STOCHASTIC DYNAMICS OF A JOSEPHSON JUNCTION THRESHOLD DETECTOR

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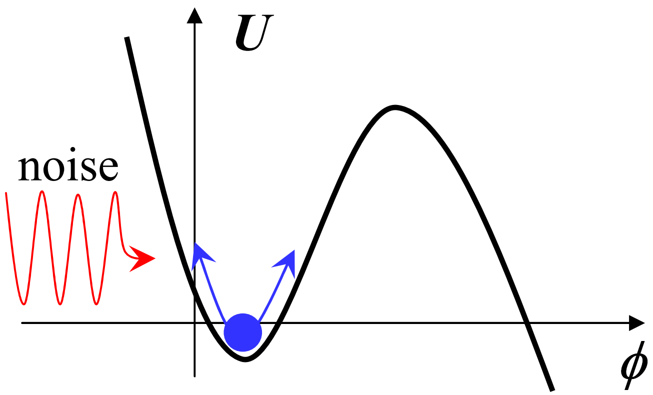
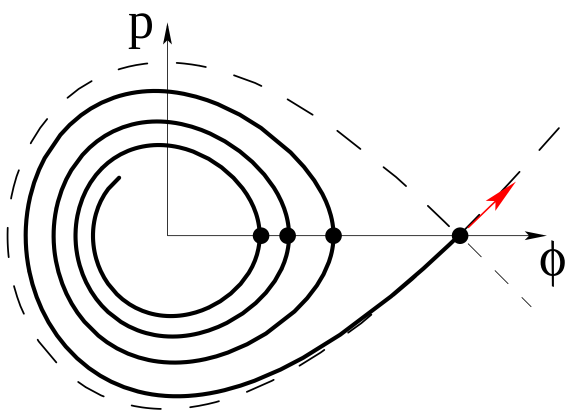




Reference:

Phys. Rev. Lett. **98**, 136803 (2007)

OUTLINE

1. **Stochastic Path Integral formalism:**
   replaces Fokker-Planck and master equations.

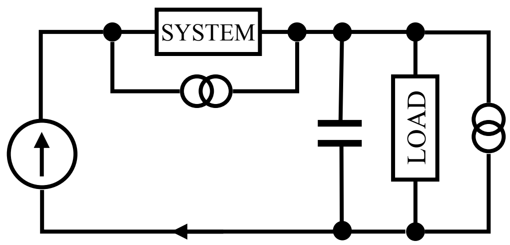
2. **Generalized Kramers' problem:**
   beyond Gaussian noise.

3. **Application to Josephson junction threshold detector:**
   dynamics in the extended phase space.

4. **Remaing problems to be solved.**

FIGHTING "CENTRAL LIMIT THEOREM"

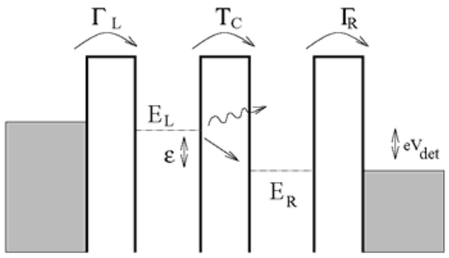
Simplified measurement circuit:

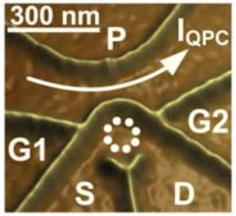


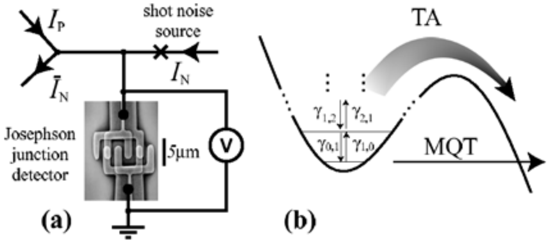
Bandwidth problem: $\tau_{RC} \gg 1/(eV)$

Solution: on-chip noise detectors!

Example 1:	**Example 2:**	**Example 3:**

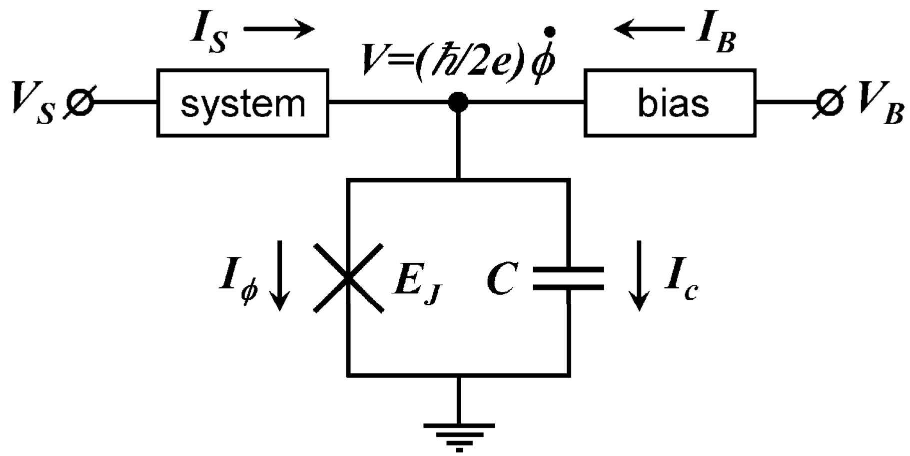






Kouwenhoven et al., (2004) Ensslin et al., (2004) Pekola et al., (2005)

JOSEPHSON JUNCTION DETECTOR



Kirchhoff's law:

$$I_S + I_B = (E_J/\Phi_0)\sin\phi + C\dot{V}$$

$$\Rightarrow \quad C\Phi_0^2\ddot{\phi} + E_J \sin\phi = \Phi_0(I_S + I_B)$$

Ideal detector:

$$\langle I_S \rangle = J_S - G_S V\,, \qquad \langle\langle I_S^n \rangle\rangle \neq 0$$

$$\langle I_B \rangle = J_B - G_B V\,, \qquad \langle\langle I_B^2 \rangle\rangle = 2TG_B$$

PHASE DYNAMICS

Equations of motion:

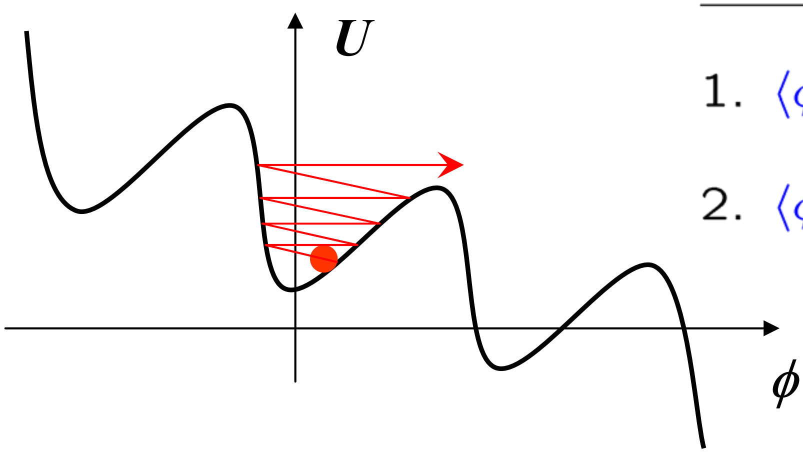
$\dot{\phi} = p/m$, where "mass": $m = \Phi_0^2 C$

$\dot{p} = -\partial U/\partial \phi + \Phi_0(I_S - J_S)$

Langevin source

Tilted periodic potential:

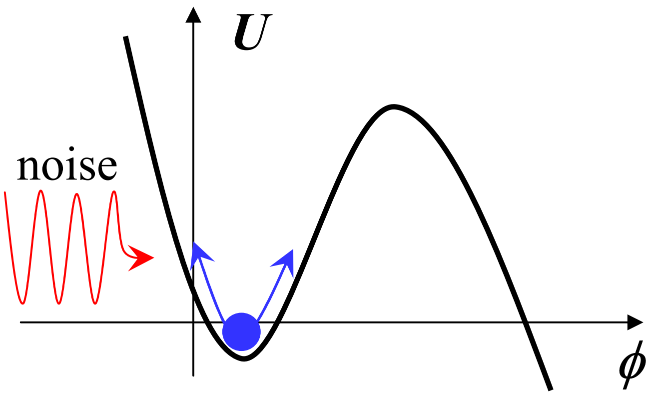
$U(\phi) = -E_J \cos\phi - \Phi_0(J_S + J_B)\phi$



Two regimes:

1. $\langle\dot{\phi}\rangle = V/\phi_0 = 0 \Rightarrow$ supercurrent
2. $\langle\dot{\phi}\rangle \neq 0 \Rightarrow$ dissipative current

GENERALIZED KRAMERS' PROBLEM



1. Hamiltonian dynamics
2. Friction
3. Noise

Hamiltonian-Langevin equations:

$$\dot{q} = \partial K(p,q)/\partial p + I_q,$$

$$\dot{p} = -\partial K(p,q)/\partial q + I_p,$$

where sources are arbitrary, $\langle\langle I_p^n I_q^m \rangle\rangle \neq 0$

WHAT IS THE ESCAPE RATE?

PREVIOUS WORKS

THEORY:

- J. Tobiska and Yu.V. Nazarov, *Phys. Rev. Lett.* **93**, 106801 (2004)
- J. Ankerhold, *Phys. Rev. Lett.* **98**, 036601 (2007)

EXPERIMENT:

- J. Pekola, *Phys. Rev. Lett.* **93**, 206601 (2004)
- A. V. Timofeev *et al.*, *Phys. Rev. Lett.* **98,** 207001 (2007)
- H. Pothier, B. Huard, N. Birge, D. Esteve, (2008).

STOCHASTIC PATH INTEGRAL

Evolution operator:

$P(p,q) = \int \mathcal{D}\Lambda \int \mathcal{D}\mathbf{R} \exp(S)$ where $R = (p,q)$

$S = \int dt' [-\Lambda \cdot \dot{\mathrm{R}} + \Lambda \cdot \{\mathbf{R}, K\} + \mathcal{H}(\Lambda, \mathbf{R})]$

$\rightarrow$ action in canonical form, Poisson bracket

$\mathcal{H}$ is generator: $\langle\langle I_p^n I_q^m \rangle\rangle = \partial_{\lambda_p}^n \partial_{\lambda_q}^m \mathcal{H}(0,0)$

Markov limit – "white" noise
$\Rightarrow$ Hamilton's equations

$\Rightarrow \quad P(p,q) = \exp[S(p,q)]$

Pilgram, Jordan, *E.S.*, Büttiker, PRL (2003)

Jordan, *E.S.*, Pilgram, J. Math. Phys. (2004)

INSTANTON SOLUTIONS

Average dynamics: $\Lambda = 0$

$\mathcal{H}(0) = 0 \Rightarrow S = 0 \Rightarrow$ most probable path.

$\delta S/\delta\Lambda = 0 \quad \Rightarrow \quad \dot{\mathbf{R}} = \{\mathbf{R}, K\} + \langle \mathbf{I} \rangle$

oscillations (for $\{\mathbf{R}, K\}$)

damping: $\langle \mathbf{I} \rangle \sim -\mathbf{R}$

Uphill solutions: $\Lambda \neq 0$

"Hamiltonian" $\Lambda \cdot \{\mathbf{R}, K\} + \mathcal{H} = 0$ is conserved

Gaussian noise, $\mathcal{H} = \langle \mathbf{I} \rangle \Lambda + (1/2)\langle\langle \mathbf{I}^2 \rangle\rangle \Lambda^2$

$\Rightarrow$ time reversal path: $-\dot{\mathbf{R}} = \{\mathbf{R}, K\} + \langle \mathbf{I} \rangle$ $\Rightarrow$ **escape!**

$$\Gamma_{\text{escape}} = \exp[S_{\text{in}}]$$

ENERGY DIFFUSION

Weak damping: Quasi-periodic motion.

Langevin equations $\Rightarrow$ $\dot{E} = \dot{q} I_p - \dot{p} I_q$. $\longleftarrow$ **new source**

$$\Rightarrow \qquad S = \int dt' [-\lambda_E \dot{E} + \mathcal{H}(\lambda_E \dot{q}, -\lambda_E \dot{p})].$$

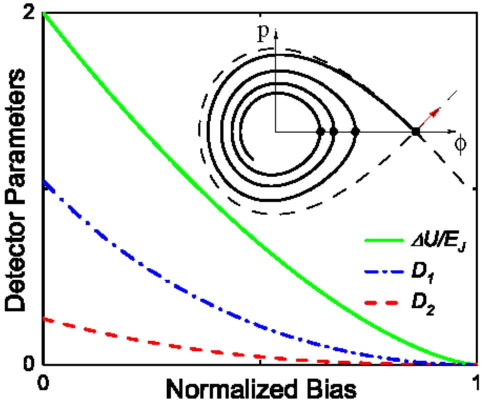
After many periods: $S = \int dt' [-\lambda_E \dot{E} + \langle \mathcal{H} \rangle_E]$

Instanton solutions:

$$\langle \mathcal{H} \rangle_E \equiv \frac{1}{T_0} \oint dt \mathcal{H} = 0 \qquad \Rightarrow \qquad \lambda_E = \lambda_{\text{in}}(E)$$

$$\Rightarrow \qquad \boxed{\log \Gamma = -\int \lambda_{\text{in}}(E) dE}$$ $\longrightarrow$ **formal solution**

APPLICATION TO JOSEPHSON DETECTOR



Escape rate:

$$\log\Gamma = -\frac{\Delta U}{T_{\rm eff}} + \frac{D_1 \Phi_0 E_J \langle\langle I_S^3\rangle\rangle_{\rm tot}}{C T_{\rm eff}^3}.$$

Noise temperature: $T_{\rm eff} \equiv \langle\langle I_S^2\rangle\rangle / 2\, G_S$.

$\langle\langle I_S^3\rangle\rangle$-contribution is small by $\hbar\omega_{\rm pl}/T_{\rm eff} \ll 1$

Total cumulant:

$$\langle\langle I_S^3\rangle\rangle_{\rm tot} = \langle\langle I_S^3\rangle\rangle - 3T_{\rm eff}\,\partial_{V_S}\langle\langle I_S^2\rangle\rangle \longrightarrow$$ **cascade correction**

Important!

$$\langle\langle I_S^3\rangle\rangle_{\rm tot} = 3C^2 G_{\rm tot}\langle\langle V^3\rangle\rangle \longrightarrow$$ **equal time correlator**

Jordan, *E.S.*, Pilgram, J. Math. Phys. (2004)

OUTLOOK

1. **Semi-classical contributions to the prefactor.**
2. **Measurement of rare event statistics.**
3. **Josephson junction bifurcation amplifier.**