# Generation of Localized Convective Flows under Parametric Disorder


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## Abstract

We study localization of thermoconvective currents in a horizontal layer under frozen parametric disorder. The phenomenon is, to some extend, similar to the Anderson localization, but here the system is an active nonlinear medium in contrast to conservative ones for classic Anderson localization. Additionally, we find an imposed advection to lead to a crucial up-advection-stream delocalization of convective currents. From the practical viewpoint, it is noteworthy, that these phenomena strongly effect the transport of a nearly indiffusive pollutant, which is also discussed.


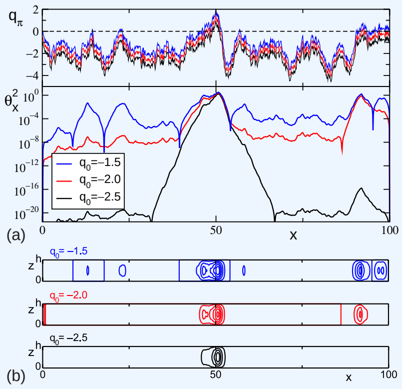


Fig. 1: (a): Establishing steady solutions to Eq.(1) at $u=0$ [$q_\pi(x)\equiv\pi^{-1}\int_{x-\pi/2}^{x+\pi/2}q(x')dx'$]. (b): The stream lines for the solutions plotted in graph (a) in the case of convection in a porous layer.

## Excitation of Localized Currents

The *large-scale natural thermal convection* between two horizontal plates is governed by a *modified Kuramoto–Sivashinsky equation*

$$\dot\theta=(-u\theta-\theta_{xxx}-q(x)\theta_x+(\theta_x)^3)_x\,,\qquad(1)$$

the temperature perturbation $\theta$ is almost uniform across the layer and depends only on the horizontal coordinate $x$; $u$ is the imposed advection velocity;

$$q(x)=q_0+\xi(x),$$

where $q_0$ is the mean supercriticality ($q_0=0$ is the convective instability threshold of the disorderless system), $\xi(x)$ is a white Gaussian noise: $\langle\xi(x)\rangle=0$, $\langle\xi(x)\xi(x')\rangle=2\delta(x-x')$. The flow strength is proportional to $\psi=\theta_x$.

For a localized flow pattern, asymptotically

$$\psi(x)\propto\begin{cases}\exp(-\gamma_-|x|), & ux>0;\\ \exp(-\gamma_+|x|), & ux<0;\end{cases}$$

$\gamma_\pm$: the *up-/downstream localization exponents*.

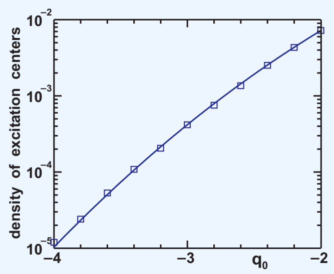


Fig. 2: The spatial density $\nu$ of excitation centers at $u=0$. The fitting line:
$\nu=(-4q_0)^{-1}(1.95\pi)^{-1/2}\exp(-1.95q_0^2/4)$.

## Localization Exponents and Nonlinear Solutions

The localization exponents are the *Lyapunov exponents* of the stochastic system

$$\theta'=\psi,\quad \psi'=\phi,\quad \phi'=-[q_0+\xi(x)]\psi-u\theta\,.\qquad(2)$$

The spectrum of Lyapunov exponents consist of three elements: $\gamma_1\ge\gamma_2\ge\gamma_3$. Due to symmetries,

$$\gamma_3(q_0,u)=-\gamma_1(q_0,-u)\,,$$

$$\gamma_2(q_0,u)=-\gamma_1(q_0,u)+\gamma_1(q_0,-u)\,.$$

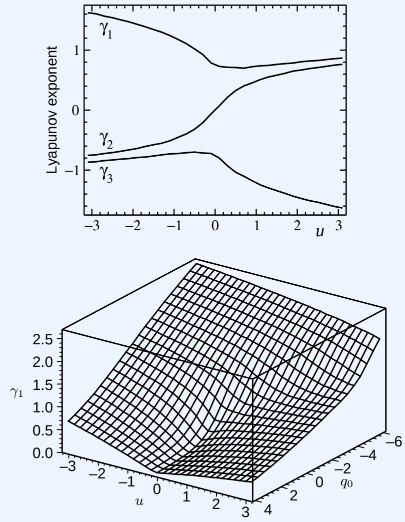


Fig. 3: Spectrum of the localization exponents (for the upper plot $q_0=-1$)

For $u>0$, up the advection stream:

$$\psi_+(x)=\gamma_2\Theta_2(x)e^{\gamma_2x}+\gamma_1\Theta_1(x)e^{\gamma_1x}\,;$$

down the advection stream:

$$\psi_-(x)=\gamma_3\Theta_3(x)e^{\gamma_3x}\,.$$

where $\Theta_i(x)\sim1$. Note, for small finite $u$, when $\gamma_2$ is also small but finite, the slowly decaying mode $\gamma_2\Theta_2(x)e^{\gamma_2x}$ may dominate in the distance from the excitation center, thus, leading to a large upstream delocalization of convective currents (Fig.4).

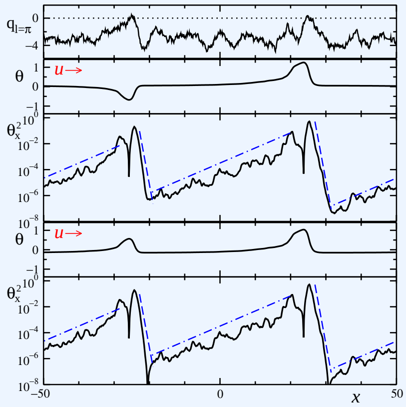


Fig. 4: Sample steady solutions to Eq.(1) at $u=0.3$, $q_0=-3.0$. Dashed lines: decay with exponent $\gamma_3=1.70$; dashdot lines: $\gamma_2=0.134$.

## Transport of a Nearly Indiffusive Pollutant

For a *steady flow*, the *establishing pollutant flux* $J$ in an extensive domain of the length $L$ with the imposed concentration difference $\delta C$ at the ends is

$$J=-\sigma\frac{\delta C}{L}\,.$$

For the case of convection in a porous layer, the *effective diffusivity*

$$\sigma=\left\langle\left(D+\frac{2\psi^2(x)}{21D}\right)^{-1}\right\rangle^{-1}$$

(here $D$ is the molecular diffusivity).

For small $D$ and sparse excitation centers ($\nu\ll1$), the effective diffusivity may be evaluated analytically,

$$\sigma=D\left(\frac{2}{D}\right)^{\frac{2\nu}{\gamma}}$$

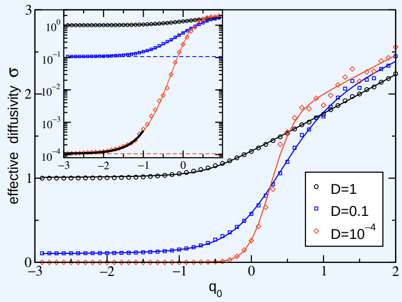


Fig. 5: Dependencies of effective diffusivity $\sigma$ on mean supercriticality $q_0$ for molecular diffusivity $D$ indicated in the plot. The bold black line in the inner plot represents the result of the analytical theory.

## Conclusions

**— In the presence of parametric disorder, spatially localized flow patterns are excited below the instability threshold of the disorderless system.**

**— An weak imposed longitudinal advection leads to a crucial upstream delocalization of convective currents (the localization length may be increased by one order of magnitude). Moderately below the instability threshold of the disorderless system, such delocalization may lead to the transition from a set of localized flow patches to an almost everywhere intense "global" flow.**

**— The advection results in localization of temperature perturbations whereas in the absence of the advection, only currents are localized.**

**— Bellow the instability threshold of the disorderless system, localized convective currents crucially enhance the diffusion of a nearly indiffusive pollutant. Additionally, the effective diffusivity allows to spectate the transition to a "global" flow.**


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## References


[1] D.S. Goldobin and E.V. Shklyaeva, *Localization and advectional spreading of convective flows under parametric disorder*, Phys. Rev. E *submitted* (2008). [preview: arXiv:0804.3741]

[2] D.S. Goldobin and E.V. Shklyaeva, *Diffusion of a passive scalar by convective flows under parametric disorder*, J. Stat. Mech. *submitted* (2008). [preview: arXiv:0805.1518]