Order and disorder in mesorings
Transmission of magnetic signals

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Lyon, 02/06/2008

Outline

Mesoscopic systems

- Cyllindrical symmetry
  – rings
  – torii
  – cyllinders
- Classical and Quantum features
  $T = 0$ pure quantum case
  $T \neq 0$ quantum – cross over – classical
- Known (adressed) problems
  – equilibrium states (single/many nodes)
  – coupling to environment (week, strong)
  – interactions among rings
  – persistent currents (in theory: too small)

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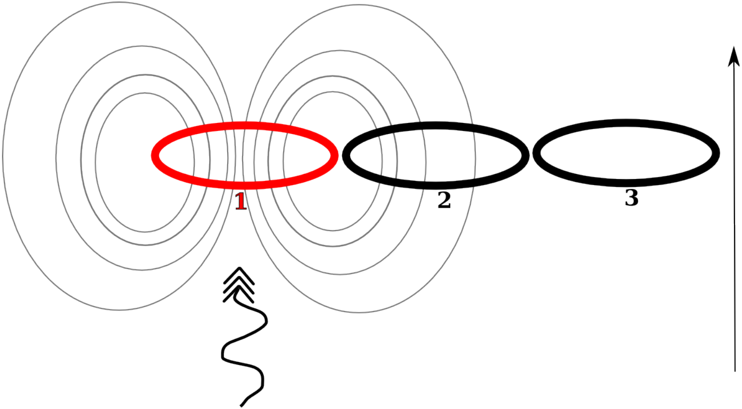
  – persistent currents (in experiment: too large)

Driven Mesorings

The flux–curent coupling



$$\phi_i + \delta_{i1}\phi_{ext}(t) + \phi_c = \sum_{k=1}^{3} \mathcal{M}_{ik} I_k, \quad i = 1, \ldots, 3$$

Evolution Equations

- Ohmic and coherent components

$$I_k = I^{nor}(\phi_k) + I^{coh}(\phi_k)$$

- Dissipative part

$$I^{nor}(\phi_k) = -\frac{1}{R}\frac{d\phi_k}{dt} + \sqrt{\frac{2k_B T}{R}}\Gamma_k(t)$$

- Persistent current

$$I^{coh}(\phi_k) = I_0 \sum_{n=1}^{\infty} A_n(T) \left[\sin(2\pi n \frac{\phi}{\phi_0}) + \sin(2\pi n(\frac{\phi}{\phi_0} + \frac{1}{2}))\right]$$

$$A_n(T) = \frac{4T}{\pi T*} \frac{\exp(-nT/T*)}{1 - \exp(-2nT/T*)} \cos(nk_F l_x)$$

Evolution Equations

- Evolution equations

$$\frac{1}{R}\sum_{k=1}^{3}\mathcal{M}_{ik}\frac{d\phi_k}{dt} = -(\phi_i + \delta_{1i}\phi_{ext}(t) + \phi_c) + \sum_{k=1}^{3}\mathcal{M}_{ik}I^{coh}(\phi_k, T) + \sqrt{\frac{2k_BT}{R}}\sum_{k=1}^{3}\mathcal{M}_{ik}\Gamma_k$$

- Langevin equations (after inversion)

$$\frac{1}{R}\frac{d\phi_n}{dt} = -\sum_{i=1}^{3}(\mathcal{M}^{-1})_{ni}(\phi_i + \delta_{1i}\phi_{ext}(t) + \phi_c) + I^{coh}(\phi_n, T) + \sqrt{\frac{2k_BT}{R}}\Gamma_n$$

- Dimensionless form

$$\frac{dx_n}{ds} = -V'(x_n, T) - \sum_{i=1(\neq n)}^{3}\lambda_{ni}x_i + \sqrt{2D}\xi_n(s)$$

$$x_n = \frac{\phi_n}{\phi_0}, \qquad s = \frac{t}{\tau}, \qquad \tau = \frac{\mathcal{L}}{R}$$

Generalized potential

- Effective potential

$$V(x_n,T) = \frac{1}{2}a_n x_n^2 - b_n x_n - I_0 \int f(y,T)dy$$
$$f(y,T) = \frac{1}{2}\left(\sin(2\pi n\frac{\phi}{\phi_0}) + \sin(2\pi n(\frac{\phi}{\phi_0}+\frac{1}{2}))\right)$$

- Coupling constants and diagonal elements

$$\lambda_{ni} = \mathcal{L}(\mathcal{M}^{-1})_{ni}, \qquad a_n = \mathcal{L}(\mathcal{M}^{-1})_{nn}$$

- Externally induced fluxes

$$b_n = \frac{\phi_{ext}(t)+\phi_c}{\phi_0}\mathcal{L}\sum_{i=1}^{3}(\mathcal{M}^{-1})_{ni}$$
$$\phi_{ext}(t) = a\sin^2(\omega t), \qquad \phi_c = 0.1$$

- Mutual inductances $\mathcal{M}_{ik} = -f(r_{ik})$

$$f(r) = \frac{8\pi R}{b(r)}\left[\left(1-\frac{b^2(r)}{2}\right)K(b(r)) - E(b(r))\right], \quad b(r) = \frac{4R^2}{4R^2+r^2}$$

Time Lag

Signal:

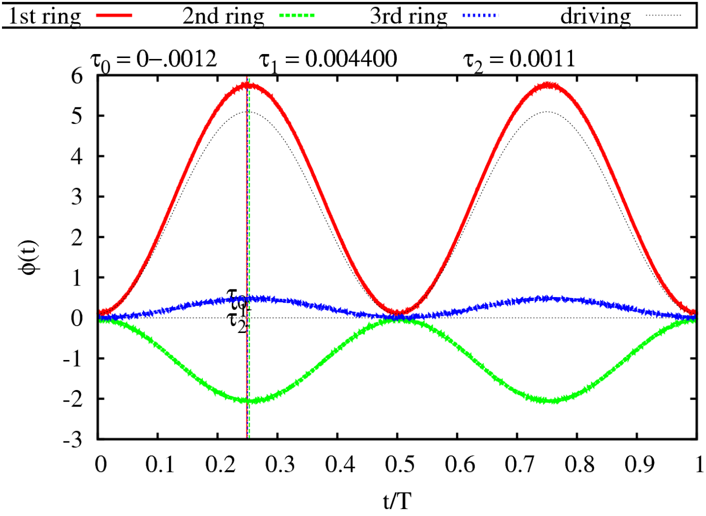
$$F_{ext}(t) = a\sin^2(\omega t) + 0.1$$

Time delay between

- the external signal and the responce of 1^{st} ring: τ_0
- the signal from 1^{st} ring and responce of 2^{nd} ring: τ_1
- the signal from 1^{st} ring and responce of 3^{nd} ring: τ_2

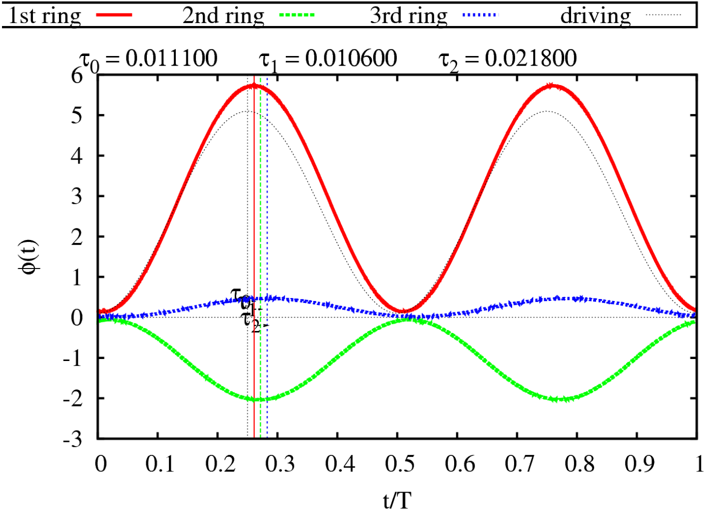
Time Lag

no delay, $\omega = 0.01$, $a = 5$, $T_0 = 0.1$



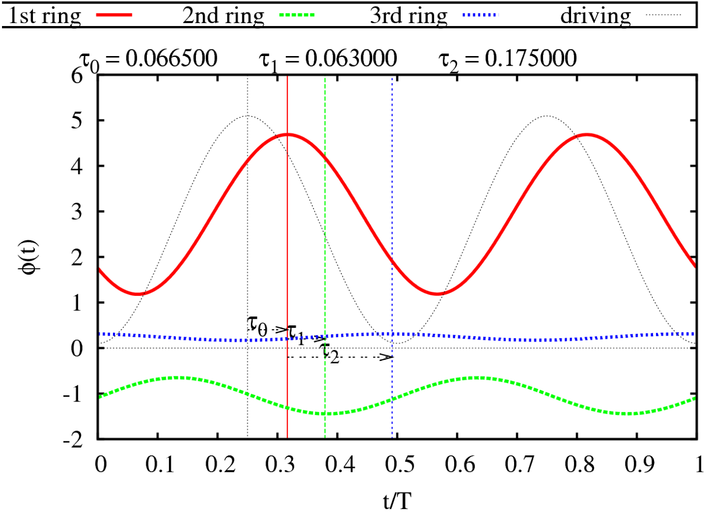
Time Lag

small delay, $\omega = 0.05$, $a = 5$, $T_0 = 0.1$



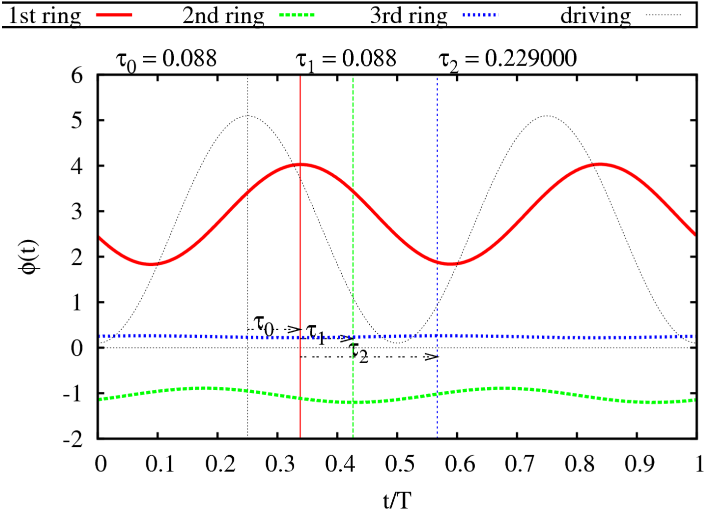
Time Lag

larger delay, $\omega = 0.5$, $a = 5$, $T_0 = 0.1$



Time Lag

large delay, $\omega = 1$, $a = 5$, $T_0 = 0.1$

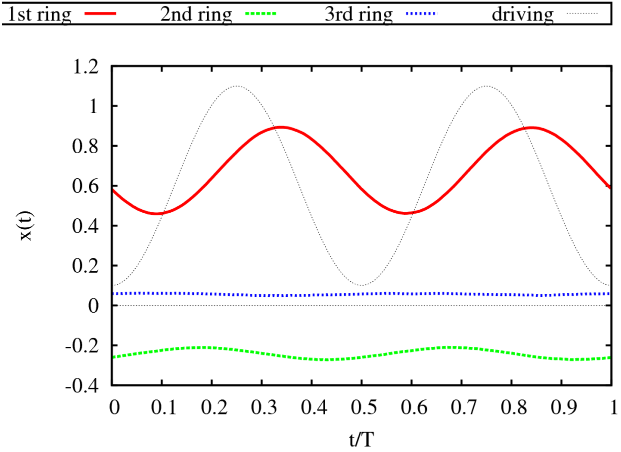


Stochastic Resonance

$\omega = 1,\ a = 1,\ T_0 = 0.1$

- Amplification factor

$$A_f(n) = \frac{a_n}{a}, \qquad A_1 = 0.434256, \quad A_2 = 0.062391, \quad A_3 = 0.012273$$

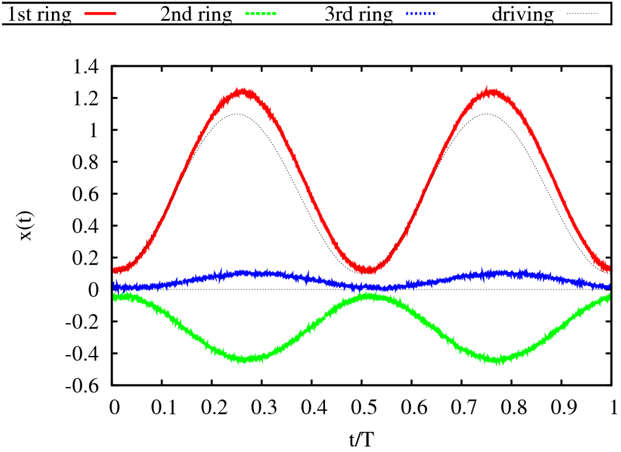


Stochastic Resonance

$\omega = 0.05$, $a = 1$, $T_0 = 0.1$

- Amplification factor

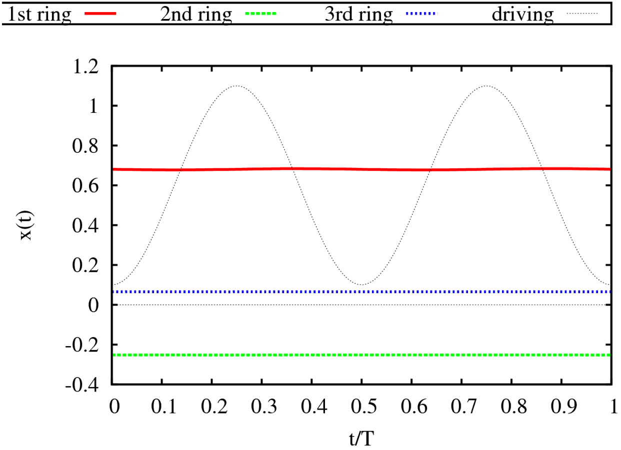
$$A_f(n) = \frac{a_n}{a}, \qquad A_1 = 1.126887, \quad A_2 = 0.403562, \quad A_3 = 0.101770$$



Flux Rectifier

$\omega = 80,\ a = 1,\ T_0 = 0.1$

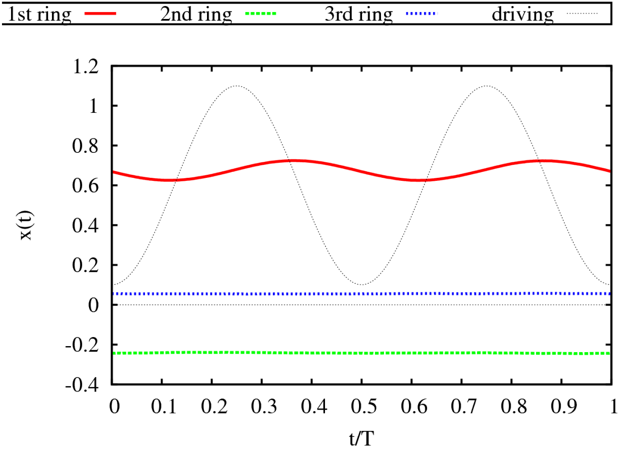
- Magnetic flux rectifier



Flux Rectifier

$\omega = 5,\ a = 1,\ T_0 = 0.1$

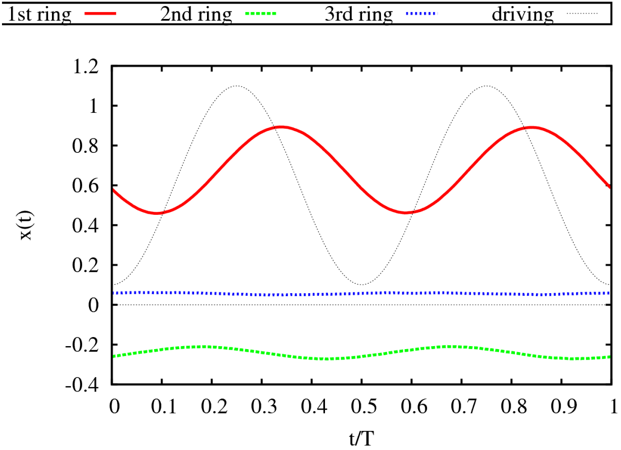
- Magnetic flux rectifier



Flux Rectifier

$\omega = 0.08$, $a = 1$, $T_0 = 0.1$

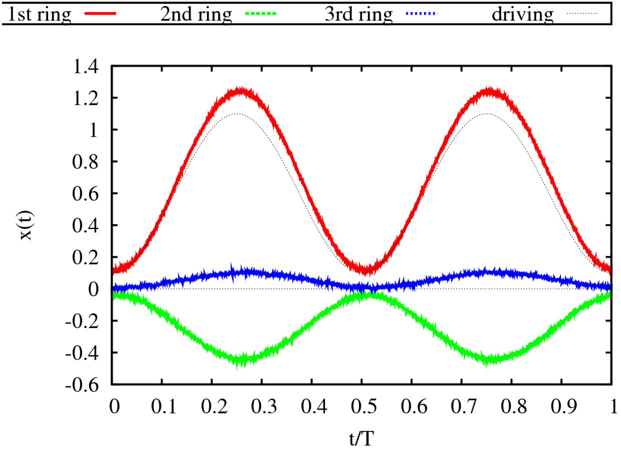
- Magnetic flux rectifier



Flux Rectifier

$\omega = 0.01$, $a = 1$, $T_0 = 0.1$

- Magnetic flux rectifier



Take home messages

Transmission of magnetic signals in mesoscopic sings

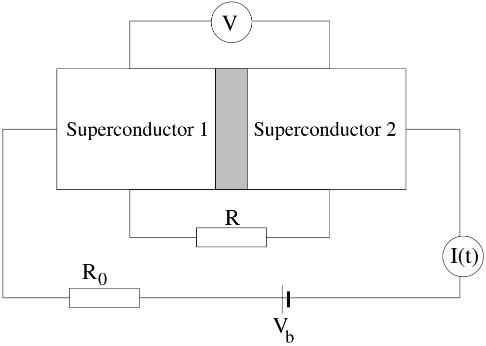
- Stochastic resonance
- Time lag due to fast driving
- Rectification of slow signals

- Phase transitions in 1D?
- Rectification of very slow signals (more rings)?
- Influence of quantum corrections on SR?

- J. Łuczka, J. Dajka, M. Mierzejewski and P. Hänggi, Acta Phys. Pol. B **37**, p1605 (2006).
- J. Łuczka, J. Dajka, M. Mierzejewski and P. Hänggi, AIP, UPoN, p197 (2005).

$$\left(\frac{\hbar}{2e}\right)^2 C\ddot{\phi} + \left(\frac{\hbar}{2e}\right)^2 \frac{1}{R}\dot{\phi} = -\frac{\hbar}{2e} I_0 \sin(\phi) + \frac{\hbar}{2e} I(t) + \frac{\hbar}{2e}\sqrt{\frac{2kT}{R}}\xi(t)$$

- $I(t) = I_d + I_a \cos(\Omega t)$
- $\mathbb{V} = (\hbar/2e)\dot{\phi}$



$$\text{classical limitation}: \quad \hbar\omega_0 \ll E_J, \quad \hbar\omega_0 \ll k_B T$$