Temporal avalanche correlations in crackling noise

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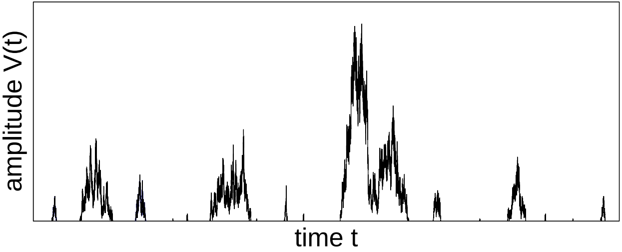
Unsolved Problems on Noise 2008

Crackling Noise

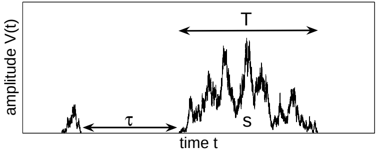
- Many slowly driven interacting systems (Barkhausen noise, plastic deformation...) exhibit “crackling noise” - an intermittent global activity signal $V(t)$:



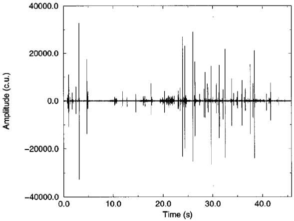
- *Avalanche* sizes $s = \int_0^T V(t)dt$ (and durations T etc.) follow power law distributions, $P(s) = s^{-\tau_s} f_c(s/s_0)$.

Crackling Noise - Temporal Correlations

- Short times - structure of individual avalanches:



- Longer times - correlations between different events:

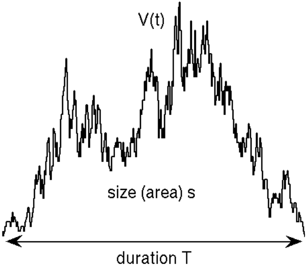


P. A. Houle and J. P. Sethna, PRE **54**, 278 (1996).

Crackling Noise - Short Time Correlations

- A simple relation between avalanche ($\langle s(T) \rangle \sim T^{\gamma_{st}}$) and power spectrum scaling (originally for Barkhausen noise):

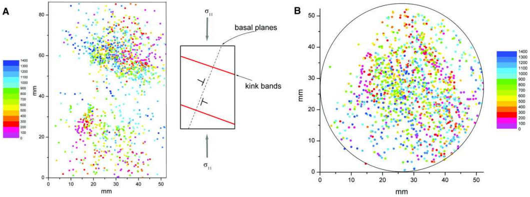
$$S(f) \sim f^{-\gamma_{st}}. \quad (1)$$



- Appears to be valid for a number of systems.

Crackling Noise - Long Time Correlations

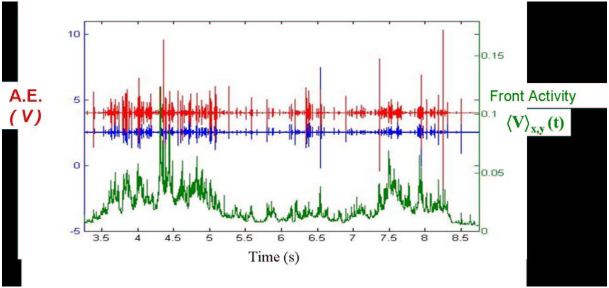
- Temporal event clustering observed in a number of systems:
  - Earthquakes - mainshocks followed by a sequence of aftershocks obeying Omori's law.
  - Acoustic emission events in plastic deformation and fracture of various materials.
- Temporal clustering usually connected to spatial clustering:



J. Weiss and D. Marsan, Science **299**, 89 (2003).

Crackling Noise - Long Time Correlations

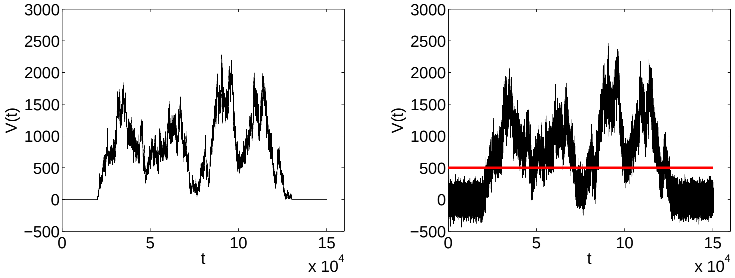
- To study correlations between different avalanches, one should first define an avalanche.
- This is not always easy:



Acoustic emission and front velocity from peeling of plexiglass plates.
From S. Santucci (Oslo).

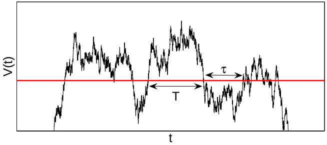
Crackling Noise - The Effect of Thresholding

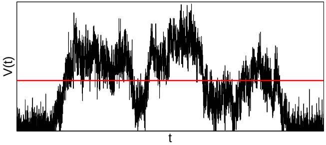
- In a typical experiment, a finite detection threshold is used (due to noise etc.).
- Thus, avalanches can be broken into apparently distinct “subavalanches”:



The Random Walk Model

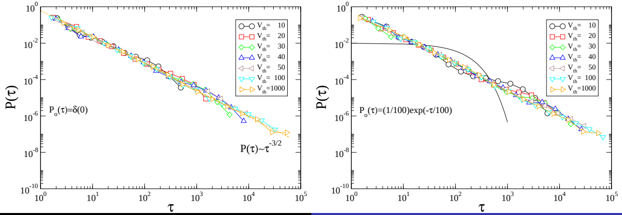
- Model the $V(t)$-signal of a single avalanche with an excursion of a random walk from the origin.
- Insert exponentially distributed waiting times τ between such "avalanches".
- Study the effect of a finite threshold level V_{th} on the observed waiting time (or quiet time) statistics, with or without additive white noise:





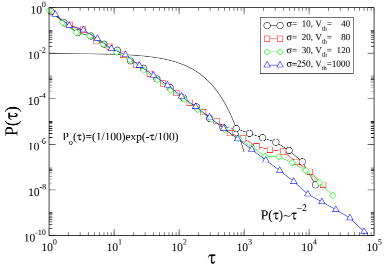
The Random Walk Model - Without Noise

- The durations T of such excursions are power law distributed,
$$P(T) \sim T^{-3/2}. \tag{2}$$
- Due to the Markovian nature of the process, the same applies also to the thresholded durations.
- Translation invariance implies that the "internal" waiting (quiet) times τ have the same distribution, $P(\tau) \sim \tau^{-3/2}$, with a cut-off due to a finite V_{th}:



The Random Walk Model - With Noise

- An experimentalist chooses the threshold level to be just above the noise level to keep maximum amount of information.
- To model this, add Gaussian white noise with standard deviation σ and use threshold $V_{th} = 4\sigma$:



- The exponent appears to change, $\tau_w \approx 2.0$.

The Manna Sandpile Model

- To see if such ideas can be extended to more realistic avalanche signals, we consider the Manna sandpile model of self-organized criticality.
- A 2d square lattice with an integer z_i assigned to each site.
- If $z_i \geq z_c = 2$, a *toppling* occurs: $z_i \rightarrow z_i - 2$ and two randomly chosen nearest neighbours receive a “grain”.
- Parallel dynamics, grains can leave the system through the open boundaries, slow drive.
- Take $V(t)$ to be the number of topplings as a function of time (time unit $\sim$ one parallel update of the lattice).

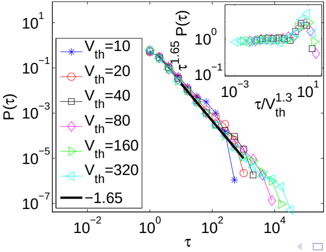
The Manna Sandpile Model - Waiting Times

- The avalanche statistics can be characterized by power law distributions, with the avalanche durations obeying

$$P(T) \sim T^{-\tau_T}, \qquad (3)$$

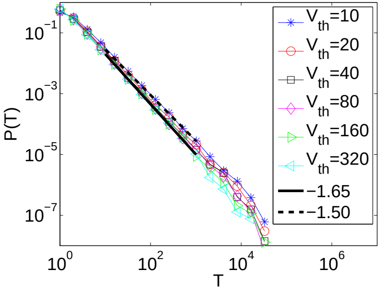
with $\tau_T \approx 1.51$.

- Upon thresholding, the observed waiting times become power-law distributed, $P(\tau) \sim \tau^{-\tau_W}$, with $\tau_W \approx 1.65$:



The Manna Sandpile Model - Avalanche Durations

- The duration distribution exponent evolves from $\tau_T \approx 1.5$ to $\tau_T \approx 1.65$ as V_{th} is increased:



- Similar observations have been made in the deterministic BTW model [M. Paczuski, S. Boettcher, and M. Baiesi, PRL **95**, 181102 (2005)].

Conclusions

- The high frequency power spectra of a wide class of avalanching systems can be understood ($S(f) \sim f^{-\gamma_{st}}$).
- Understanding longer time correlations - clustering of avalanches - is more difficult.
- A finite detection threshold could explain the observations in some cases, such as for solar flares.
- A simple random walk model illustrates how power law distributed waiting times between avalanches can arise from thresholding - the effect of noise remains to be explained ($\tau_w \approx 2.0$?).
- A similar symmetry between waiting times and avalanche durations is observed in the Manna model for high thresholds - the duration exponent changes from the non-thresholded case.