \quad \tau_1 \ll \tau \ll \tau_2$$

$$\longrightarrow \quad S_I(\omega) \propto 1/\omega, \quad \tau_2^{-1} \ll \omega \ll \tau_1^{-1}$$

Surdin (1939)

Realization: Activated processes, $\tau = \tau_0 e^{E/kT}$, distirbution $F(E) \approx \text{const}$, $p_I(\tau) = F/(d\tau/dE) = kTF/\tau$

$$S_I(\omega) \propto T/\omega$$

Van der Ziel, 1950

Other realizations of $p_I(\tau) \propto 1/\tau$:

Tunneling between the surface layer and traps within oxide layer, McWhorter (1957)

Two-level tunneling systems in amorphous media, Kogan&Nagaev (1984)

Various dynamic defects

Typically, in materials with metallic conductance flicker noise increases with temperature.

What happens in the materials with hopping conductance?

Semiconductor Thermistors

physics/0503086

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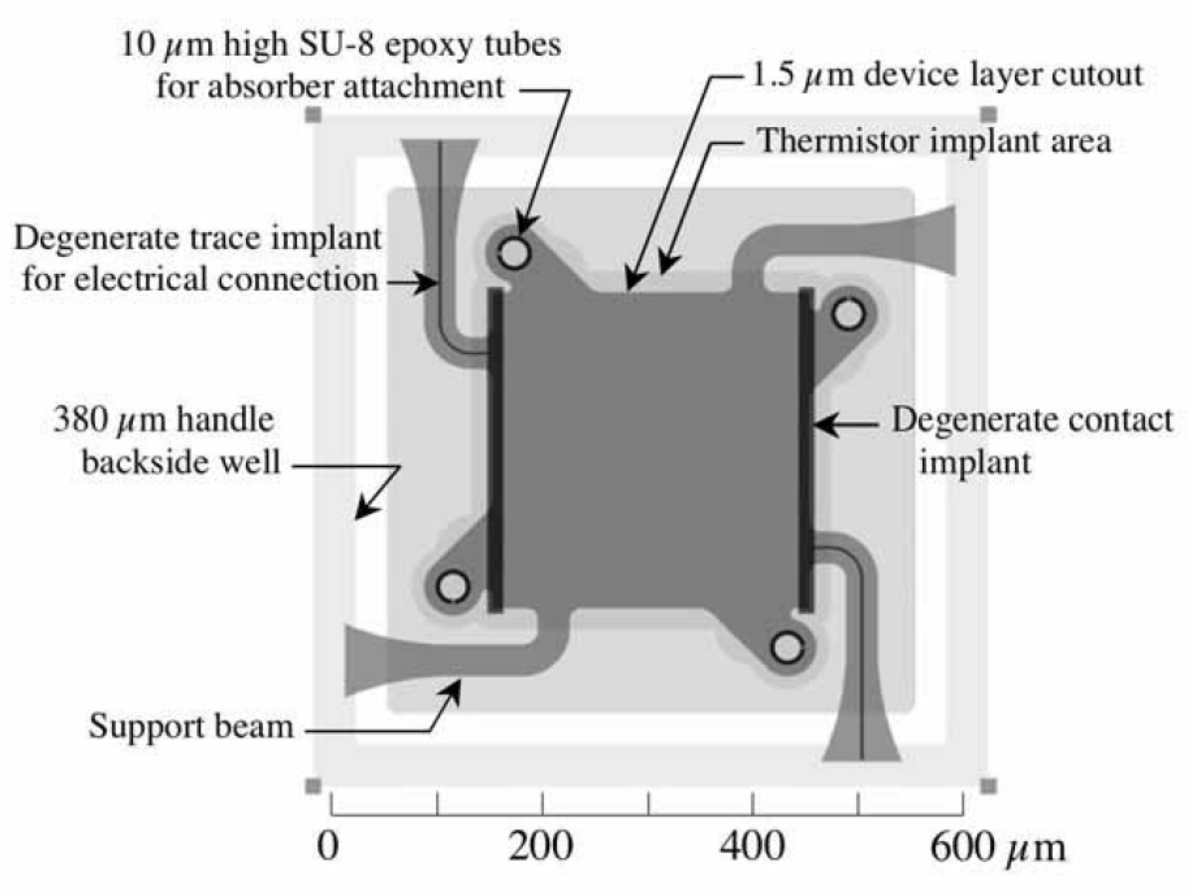
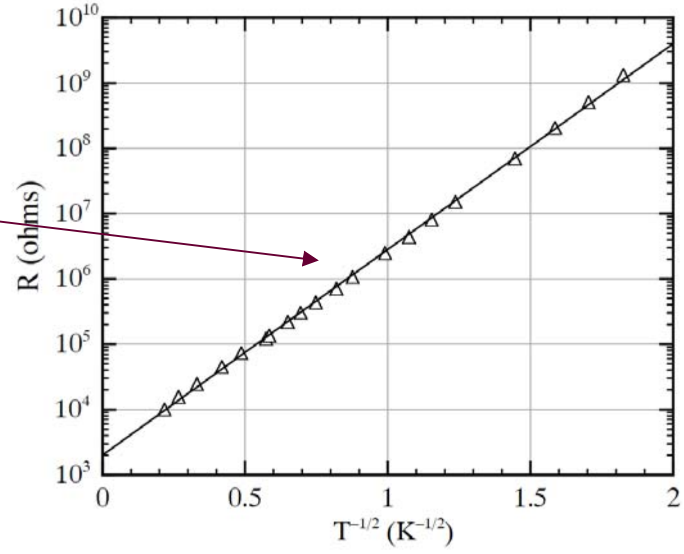


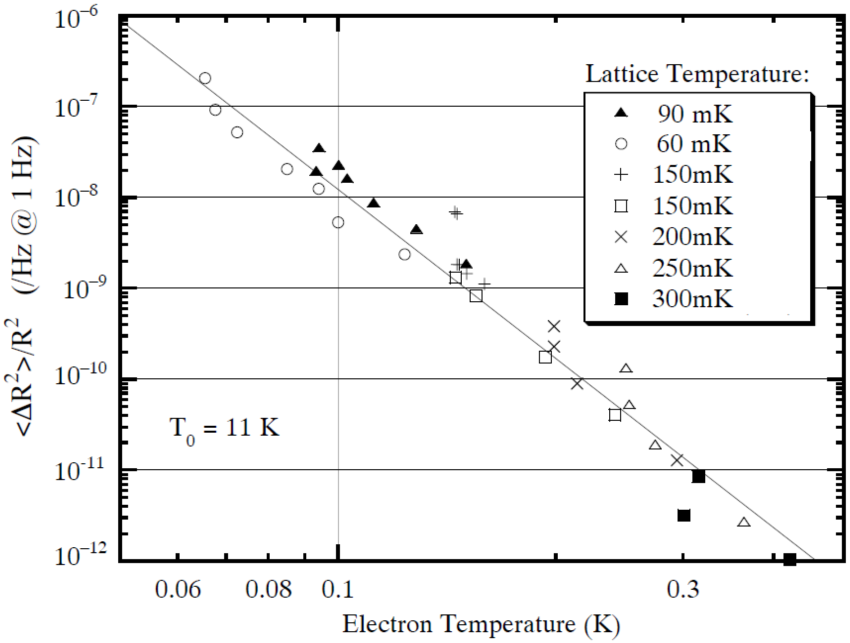
Structure of one pixel of a monolithic X-ray detector array with ion-implanted thermistors.

Ion-implanted Si

Ion implanted Si for X-ray detectors,
Efros-Shklovskii variable range hopping

$$R \propto \exp\left(\sqrt{T_0/T}\right)$$





Empirical fitting function:

$$\alpha_{\mathrm{Hooge}} = 0.034\left(\frac{T_0}{1\mathrm{K}}\right)^{2.453} \cdot \left(\frac{T_e}{0.153\mathrm{K}}\right)^{-(5.2+0.9\log_{10}(T_0/1\mathrm{K}))}$$

Fig. 17. Resistance fluctuations in a thin ion-implanted Si thermometer as a function of the electron temperature T_e for several values of T_{lattice} and bias. The fluctuations appear to depend only on T_e, which increases with bias power for a given T_{lattice}. (from [42])

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Observation of Non-Gaussian Conductance Fluctuations at Low Temperatures in Si:P(B) at the Metal-Insulator Transition

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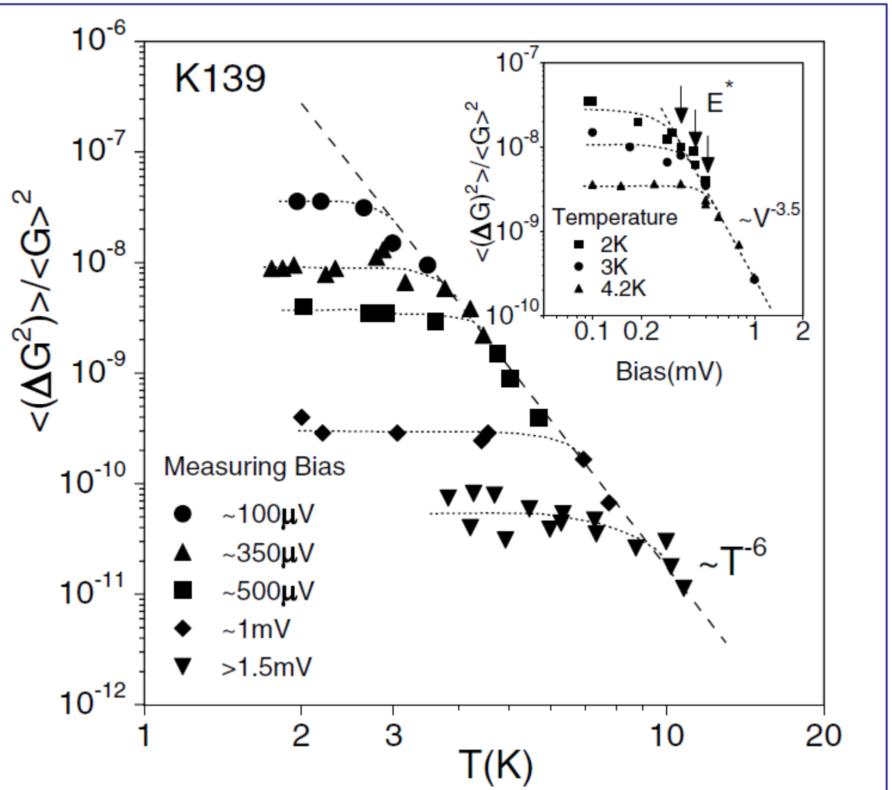
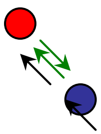


FIG. 2. $\langle \Delta G^2 \rangle / G^2$ vs T in K139 measured at different biasing voltages, showing a $T^{-\beta}$ dependence of noise at the lowest measuring bias. The inset shows how $\langle \Delta G^2 \rangle$ deviates from a G^2 dependence at larger measuring biases.

Hopping: Conductance mechanism



Hopping rate: $$\nu_{ij} = \nu_0 \exp\left(-\frac{2r_{ij}}{a} - \frac{|\varepsilon_i|+|\varepsilon_j|+|\varepsilon_i-\varepsilon_j|}{2T}\right)$$

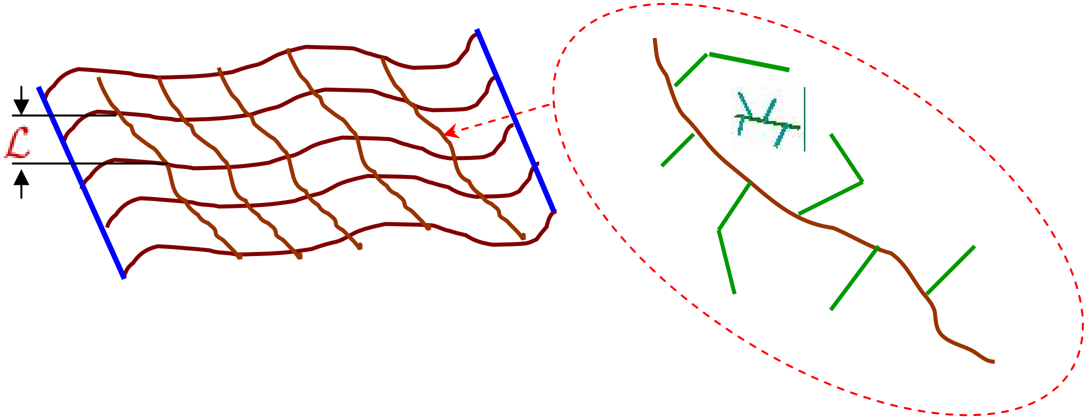
Abrahams-Miller resistor: $$R_{ij} = R_0 e^{\xi_{ij}}, \quad \xi_{ij} \equiv \frac{2r_{ij}}{a} + \frac{\epsilon_{ij}}{T}.$$

Exponentially broad distribution of resistances → percolation theory

Sites with $\xi_{ij} \leq \xi$ are considered as connected.

At some critical value, $\xi = \xi_c$, connected cluster spans the whole sample.

Conductance → percolation cluster with $\xi_c \leq \xi_{ij} \leq \xi_c + 1$



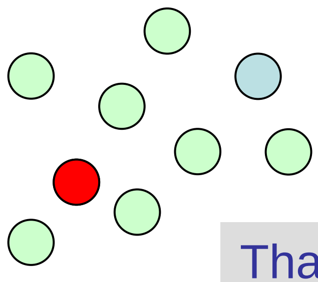
Majority of sites are not connected to the backbone cluster

However, they can randomly switch in time due to interaction with the thermal bath

Questions:

- Sources of low-frequency noise: How do they look?
- How do they influence the critical cluster, i. e., conductance?
- How does the noise spectrum look?
- How noise intensity depends on temperature?

Noise generators – hops within isolated clusters

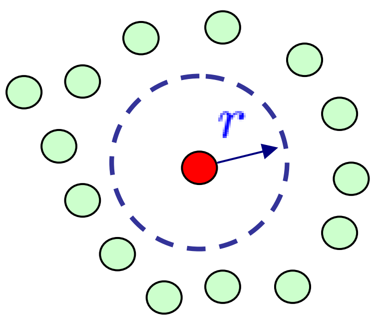


Since $\nu \propto \exp\left(-\frac{2r}{a} - \frac{\epsilon}{T}\right)$, why not consider a remote isolated pair?

Kozub,1996

That will not work, there are other sites, which can facilitate transition.

Thus the hopping rate is limited from below. Shklovskii, 2003



Only the donors surrounded by low-probable "pores" can provide very low hopping rates.

To provide the hopping rate less than ν one needs

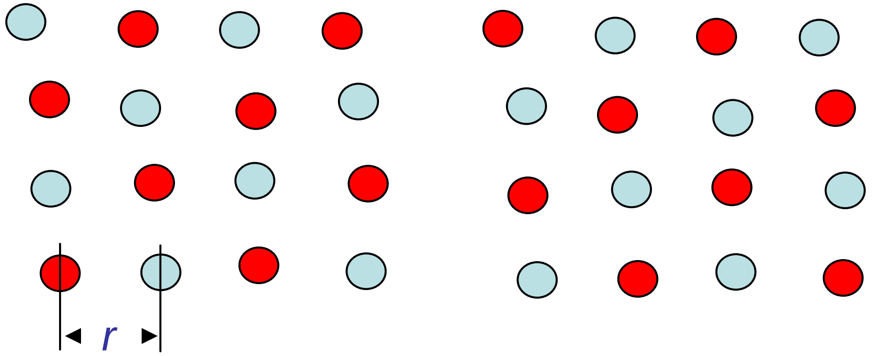
$$r > \frac{a}{2} \ln \frac{\nu_0}{\nu}, \quad \frac{\epsilon_{ij}}{T} > \ln \frac{\nu_0}{\nu}$$

This is exponentially low probable: $-\ln W(\nu) \approx \ln^6 \frac{\nu_c}{\nu}, \quad \nu_c = \nu_0 e^{-\xi_c}$

Is there any competing mechanism?

Noisy chess-board clusters

- Many-electron fluctuators, which possess small relaxation rates with much larger probability then single-electron traps.
- Formed by the rare fluctuations in positions and energies of donors.



Two metastable configurations with approximately same energies.

All donors are within the energy band of $2e^2/\kappa r$ around the Fermi level

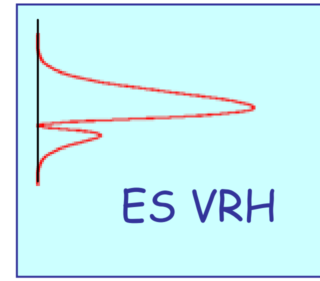
Localized electrons feel the Coulomb field produced by other donors, as well as random potential, produced by acceptors and other donors, which do not belong to the cluster.

The disorder potential, ϕ, is assumed to be relatively weak, such that the random shift of site energies is less than the Coulomb charging energy.

Probability density: Efros-Shklovskii VRH

of transition energy Δ, cell size r, and number of donors $2N$.

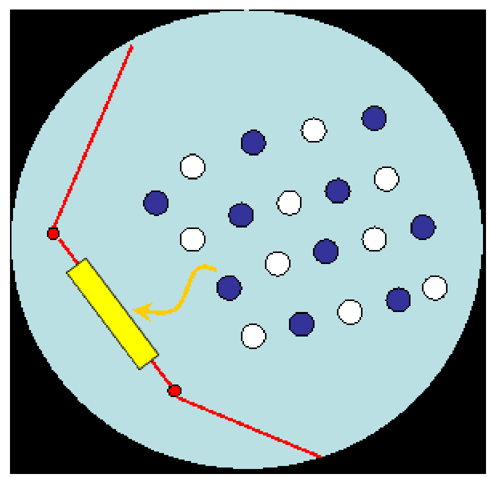
$$P(N,\Delta,r) \approx \frac{e^{-\lambda N}}{r(Nr^3)(e^2\sqrt{N}/\kappa r)}$$



Transition rate: Two competing mechanisms

$$\nu_B = \nu_0\, e^{-2Nr/a}$$

$$\nu_A = \nu_0\, e^{-N^{2/3}e^2/\kappa r T}$$



Coherent transition

Moving “domain wall”

Distribution of clusters over their relaxation rates

$$f(\nu,\Delta) = \int_0^\infty dr \sum_{N=1}^{\infty} P(N,r,\Delta) \times\delta\left[\nu - \nu_0\left(e^{-N^{2/3}e^2/\kappa r T} + e^{-2Nr/a}\right)\right]$$

Maximal contribution is given by the values of $N_c(T,\nu) \gg 1$ and $r_c(T,\nu) \gg a$ for which both mechanisms are comparable.

Queues are organized in a way to produce equal streams through doors of different widths

Result:

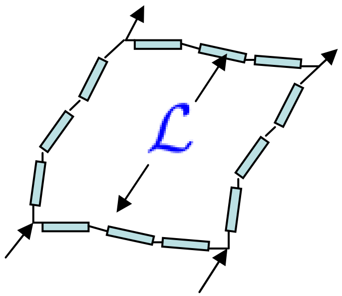
$$f(\nu,\Delta) \propto \frac{1}{\nu}\left(\frac{\nu}{\nu_0}\right)^{\beta}, \quad \beta = \frac{6\lambda}{5}\left(\frac{T}{T_0}\right)^{1/5} \ln^{1/5}\frac{\nu}{\nu_0} \ll 1$$

$\nu_{\min}$ cannot be practically distinguished from zero for temperatures of interest

Conductivity Noise

How charge fluctuations in the chess-board clusters are "read out"?

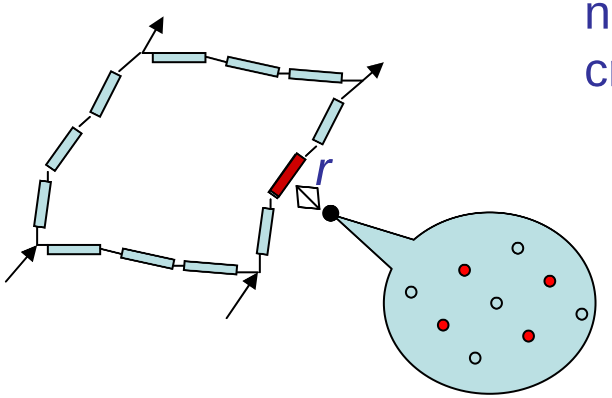
Reminder:



Backbone cluster - random lattice formed by quasi-one-dimensional links of the approximate length $\mathcal{L} \gg a$

Each chain has the largest (critical) hop, which dominates the resistance. The critical resistor has resistance $R_h = R_0 e^{(T_0/T)^{1/2}}$

Main idea (Kozub, 1996): In each cell of the backbone cluster, the conductivity noise is determined by a fluctuator adjacent to the critical resistor.



Transitions in a chess-board cluster lead to fluctuations of its dipole moment that leads to the energy fluctuations of the sites involved in the critical hop.

Estimate of noise

The transition in a chess-board cluster leads to a fluctuation f the dipole moment, and a fluctuation of the activation energy, δE, of the critical hop

The critical resistor drastically changes if $\delta E \approx T$.

In this case, the chess-board cluster removes the whole link.

Result for the Hooge parameter (in the Efros-Shklovskii regime):

$$\alpha(\omega, T) \approx n_d a^3 \left(\frac{T_0}{T}\right)^{11/4} \exp\left[-\lambda \left(\frac{T}{T_0}\right)^{3/5} \ln^{6/5} \frac{\nu_0}{\nu}\right]$$

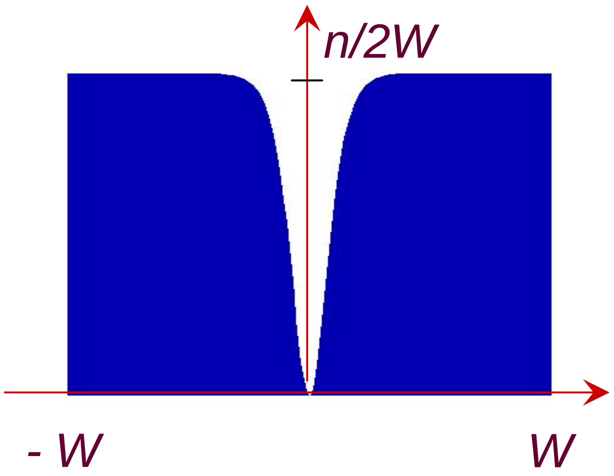
Noise strongly decreases with temperature.

Compatible with experimental data.

For the ES hopping regime, the slow fluctuators, defined by the single electron pores have much smaller density and they can be always neglected.

Chess-board aggregates are the source of 1/f-noise !

Mott hopping conductance



Relatively strong disorder – the bandwidth *W* is larger than the width of the Coulomb gap

$$W \gg e^2 n^{1/3}/\kappa$$

$$G_M = G_0\, e^{-(T_M/T)^{1/4}} \qquad \text{(3D)}$$

In the case of the Mott law single-electron hops become competitive.

Though the Coulomb interaction is irrelevant to the Mott variable range conductivity, the very low frequency 1/*f*-noise is still determined by the chessboard clusters bound by the Coulomb interaction.

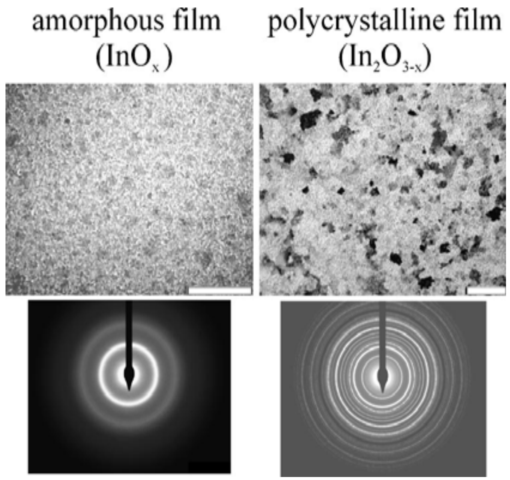
Single electron hops dominate at higher frequencies close to the frequency of Mott hops.

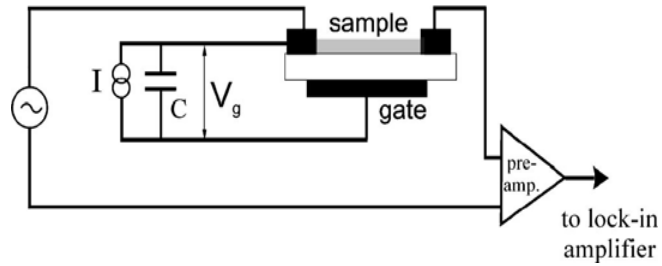
They also become more important with decreasing dimensionality because in a low-dimensional system it is easier to create a pore, but it is more difficult to create a slow chessboard cluster.

In general, the situation is rather complicated

d	T	$g(\epsilon)$	$-\ln\sigma/\sigma_0$	$T_{\mathrm{M/ES}}$	$-(1/\lambda)\ln\alpha_H$	$-\ln\alpha_H^{\mathrm{pore}}$	$\alpha_H < \alpha_H^{\mathrm{pore}}$
2	$T < \frac{T_{\mathrm{ES}}^3}{T_{\mathrm{M}}^2}$	$\frac{2}{\pi}\frac{\epsilon\kappa^2}{e^4}$	$\left(\frac{T_{\mathrm{ES}}}{T}\right)^{1/2}$	$T_{\mathrm{ES}} \sim \frac{e^2}{k_B\kappa a}$	$\left(\frac{T}{T_{\mathrm{ES}}}\right)^{2/3}\ln^{4/3}\frac{\nu_0}{\omega}$	$\left(\frac{T}{T_{\mathrm{ES}}}\right)^{2}\ln^{4}\frac{\nu_0}{\omega}$	n/a
2	$\frac{T_{\mathrm{ES}}^3}{T_{\mathrm{M}}^2} < T$	g_0	$\left(\frac{T_{\mathrm{M}}}{T}\right)^{1/3}$	$T_{\mathrm{M}} \sim \frac{1}{k_B g_0 a^2}$	$\left(\frac{T}{T_{\mathrm{ES}}}\right)^{2/3}\ln^{4/3}\frac{\nu_0}{\omega}$	$\frac{T}{T_{\mathrm{M}}}\ln^{3}\frac{\nu_0}{\omega}$	$\ln\frac{\nu_0}{\omega} < \left(\frac{T_{\mathrm{M}}^3}{TT_{\mathrm{ES}}^2}\right)^{1/5}$
3	$T < \frac{T_{\mathrm{ES}}^2}{T_{\mathrm{M}}}$	$\frac{3}{\pi}\frac{\epsilon^2\kappa^2}{e^4}$	$\left(\frac{T_{\mathrm{ES}}}{T}\right)^{1/2}$	$T_{\mathrm{ES}} \sim \frac{e^2}{k_B\kappa a}$	$\left(\frac{T}{T_{\mathrm{ES}}}\right)^{3/5}\ln^{6/5}\frac{\nu_0}{\omega}$	$\left(\frac{T}{T_{\mathrm{ES}}}\right)^{3}\ln^{6}\frac{\nu_0}{\omega}$	n/a
3	$\frac{T_{\mathrm{ES}}^2}{T_{\mathrm{M}}} < T$	g_0	$\left(\frac{T_{\mathrm{M}}}{T}\right)^{1/4}$	$T_{\mathrm{M}} \sim \frac{1}{k_B g_0 a^3}$	$\left(\frac{T}{T_{\mathrm{ES}}}\right)^{3/5}\ln^{6/5}\frac{\nu_0}{\omega}$	$\frac{T}{T_{\mathrm{M}}}\ln^{4}\frac{\nu_0}{\omega}$	$\ln\frac{\nu_0}{\omega} < \left(\frac{T_{\mathrm{M}}^{5/2}}{TT_{\mathrm{ES}}^{3/2}}\right)^{1/7}$

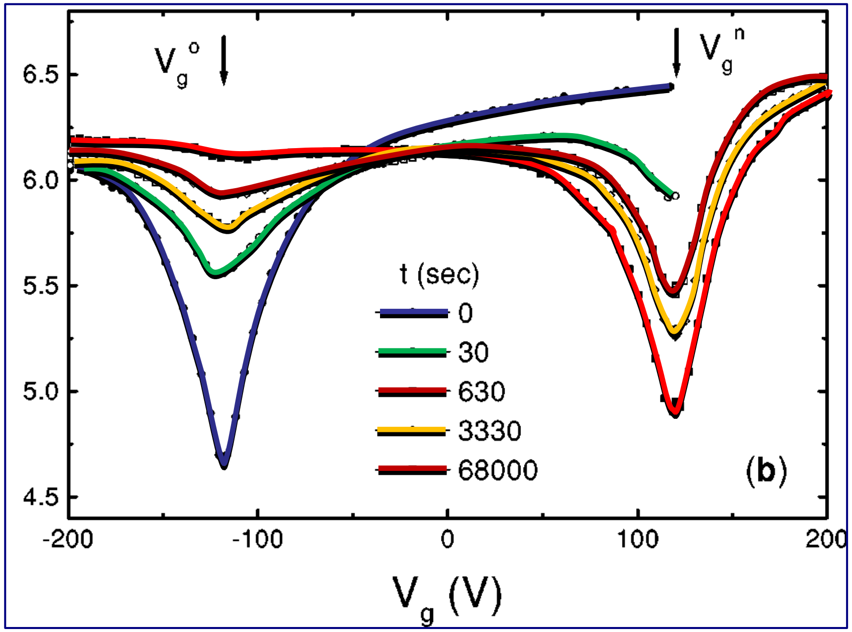
Memory effects: Double-dip experiments





Z. Ovadyahu et al.

1. The sample is allowed to equilibrate at the measuring temperature with a voltage V_g^0 held at the gate.
2. Then, a $G(V_g)$ trace is taken by sweeping V_g across a voltage range straddling V_g^0. The resulting $G(V_g)$ exhibits a minimum centered at V_g^0 which reflects an inherent feature of a hopping system—its equilibrium conductance at a local minimum.
3. At the end of this sweep, a new gate voltage, V_g^n, is applied and maintained at the gate between subsequent V_g sweeps.

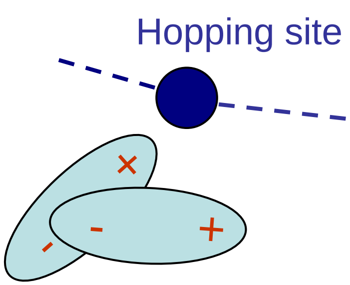


Before the experiments began, the sample was kept at the measurement temperature 4.1 K with for about 20 h

A. Vaknin, Z. Ovadyahu, M. Pollak (2002)

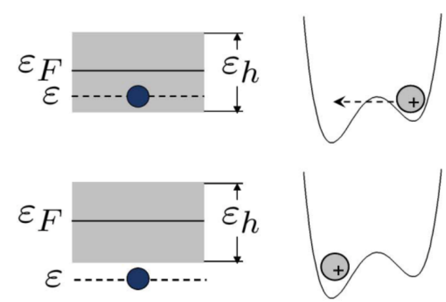
Each such sweep reveals two minima: one at V_g^0 , which fades away with time, and the other at V_g^n whose magnitude increases with time.

Our explanation



The chess-board clusters get polarized by the electrons and, in turn, form a polaron gap at the hopping sites decreasing hopping conductance.

The slow dynamics of conductance is then due to slow rearrangement of polaron clouds around the hopping sites.



Sketch of theory

The polaron gap, *U*, excludes from conductance all the states with *U* exceeding the hopping bandwidth ε_h

$$\frac{\delta G(t)}{G} \sim -\int_{\varepsilon_h}^{\infty} \mathcal{F}(U,t)\, dU \,.$$

The distribution of the polaron shifts, $\mathcal{F}(U)$, can be calculated basing on the model of chess-board aggregates

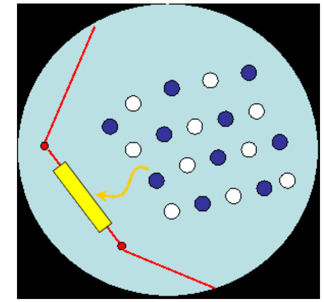
Result for the memory dip:

$$\frac{\delta G}{G} \sim -\frac{e^6 P_0\, Q(t)}{\kappa^3[(e\delta V_g)^2+\varepsilon_h^2]}\,, \quad Q(t) \equiv \int \frac{d\tau\, (\nu_0\tau)^{-\alpha}}{\tau N_c(\tau)}$$

Explains main features of experimental findings: shape, temperature dependence, time evolution.

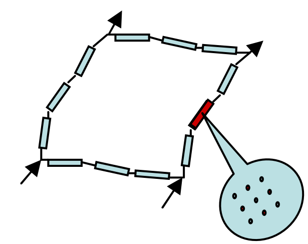
Conclusions

- A novel mechanism of the 1/*f*-noise in doped semiconductors in the hopping regime.



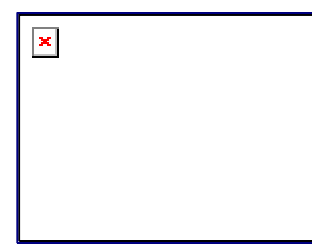
  This mechanism is associated with many-electron transitions of the chess-board clusters, the rate of which decreases exponentially with the cluster size.

  This exponential dependence results in the close to $1/\nu$ distribution of clusters over their relaxation rates,



- The slow fluctuations of cluster states modulate the critical resistors forming the backbone cluster leading to the 1/*f*-noise in the electronic conductivity.

- The model allows to understand temperature and frequency dependences of flicker noise in bolometers based on hopping conductance.



- It semi-quantitatively explains the double-dip memory effects.

Open questions and outlook

Interplay of Coulomb disorder, structural disorder, and electron-electron interaction

Fully quantitative theory of noise and memory effects

Numerical modeling of glassy behavior, its relevance to noise and memory effects (in progress)

Generalization for granular material (the problem of dielectric constant)

Thank you for attention!