Problems of noise modeling in the presence of total current branching in HEMTs and FETs channels

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Outline

- Current branching
- Circuit model
- Analytical model
- Results
- Open problems

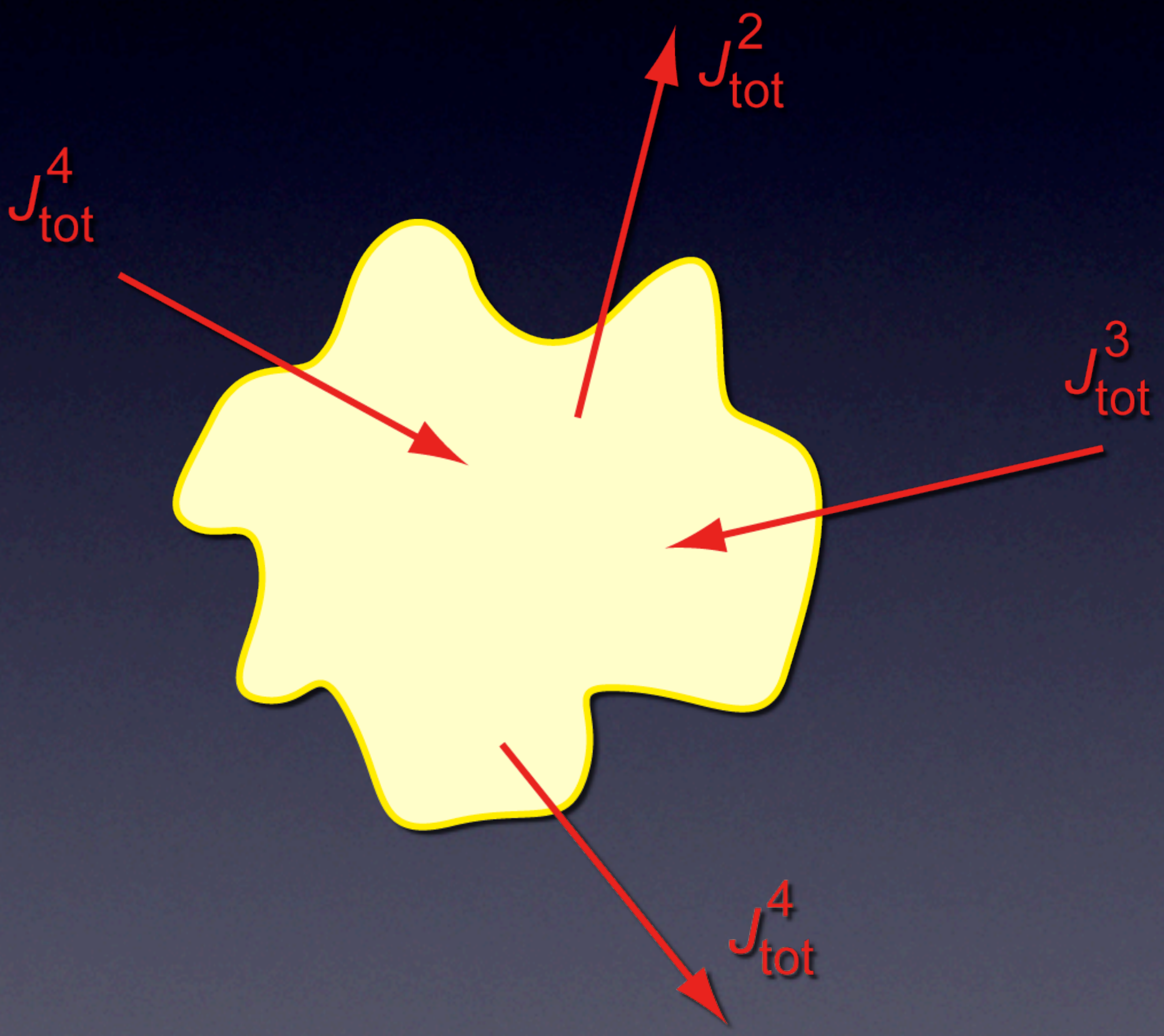
Conservation of total current

- Maxwell eqs. → conservation of total current → Kirchhoff law

$$\oint_S \mathbf{j}_{tot}\, d\mathbf{S} = 0$$

$$\mathbf{j}_{tot} = \boxed{\varepsilon\varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}} + \boxed{\mathbf{j}_d}$$

displacement drift



No branching → 2 terminals → Diode

$\mathbf{j}_d \parallel \mathbf{E}$ ⇒ $$\frac{\partial [S(x) j_{tot}(x,t)]}{\partial x} = 0$$

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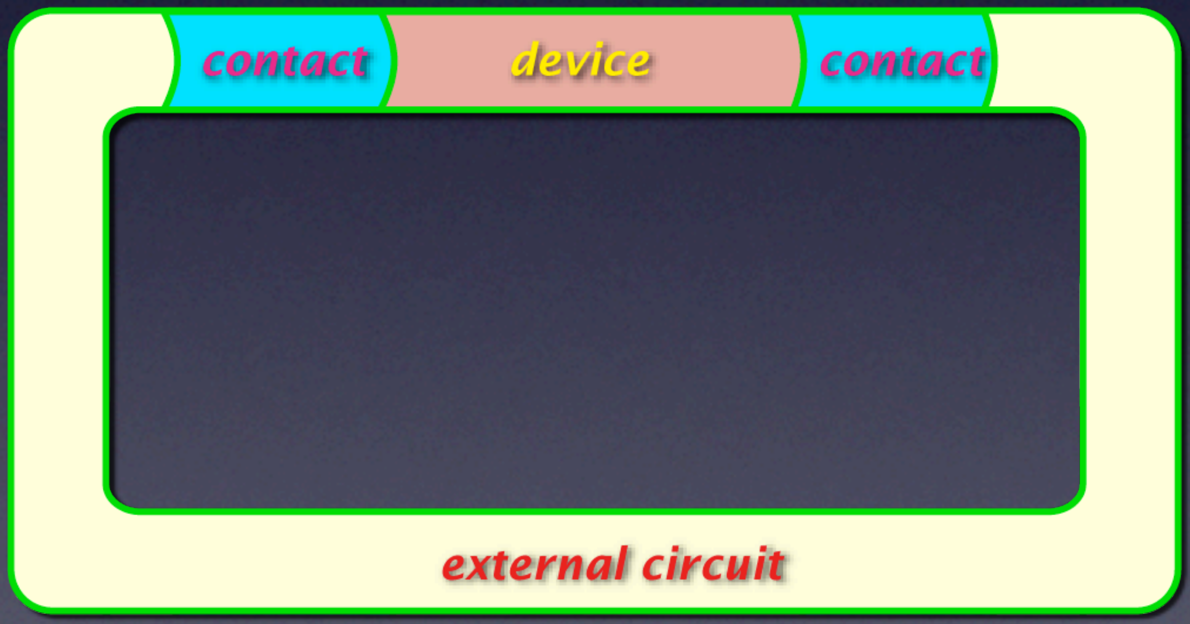

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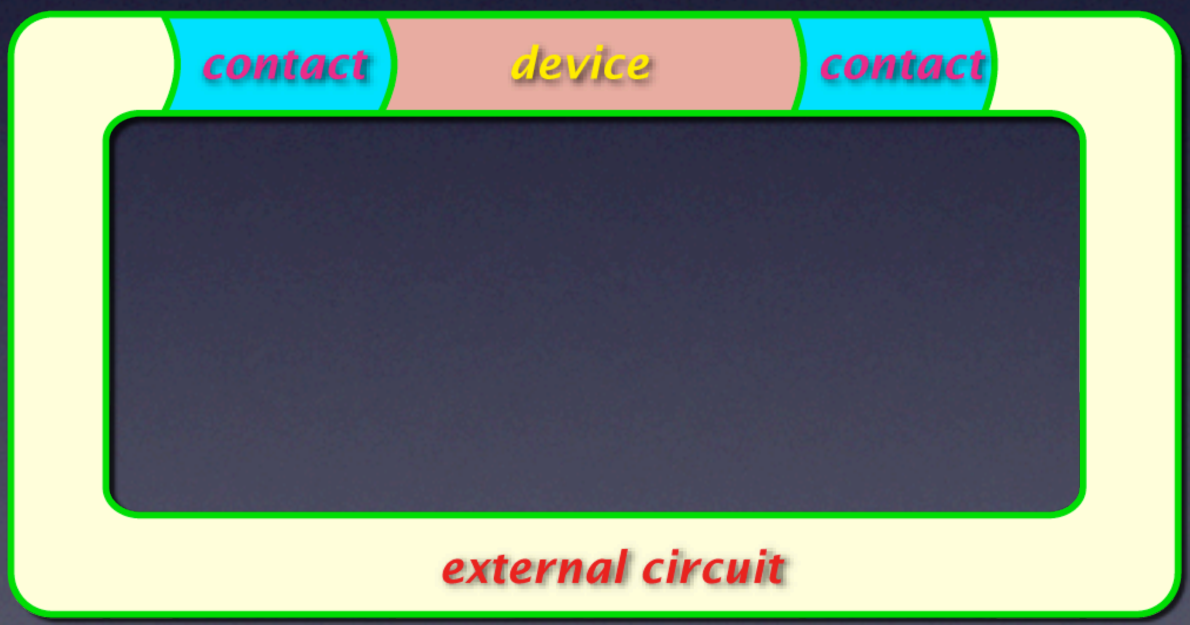
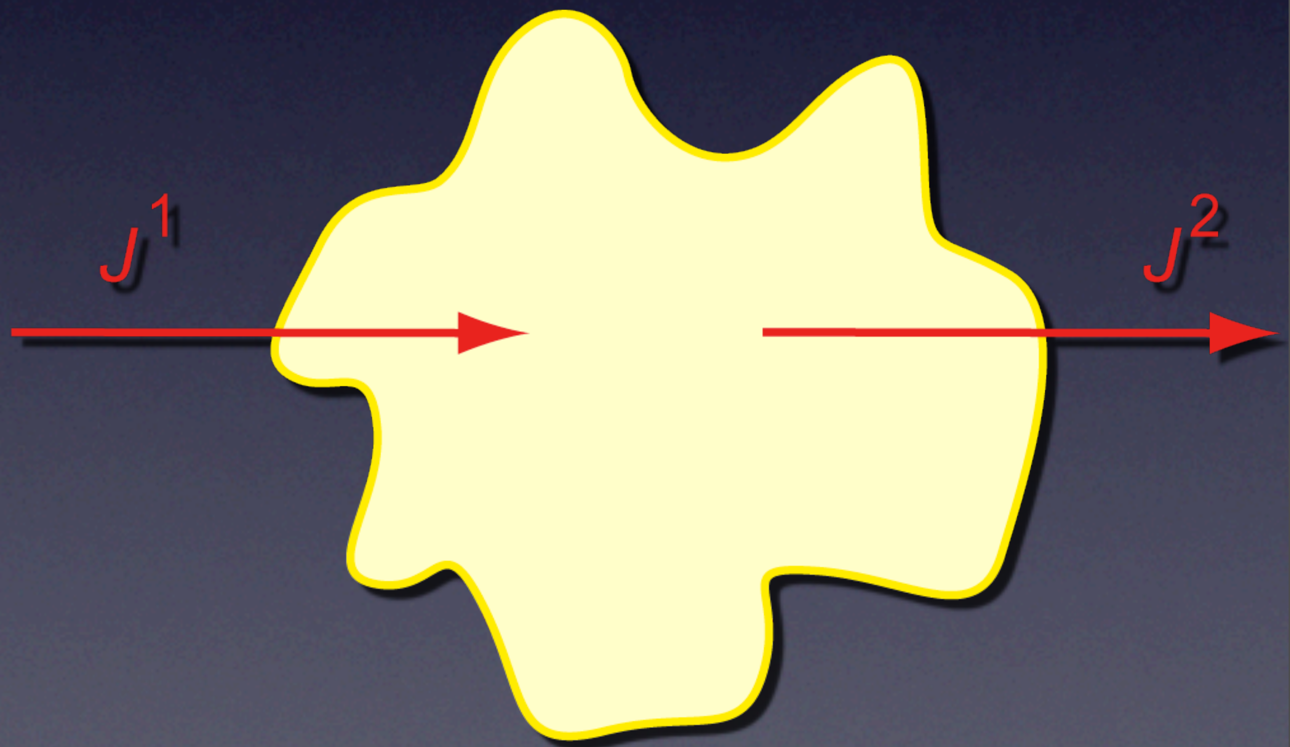
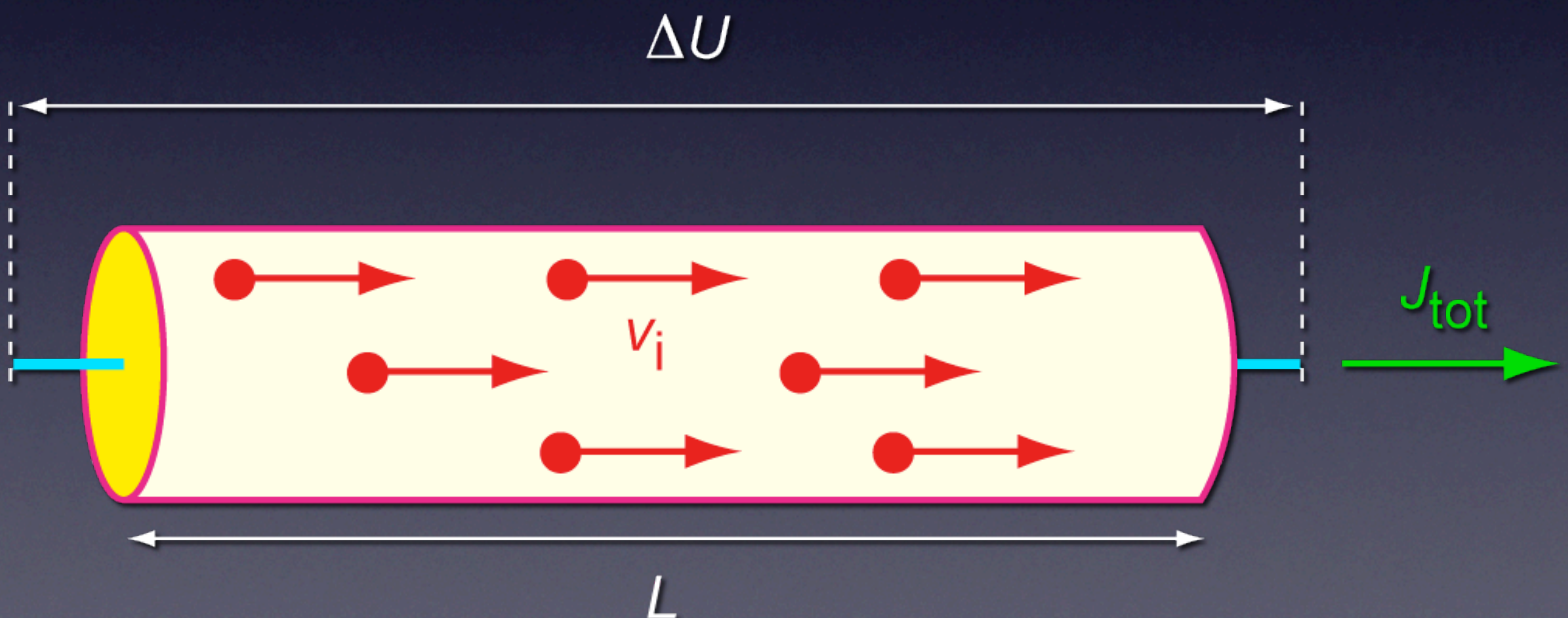




1st consequence: Ramo-Shockley theorem

$$I_{tot} = \frac{e}{L}\sum_{i=1}^{N} v_i - \epsilon\epsilon_0 \frac{\partial}{\partial t}\Delta U$$



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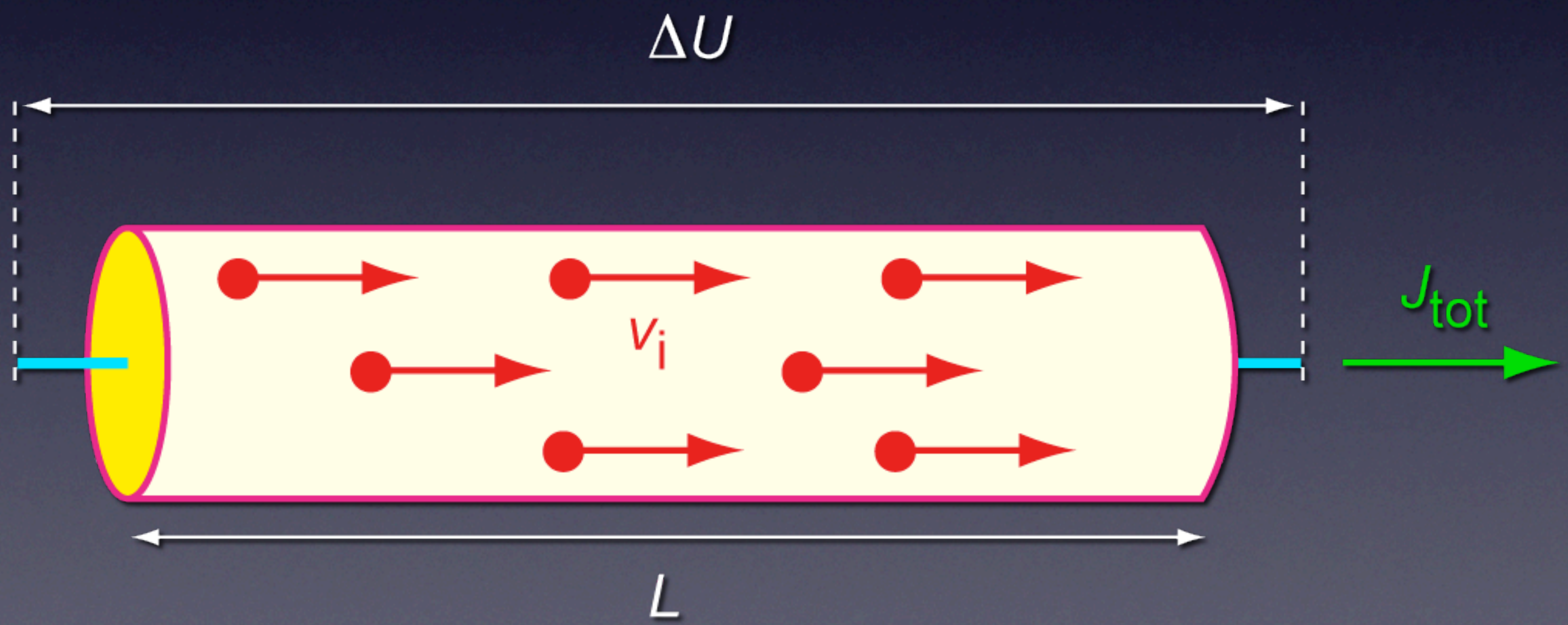

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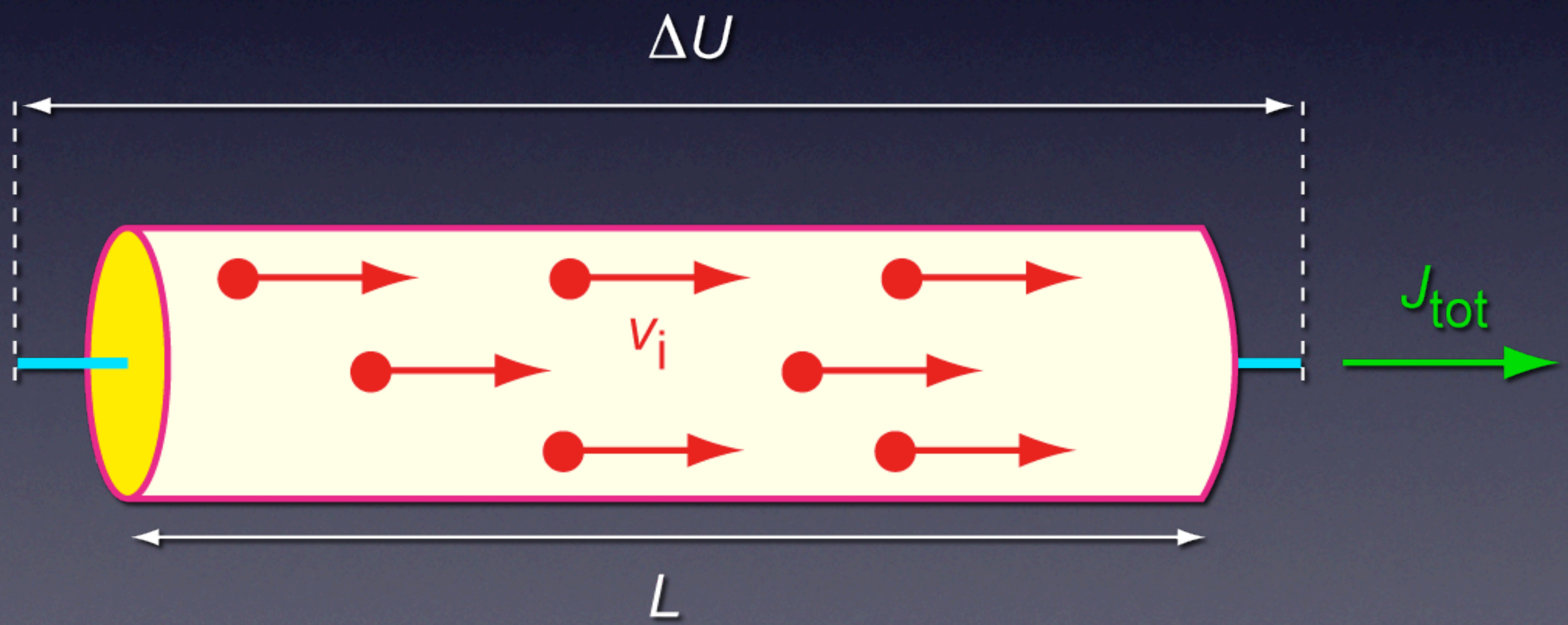
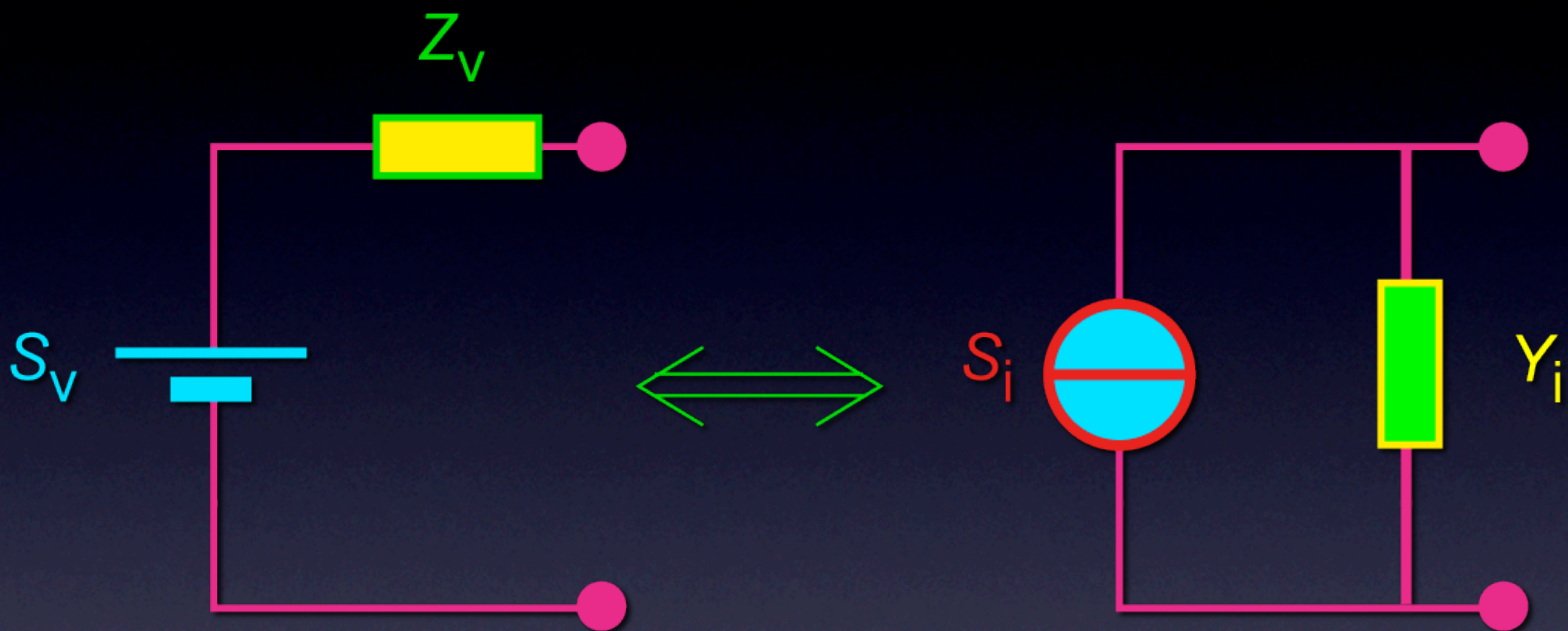
$$I_{tot} = \boxed{\frac{e}{L}\sum_{i=1}^{N} v_i} - \boxed{\epsilon\epsilon_0 \frac{\partial}{\partial t}\Delta U}$$

drift displacement



2nd consequence: Thevenin-Norton theorem

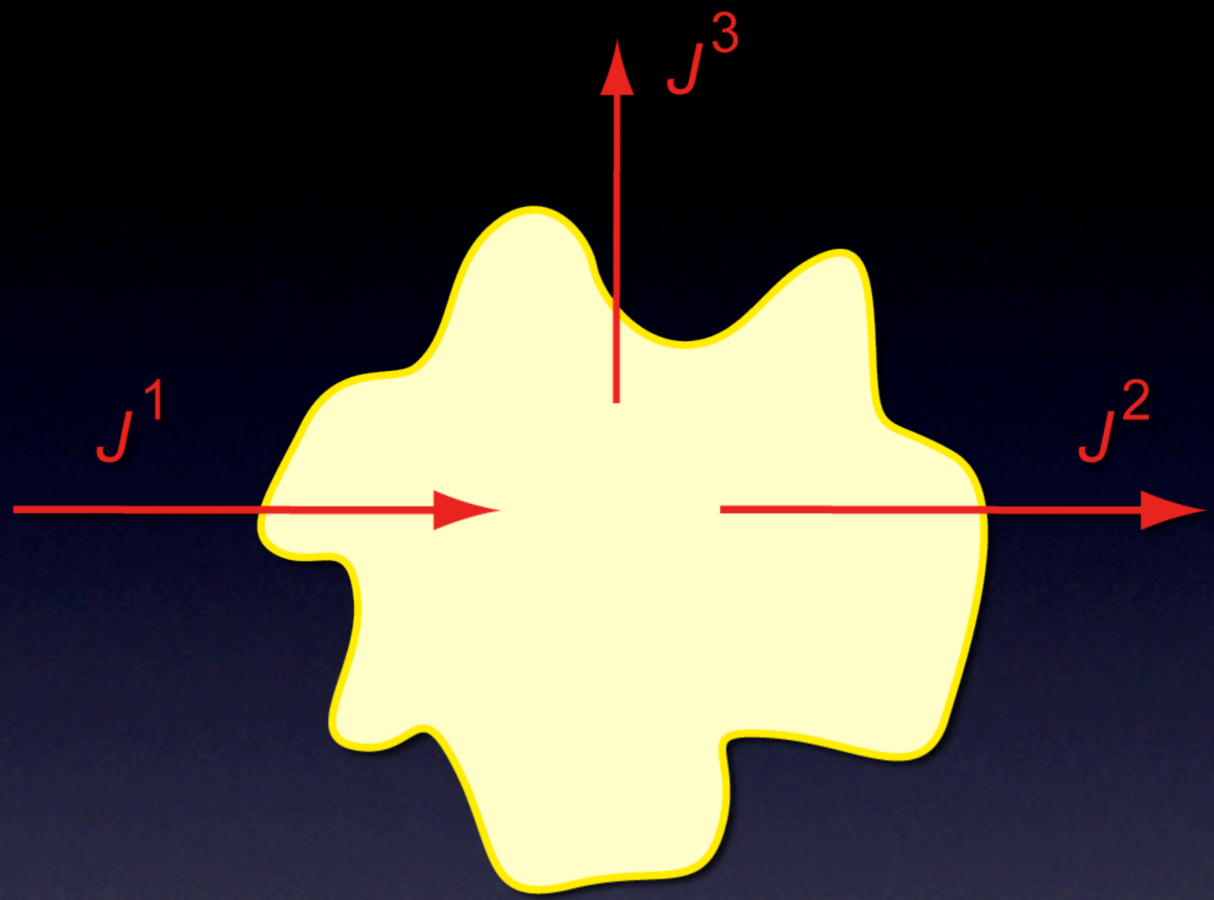


$$\overline{v^2} = 4KTR\Delta f$$

$$\overline{i^2} = 4KTG\Delta f$$

Branching → 3 terminals → transistor

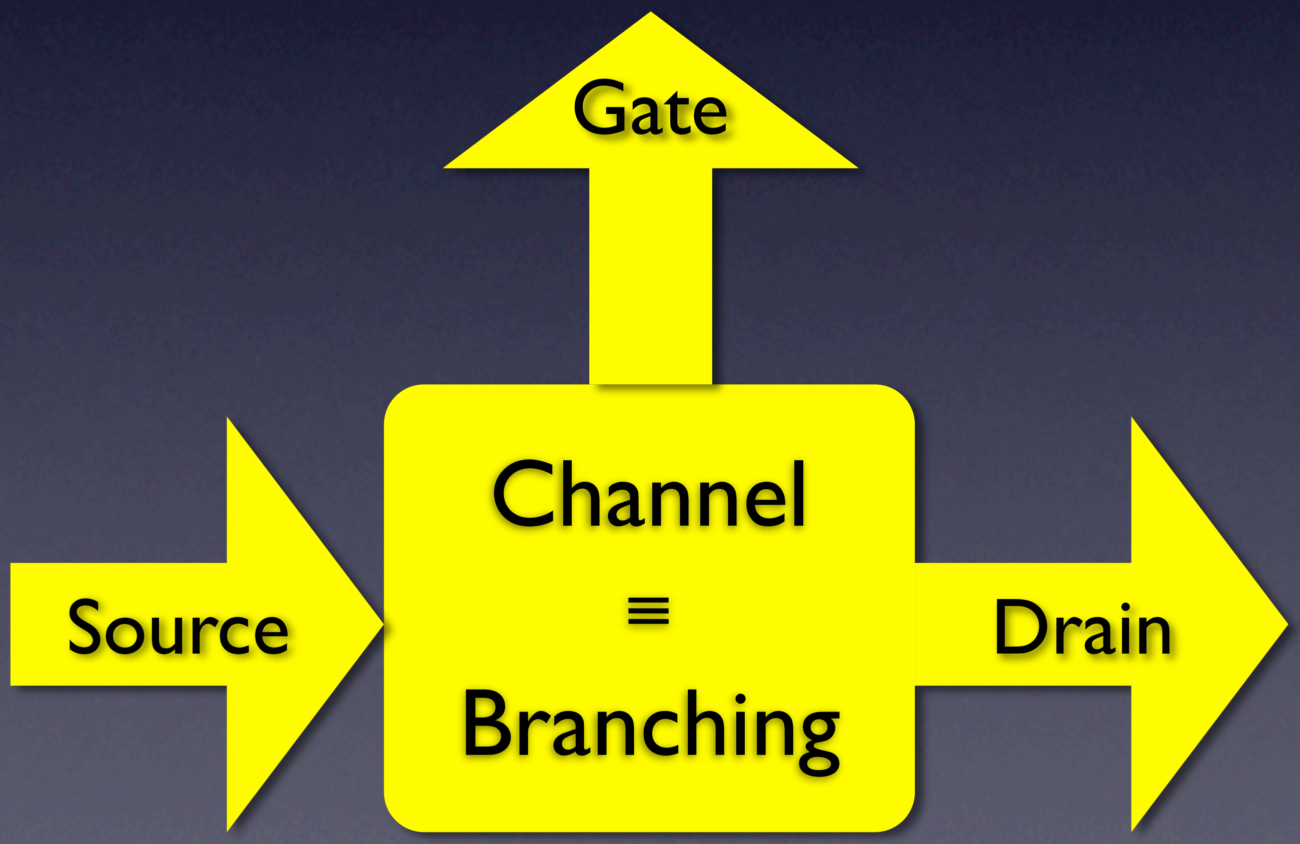
$$\mathbf{j}_d \not\parallel \mathbf{E}$$



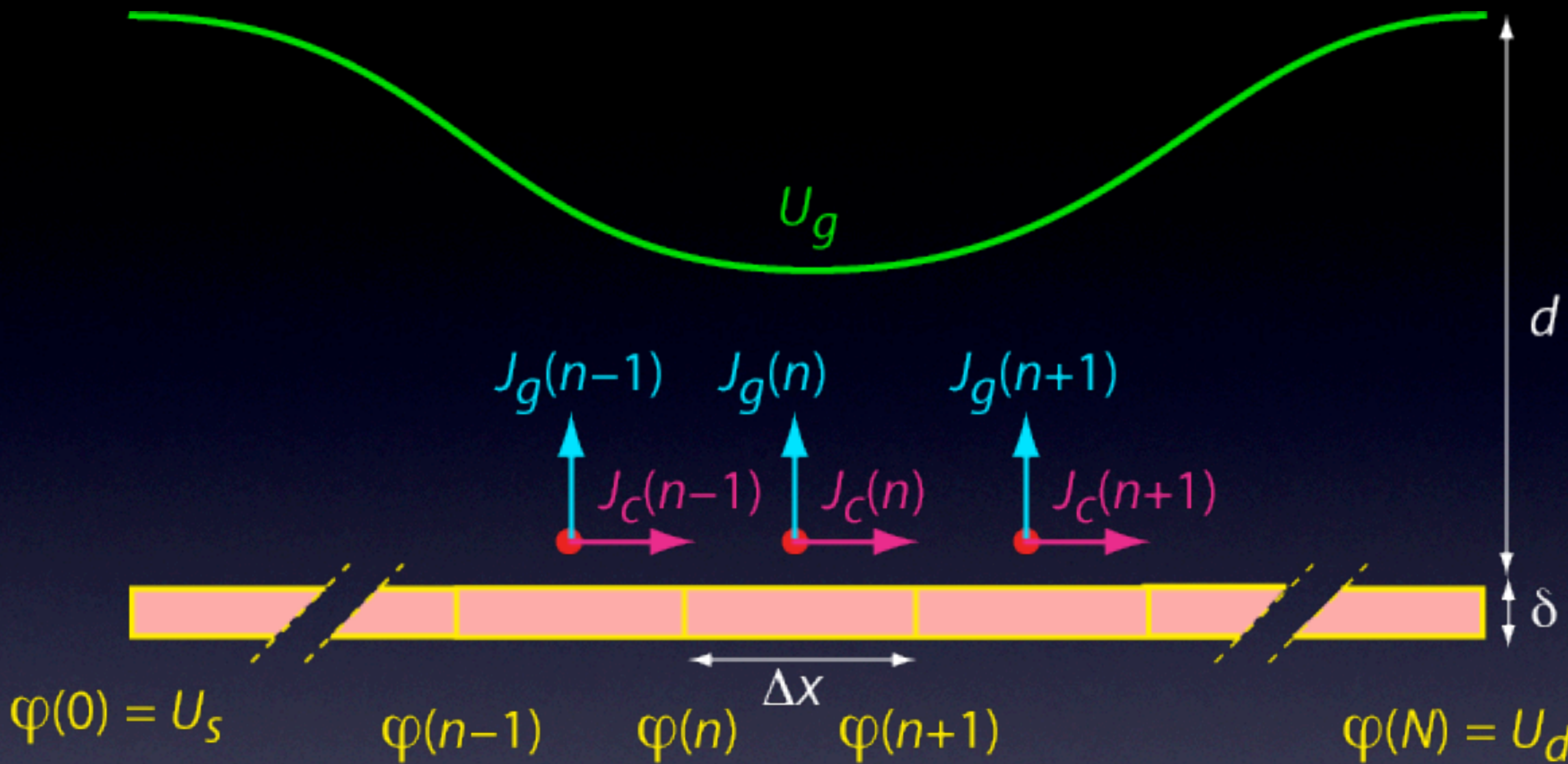
- Total current conservation ≡ zero-sum rule (Kirchhoff)
- Typical of field-effect transistors
- Channel ≡ total current branching between source-drain and channel-gate directions

Aim of the study

Discuss problems related to the intrinsic noise in FET channels induced by the continuous branching of the total current between 3 terminals



Circuit model



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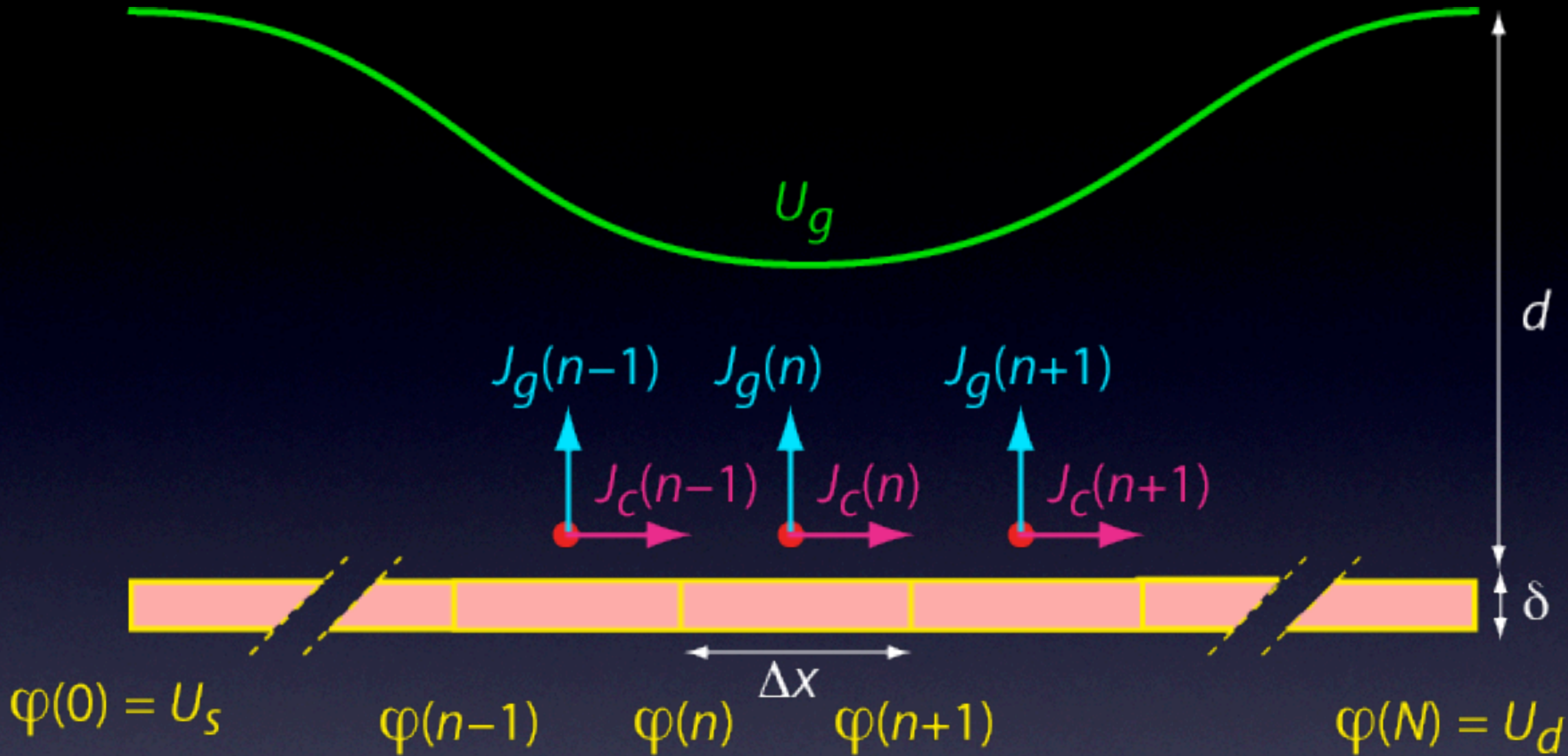
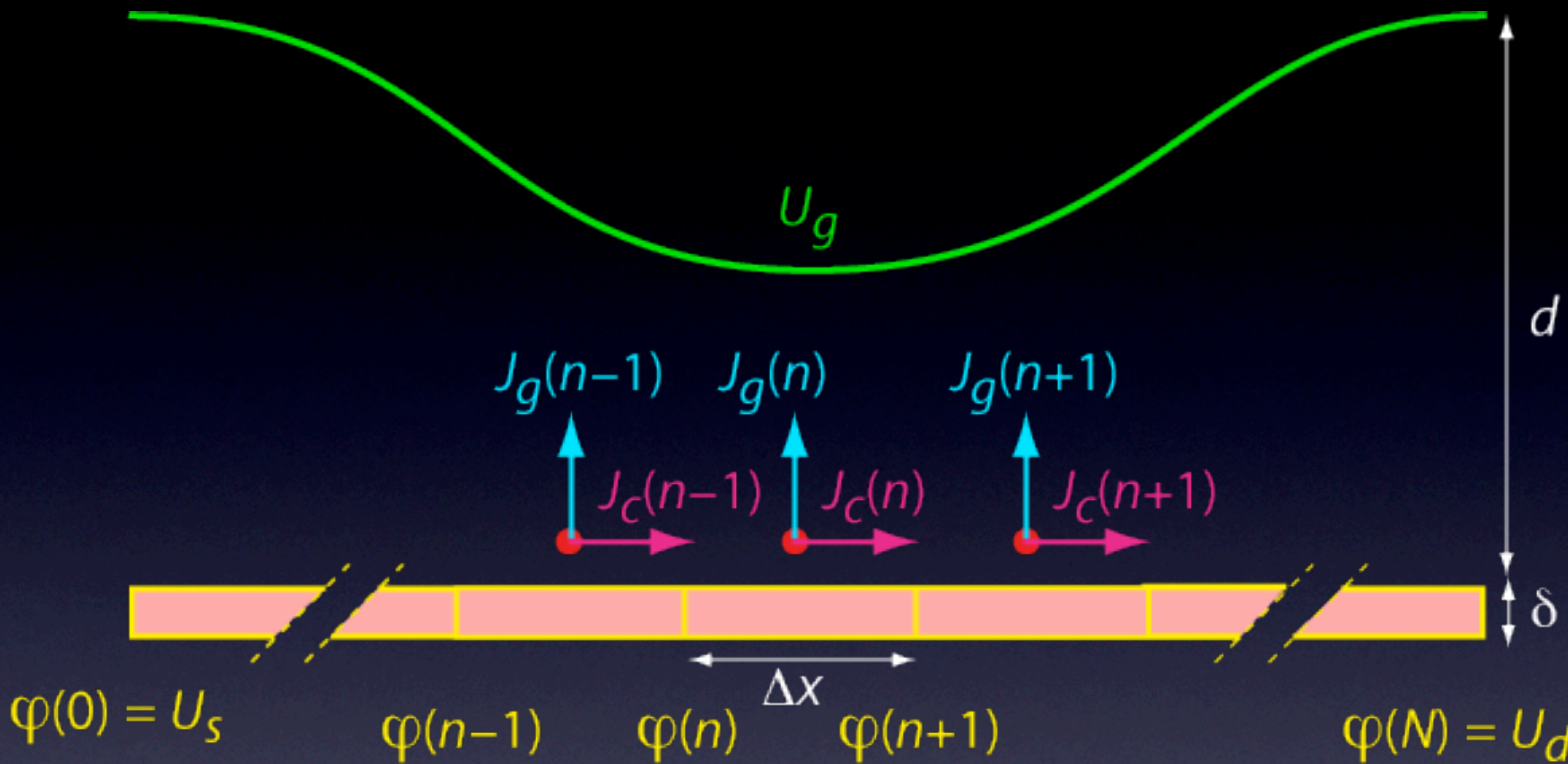


$$J_c(n) - J_c(n-1) + J_g(n) = 0$$

Circuit model



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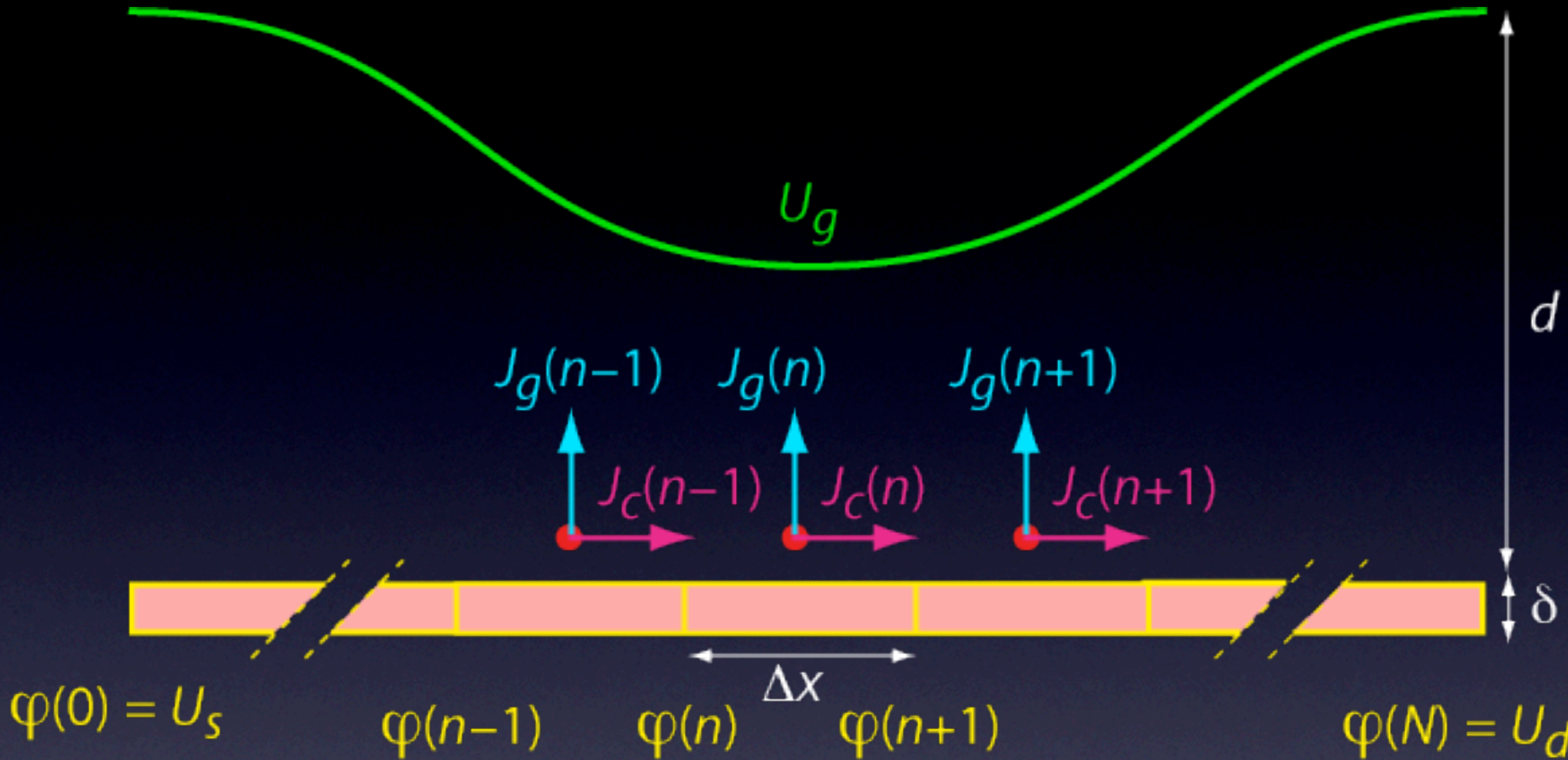

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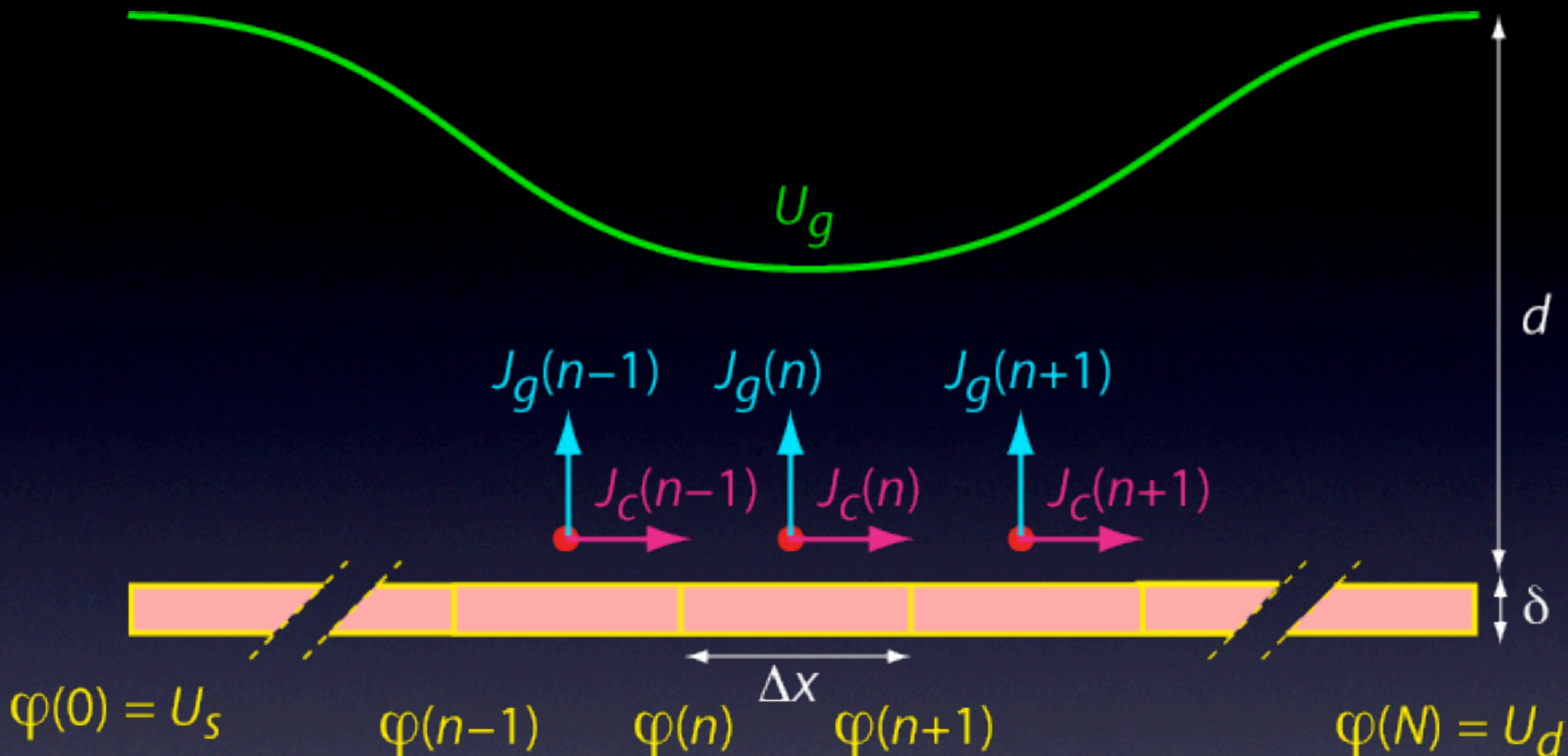

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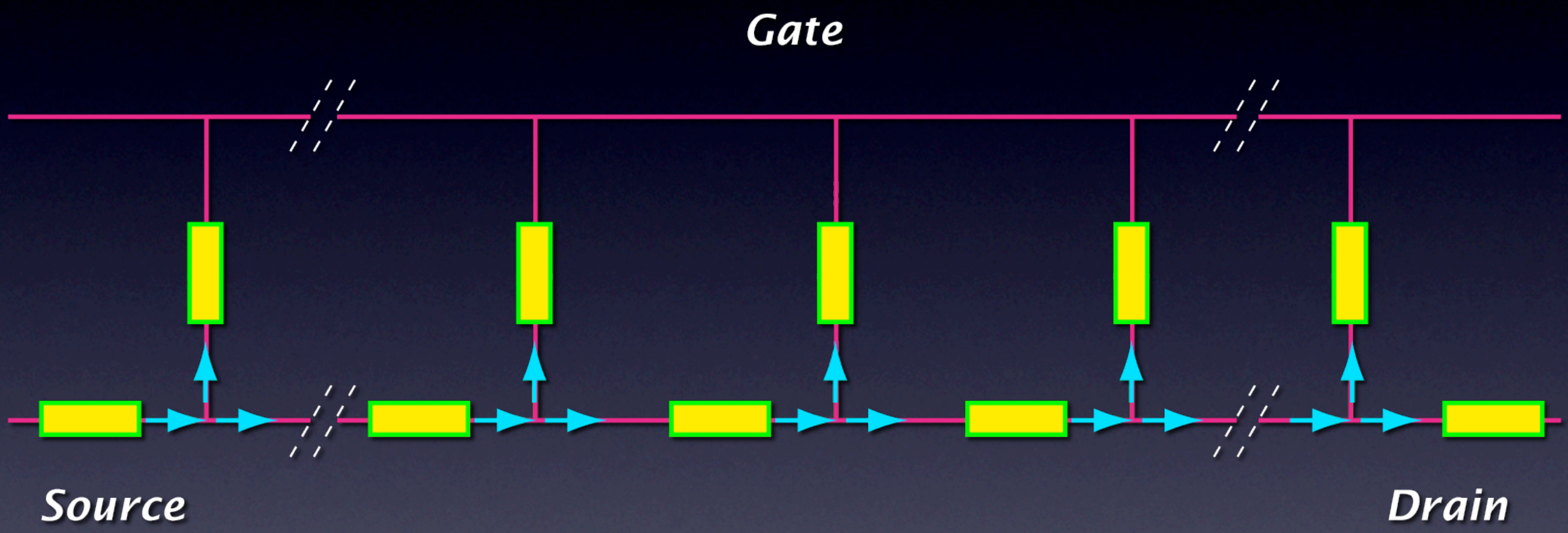
Circuit model: impedance network



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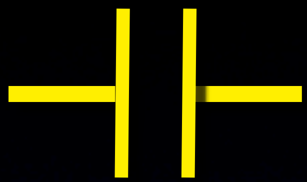
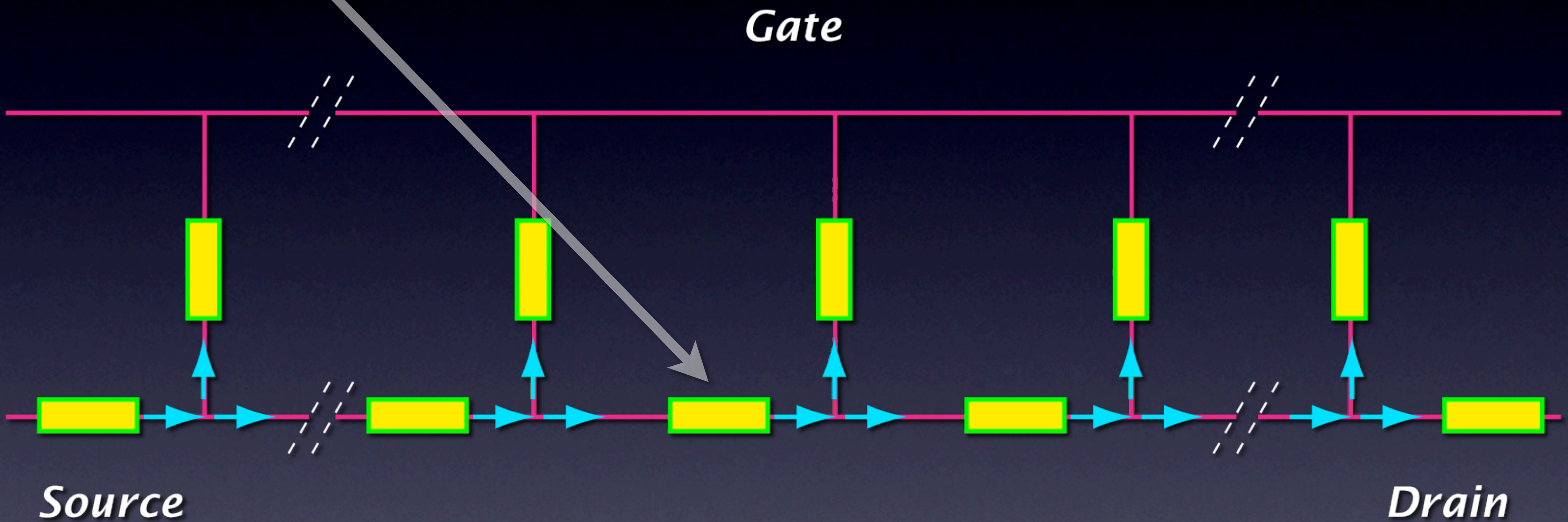


$$C_c(n) = \frac{\epsilon_c \epsilon_0 W \delta}{\Delta x}$$



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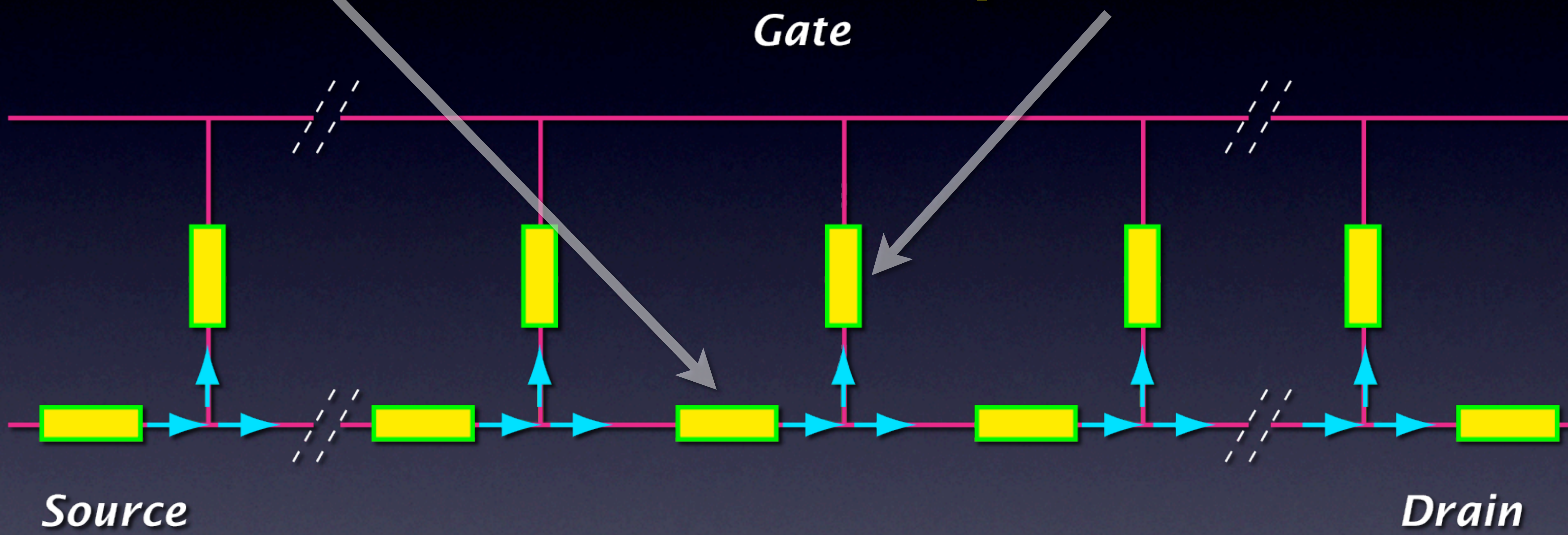
$$C_g(n) = \frac{\epsilon_d \epsilon_0 W \Delta x}{d(n)}$$



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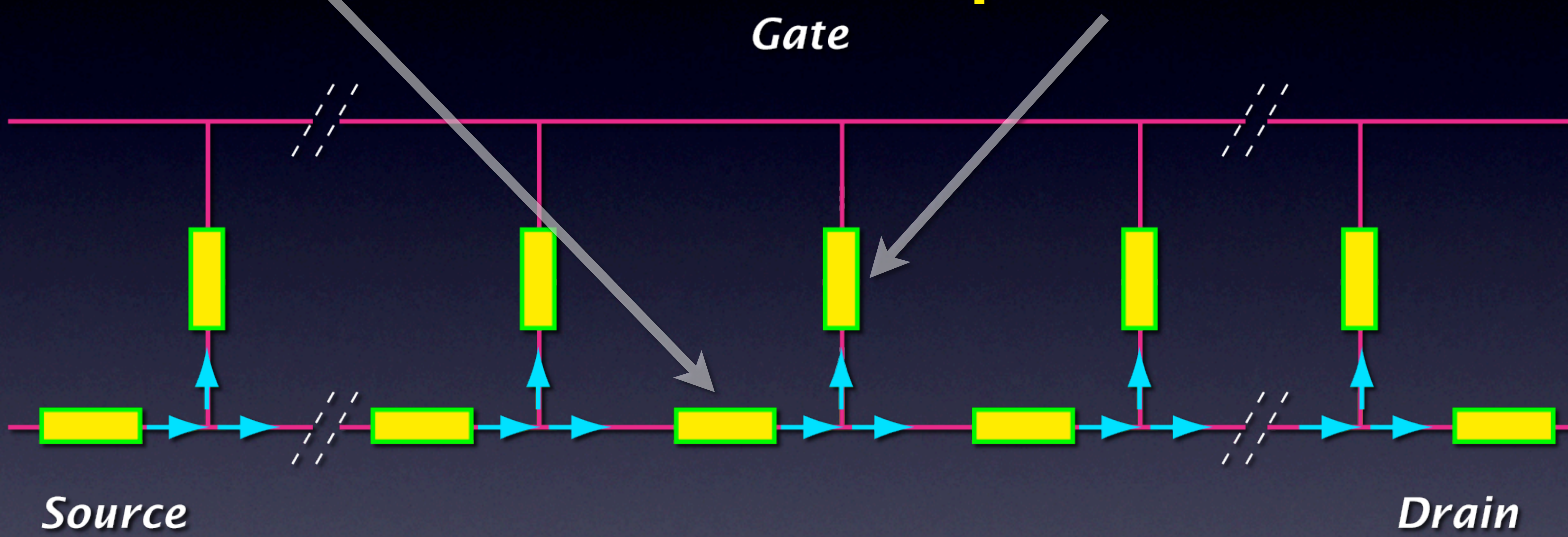

$$J_i^{disp}(n) = -C_i(n) \frac{d}{dt} \begin{cases} [\varphi(n+1) - \varphi(n)] & \text{, i=c} \\ [U_G - \tilde{\varphi}(n)] & \text{, i=g} \end{cases}$$

Discretization of current conservation

$$\left[C_c(n)\frac{d}{dt}[\varphi(n+1)-\varphi(n)]+J_c^d(k)\right]-$$

$$\left[C_c(n-1)\frac{d}{dt}[\varphi(n)-\varphi(n-1)]+J_c^d(n-1)\right]=$$

$$-\left[C_g(n)\frac{d}{dt}[U_G-\tilde{\varphi}(n)]+J_g^d(n)\right]$$

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$$\left[C_c(n)\frac{d}{dt}[\varphi(n+1)-\varphi(n)]+J_c^d(k)\right]$$ – Out source→drain

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Discretization of current conservation

$$\boxed{\left[C_c(n)\frac{d}{dt}[\varphi(n+1)-\varphi(n)]+J_c^d(k)\right]} - \text{Out source}\rightarrow\text{drain}$$

$$\boxed{\left[C_c(n-1)\frac{d}{dt}[\varphi(n)-\varphi(n-1)]+J_c^d(n-1)\right]} = \qquad \text{In}$$

$$-\boxed{\left[C_g(n)\frac{d}{dt}[U_G-\tilde{\varphi}(n)]+J_g^d(n)\right]} \qquad \text{Out channel}\rightarrow\text{gate}$$

Discretization of current conservation

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$$-\left[C_g(n)\frac{d}{dt}[U_G-\tilde{\varphi}(n)]+J_g^d(n)\right]$$ Out channel→gate

by taking the limit $\Delta x \to 0$
and normalizing to the cross-section $S = W\delta$ ...

Conservation of total current in differential form

Total $$\frac{d}{dx}[j_c(x)] + \frac{1}{\delta} j_g(x) = 0$$

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Channel $$\boxed{j_c(x)} = -\epsilon_c \epsilon_0 \frac{d}{dt} \frac{\partial}{\partial x} \varphi(x) + j_c^d(x)$$

Conservation of total current in differential form

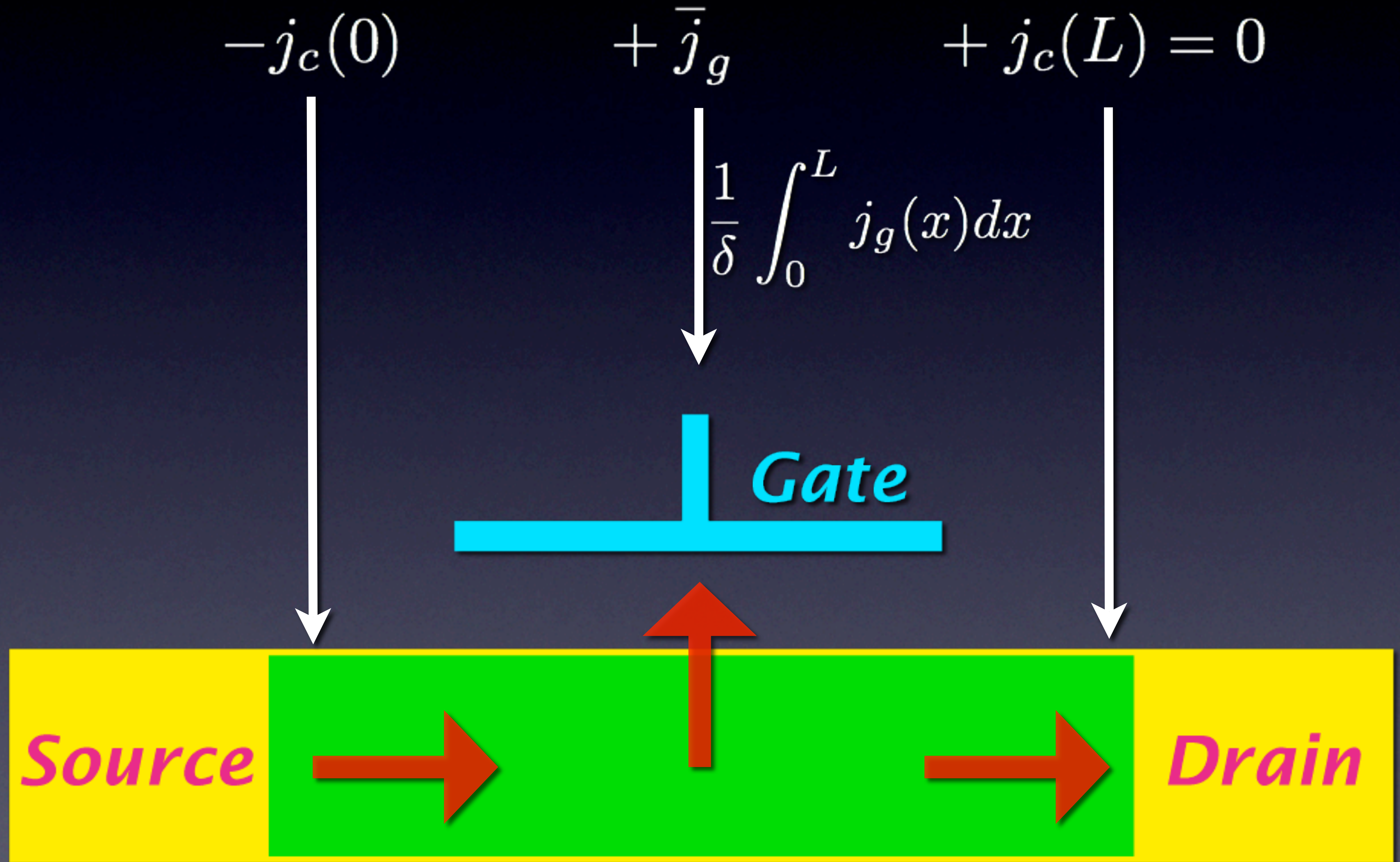
Total
$$\frac{d}{dx}[j_c(x)] + \frac{1}{\delta} j_g(x) = 0$$

Channel
$$\boxed{j_c(x)} = -\epsilon_c \epsilon_0 \frac{d}{dt}\frac{\partial}{\partial x}\varphi(x) + j_c^d(x)$$

Gate
$$\boxed{j_g(x)} = -\frac{\epsilon_d \epsilon_0}{d(x)}\frac{d}{dt}[U_G - \varphi(x)] + j_g^d(x)$$

Conservation of total current in integral form

By integrating between source (x=0) and drain (x=L)



Quasi-2D Poisson equation

By using the charge conservation law

$$\frac{\partial}{\partial t}\rho^{3D}(x) + \frac{\partial}{\partial x} j_c^d(x) + \frac{j_g^d(x)}{\delta} = 0$$

$$\Downarrow$$

$$\epsilon_c \frac{\partial^2}{\partial x^2}\varphi(x) + \frac{\epsilon_d}{d(x)\delta}[U_g - \varphi(x)] = -\frac{1}{\epsilon_0}\rho^{3D}(x)$$

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Longitudinal field

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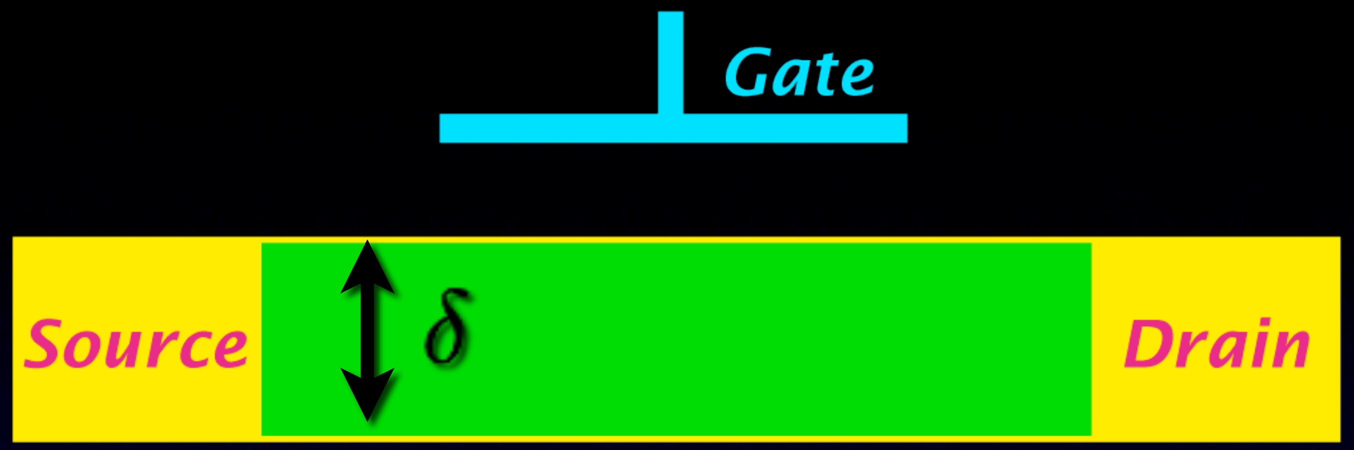
1st limit of quasi-2D Poisson equation

Narrow channel $\delta \to 0$



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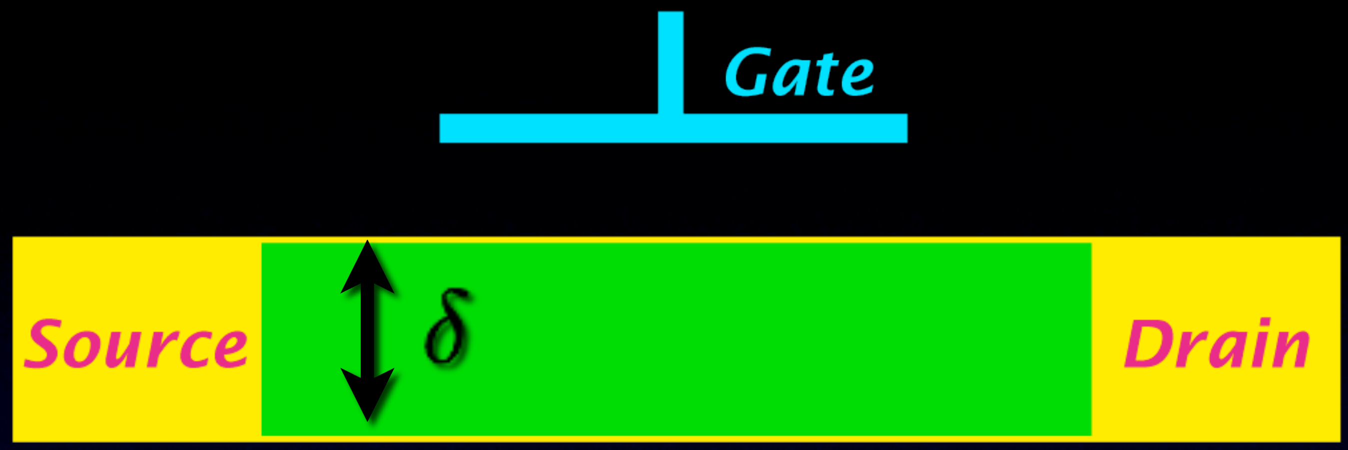

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$$\delta\ \epsilon_0 \epsilon_c \frac{\partial^2}{\partial x^2}[\varphi(x)] + \frac{\epsilon_0 \epsilon_d}{d(x)}[U_g - \varphi(x)] = -\rho^{3D}(x)\delta$$

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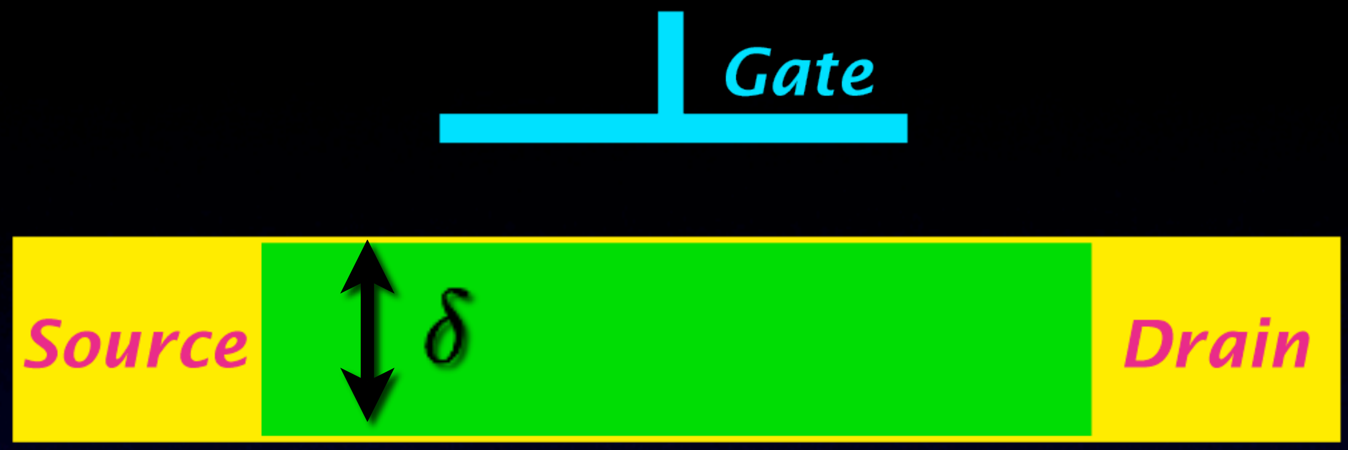

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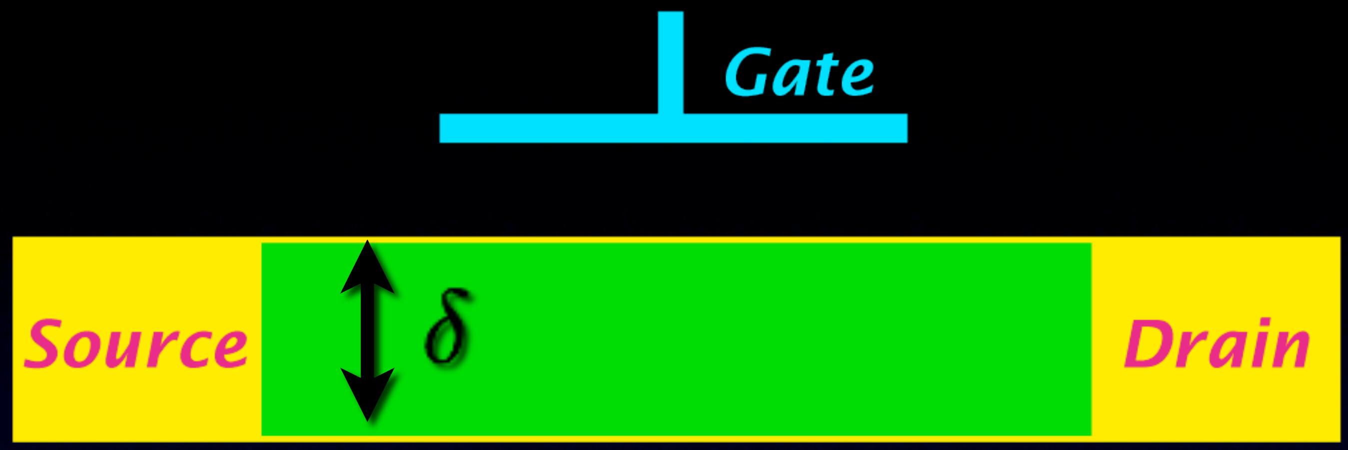

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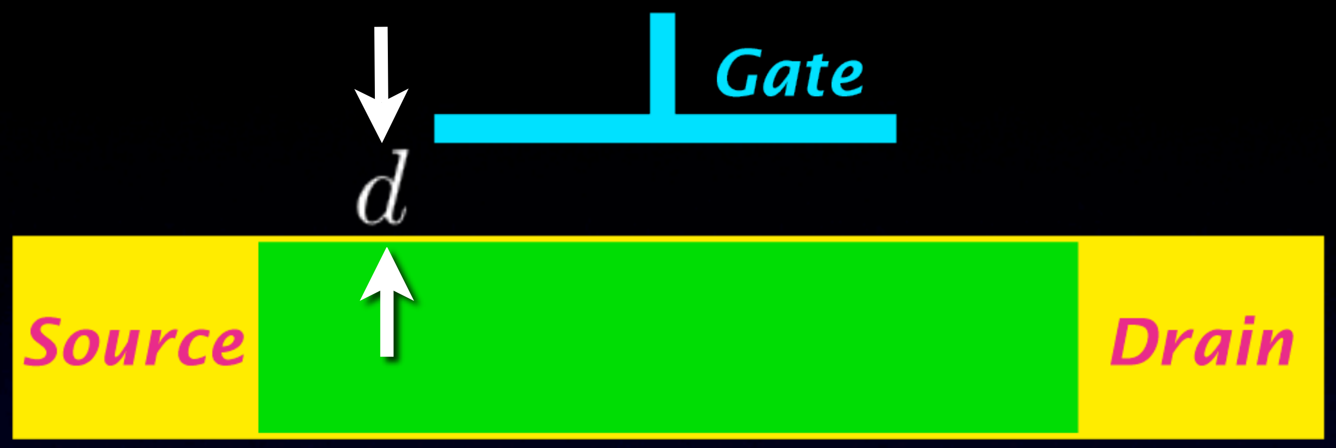
$$C_g V_0 = -eN^{2D}(x)$$

Gradual channel approximation

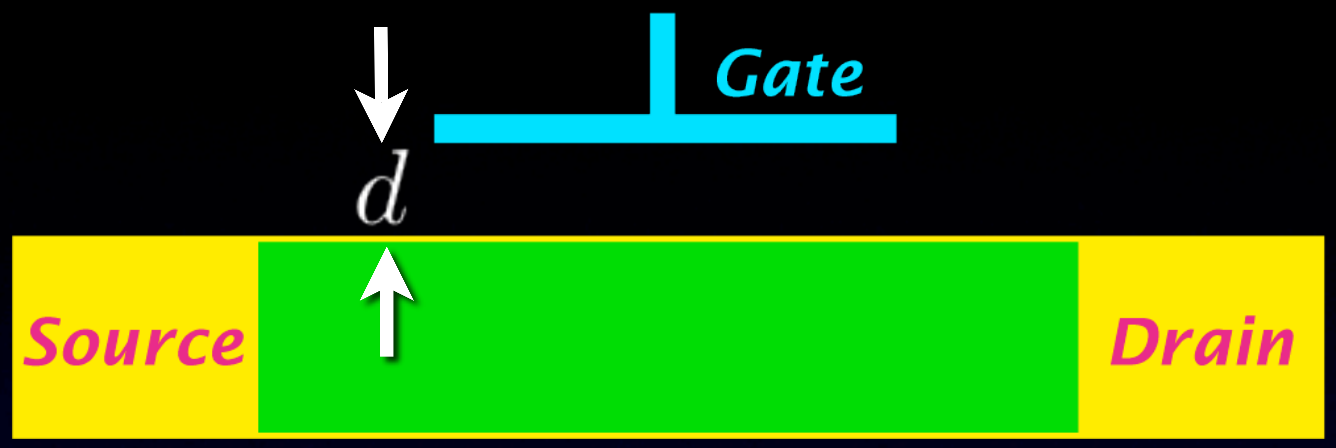
2nd limit of quasi-2D Poisson equation

Ungated transistor $d(x) \to \infty$



2^{nd} limit of quasi-2D Poisson equation

Ungated transistor $d(x) \to \infty$



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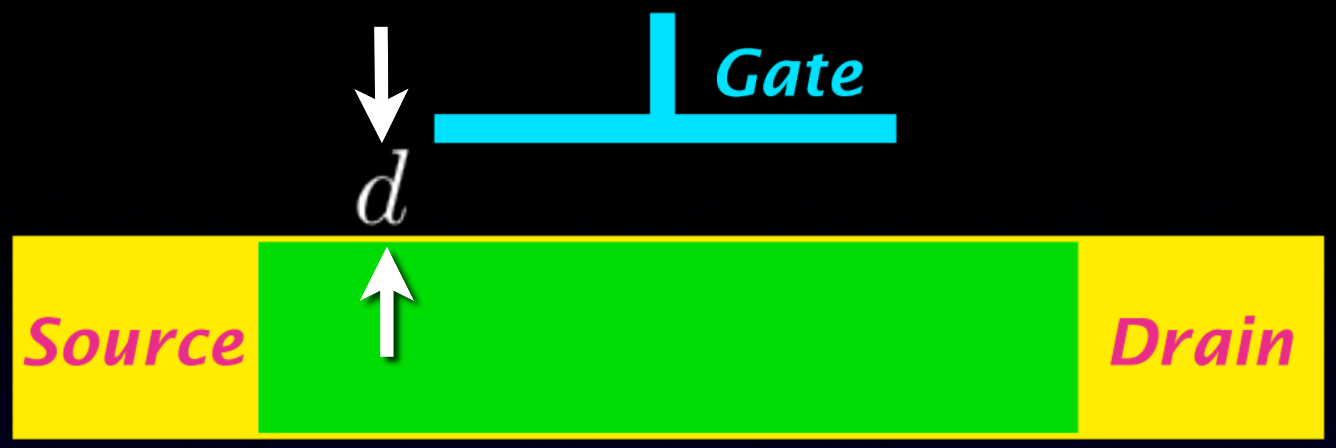
2^{nd} limit of quasi-2D Poisson equation



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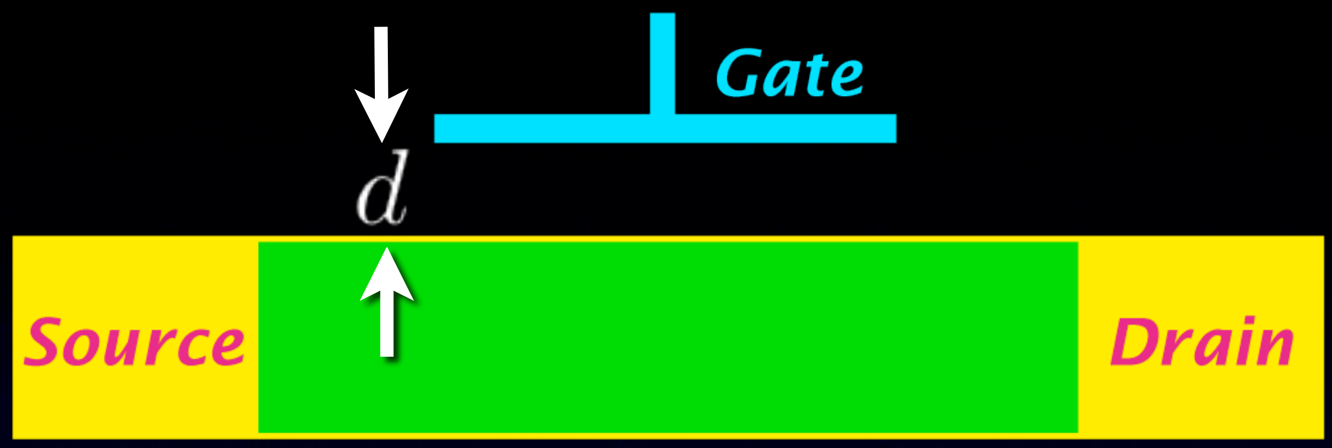

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1D Poisson equation

Closure relations

- No transport to gate → $j_g^d(x) = 0$
- Drift current in the channel → $j_c^d(x) = en^{3D}(x)v(x)$
- Velocity from hydrodynamic approach

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$$\frac{\partial}{\partial t}v + \frac{\partial}{\partial x}\left[\frac{v^2}{2} + \frac{e}{m}\varphi(x)\right] + e\nu D\frac{\partial}{\partial x}n^{3D}(x) = -\nu v + \tilde{f}$$

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Velocity relaxation rate

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Velocity
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Velocity relaxation rate

Langevin force

Analytical Model

- Carrier concentration in the channel n_0
- No drift velocity $v_0 = 0$
- Linearization of equations
- Neglecting diffusion current

Linearized equations in frequency domain

- Potential $$\frac{d^2}{dx^2}\Delta\varphi(x) + \lambda^2[\Delta U_g - \Delta\varphi] = -\frac{e}{\epsilon\epsilon_0}\Delta n$$
- Concentration $$e\Delta n = -\frac{1}{i\omega}\frac{d}{dx}\Delta j_d$$
- Drift current $$\Delta j_d(x) = en_0\Delta v = \epsilon\epsilon_0\frac{\omega_p^2}{i\omega+\nu}\left[\tilde{f} - \frac{\partial}{\partial x}\Delta\varphi\right]$$

Self-consistent fluctuating potential

$$\frac{d^2}{dx^2}\varphi - \beta^2\varphi = -\beta^2\Delta U_g + \frac{d}{dx}\tilde{f}$$

where $\beta^2 = \lambda^2[i\omega(i\omega+\nu)/(\omega_p^2 + i\omega(i\omega+\nu))]$

Solved using Green functions

$$G(x,x_0) = \frac{1}{\beta}\left[-\frac{sh\beta(L-x_0)}{sh\beta L}sh\beta x_0 + Z(x-x_0)sh\beta(x-x_0)\right]$$

Solution in common source configuration

$$\Delta \mathbf{J} = \begin{bmatrix} \Delta J_d \\ \Delta J_g \end{bmatrix}, \quad \Delta \mathbf{U} = \begin{bmatrix} \Delta U_d \\ \Delta U_g \end{bmatrix} \qquad \Delta \mathbf{J} = \hat{Y} \Delta \mathbf{U} + \xi_J$$

Admittance

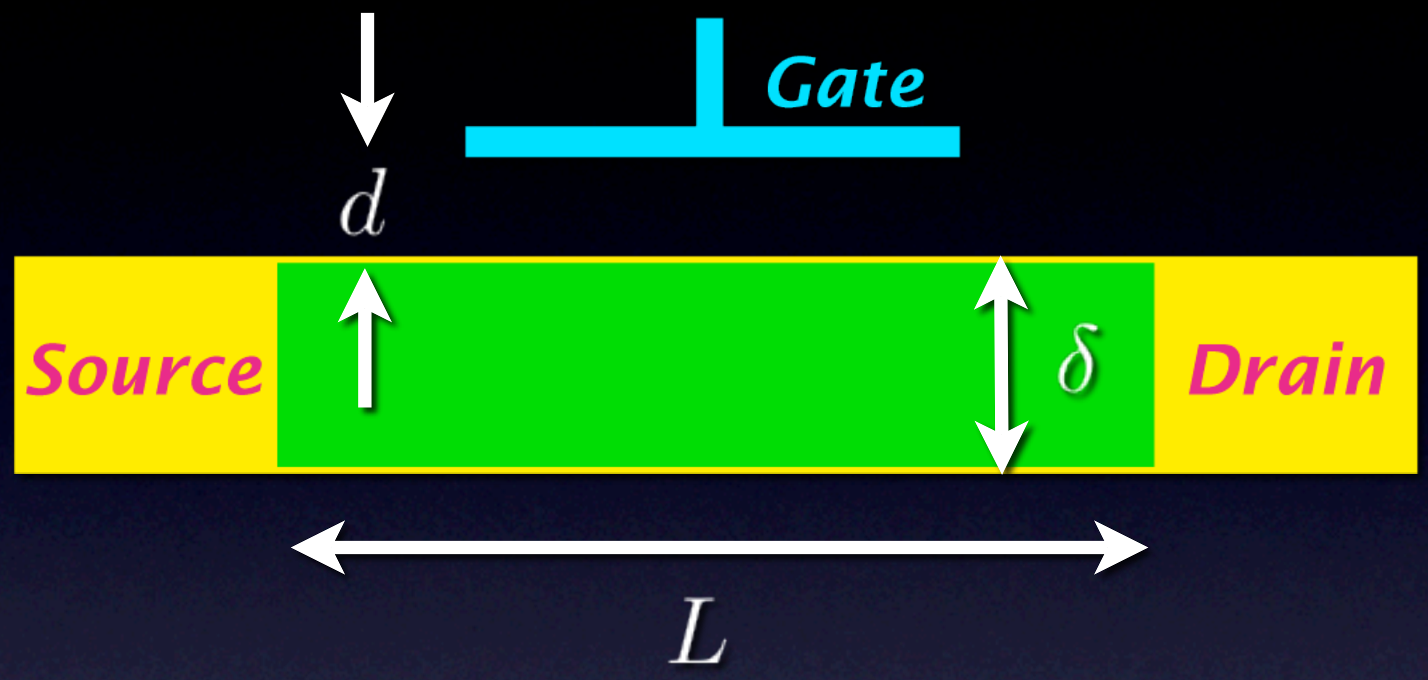
$$\hat{Y} = \epsilon\epsilon_0 \frac{\omega_p^2 + i\omega(i\omega + \nu)}{i\omega + \nu} \, \frac{\beta sh\beta \frac{L}{2}}{ch\beta \frac{L}{2}} \begin{pmatrix} -\frac{ch\beta L}{ch\beta L - 1} & 1 \\ 1 & -2 \end{pmatrix}$$

Norton generator of current noise

$$\xi_J = \epsilon\epsilon_0 \frac{\omega_p^2 \beta}{i\omega + \nu} \int_0^L \begin{bmatrix} ch\beta x_0 / sh\beta L \\ sh\beta(\frac{L}{2} - x_0)/ch\beta \frac{L}{2} \end{bmatrix} \tilde{f}(x_0) dx_0$$

Model structure

$$\begin{cases} L = 1000 \text{ nm} \\ \delta = 15 \text{ nm} \\ d = 15 \text{ nm} \\ N_D^{3D} = 8 \times 10^{17} \text{ cm}^{-3} \\ 30 + 30 \text{ nm ungated} \\ \nu = 3 \times 10^{12} \text{ s}^{-1} \\ m^* = 0.048 m_0 \end{cases}$$



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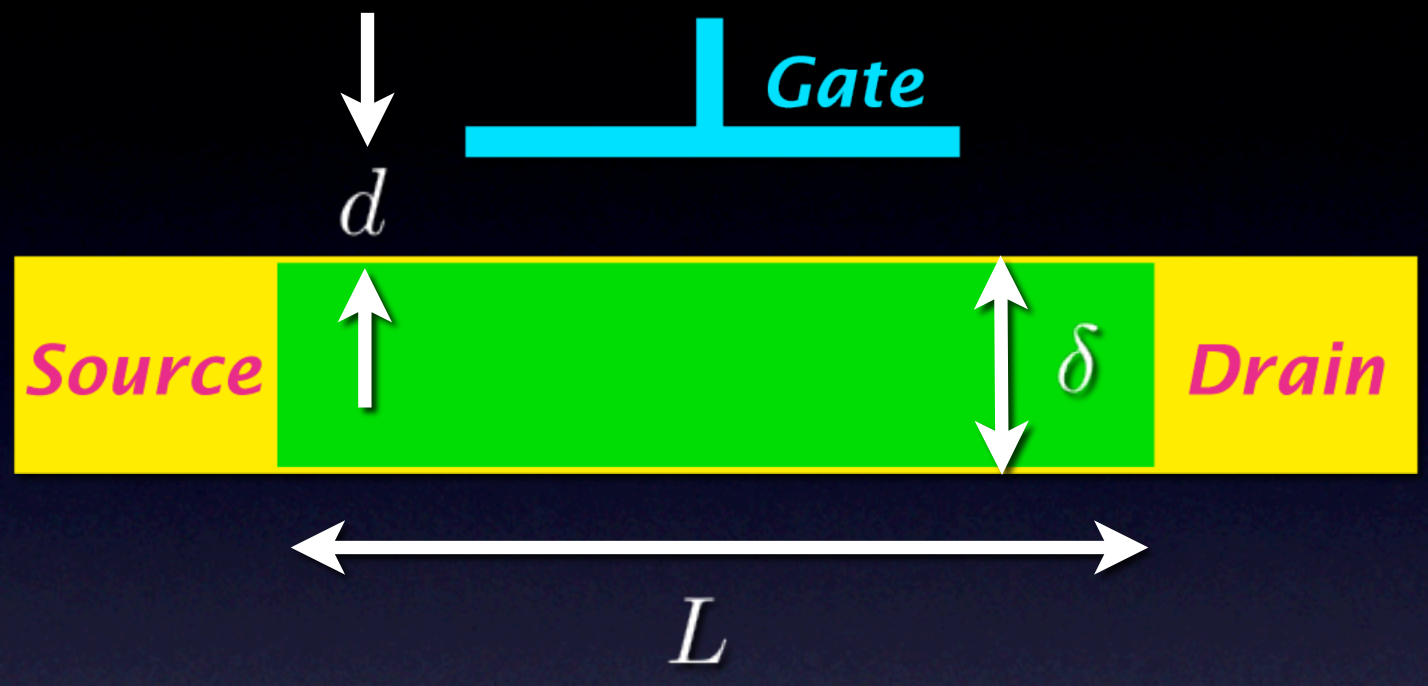
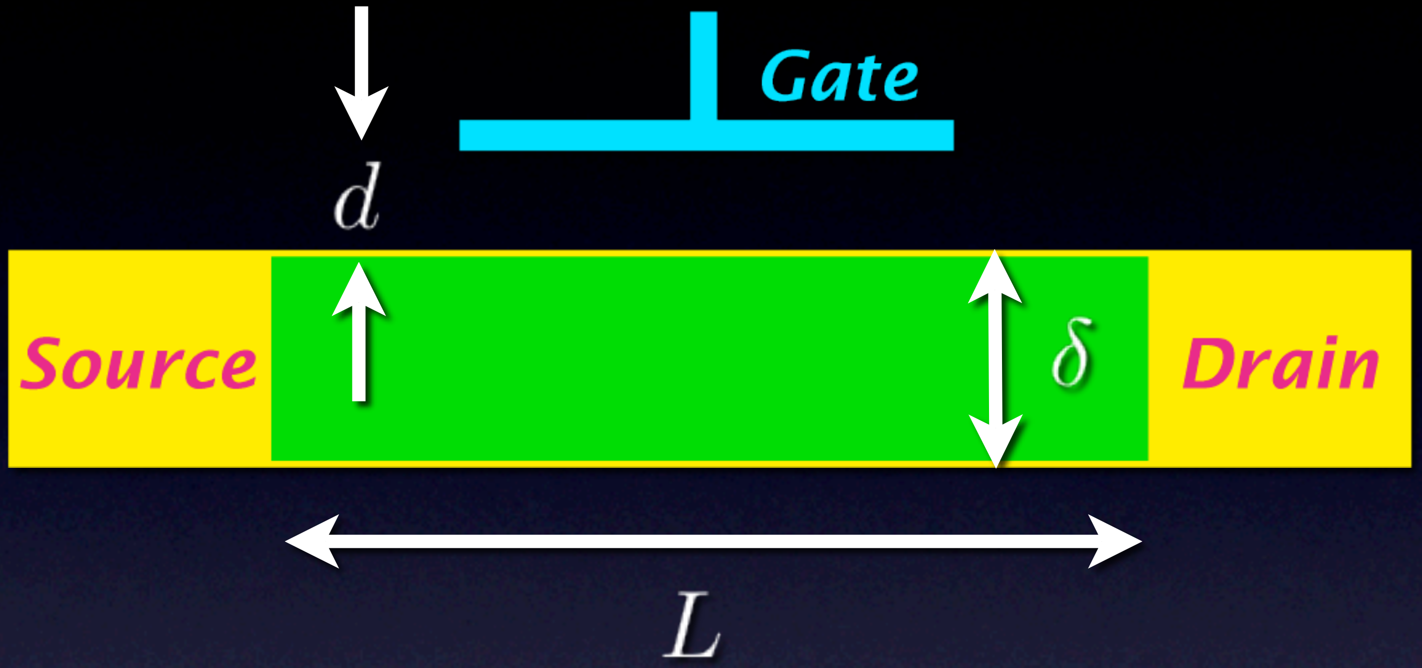


$$S_{\xi\xi}(\omega) = \int_0^L n^{3D} \left|G_\xi(\omega, x_0)\right|^2 S_{ff} \, dx_0$$

Model structure

$$\begin{cases} L = 1000 \text{ nm} \\ \delta = 15 \text{ nm} \\ d = 15 \text{ nm} \\ N_D^{3D} = 8 \times 10^{17} \text{ cm}^{-3} \\ 30 + 30 \text{ nm ungated} \\ \nu = 3 \times 10^{12} \text{ s}^{-1} \\ m^* = 0.048 m_0 \end{cases}$$

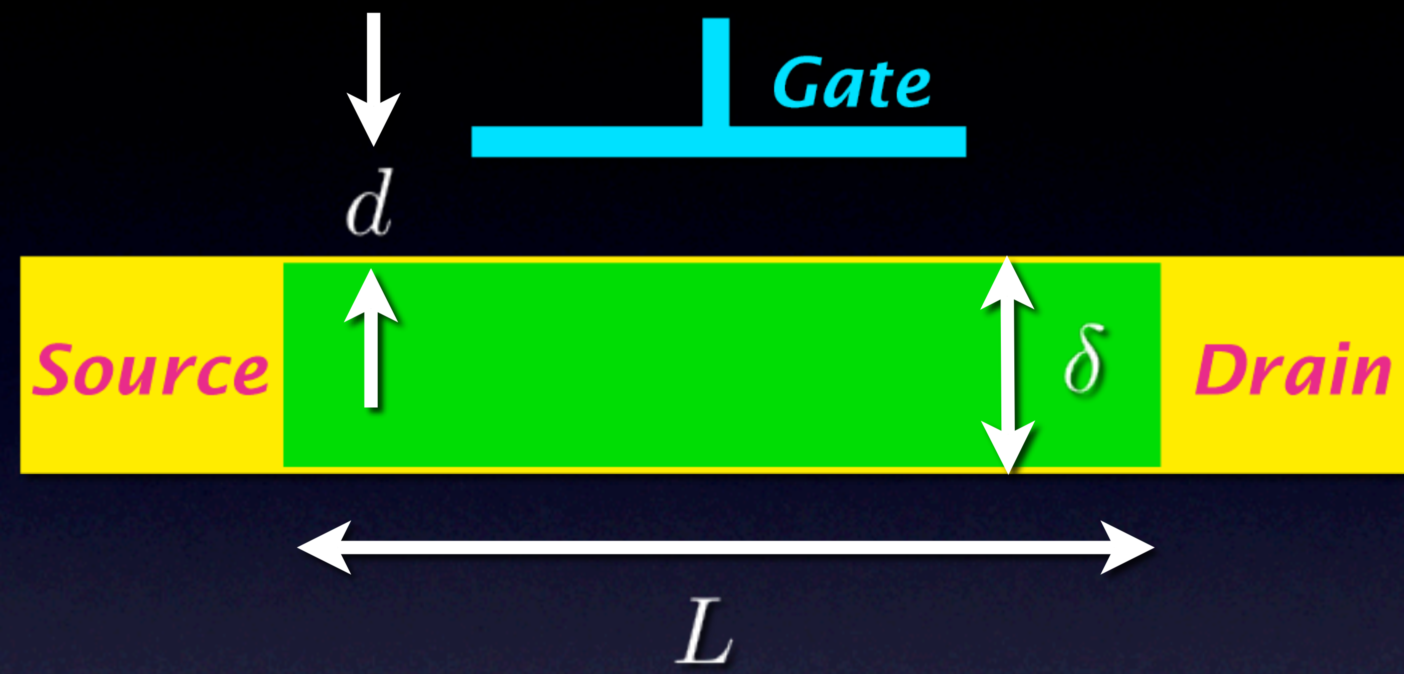


$$S_{\xi\xi}(\omega) = \int_0^L n^{3D} \boxed{|G_\xi(\omega, x_0)|^2} S_{ff} \, dx_0$$

Transfer field

Model structure

$$
\begin{cases}
L = 1000 \text{ nm} \\
\delta = 15 \text{ nm} \\
d = 15 \text{ nm} \\
N_D^{3D} = 8 \times 10^{17} \text{ cm}^{-3} \\
30 + 30 \text{ nm ungated} \\
\nu = 3 \times 10^{12} \text{ s}^{-1} \\
m^* = 0.048 m_0
\end{cases}
$$



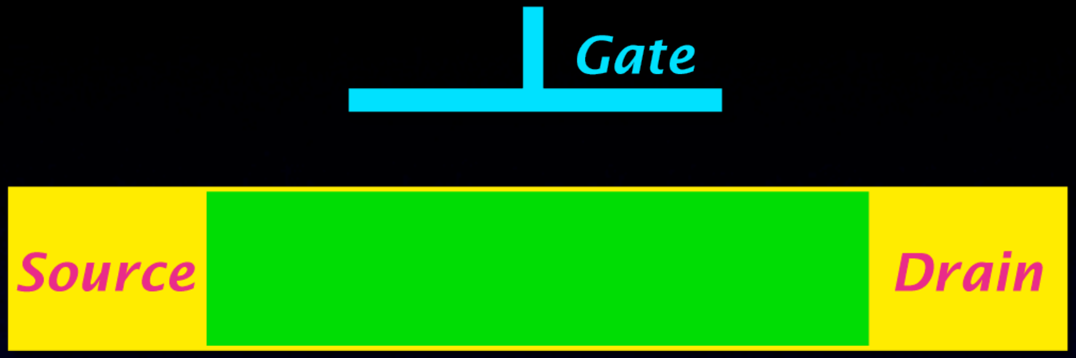
$$S_{\xi\xi}(\omega) = \int_0^L n^{3D} \, \underbrace{|G_\xi(\omega, x_0)|^2}_{\text{Transfer field}} \, \underbrace{S_{ff}}_{\text{Spectral density of Langevin force}} \, dx_0$$

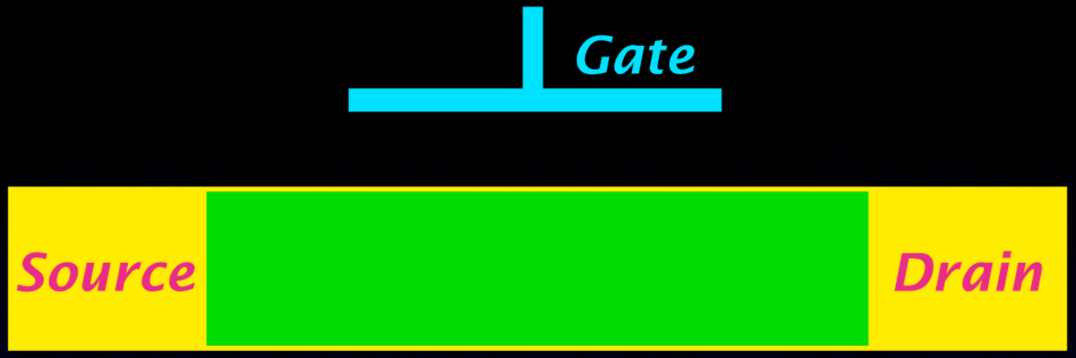
Transfer field

Spectral density of Langevin force

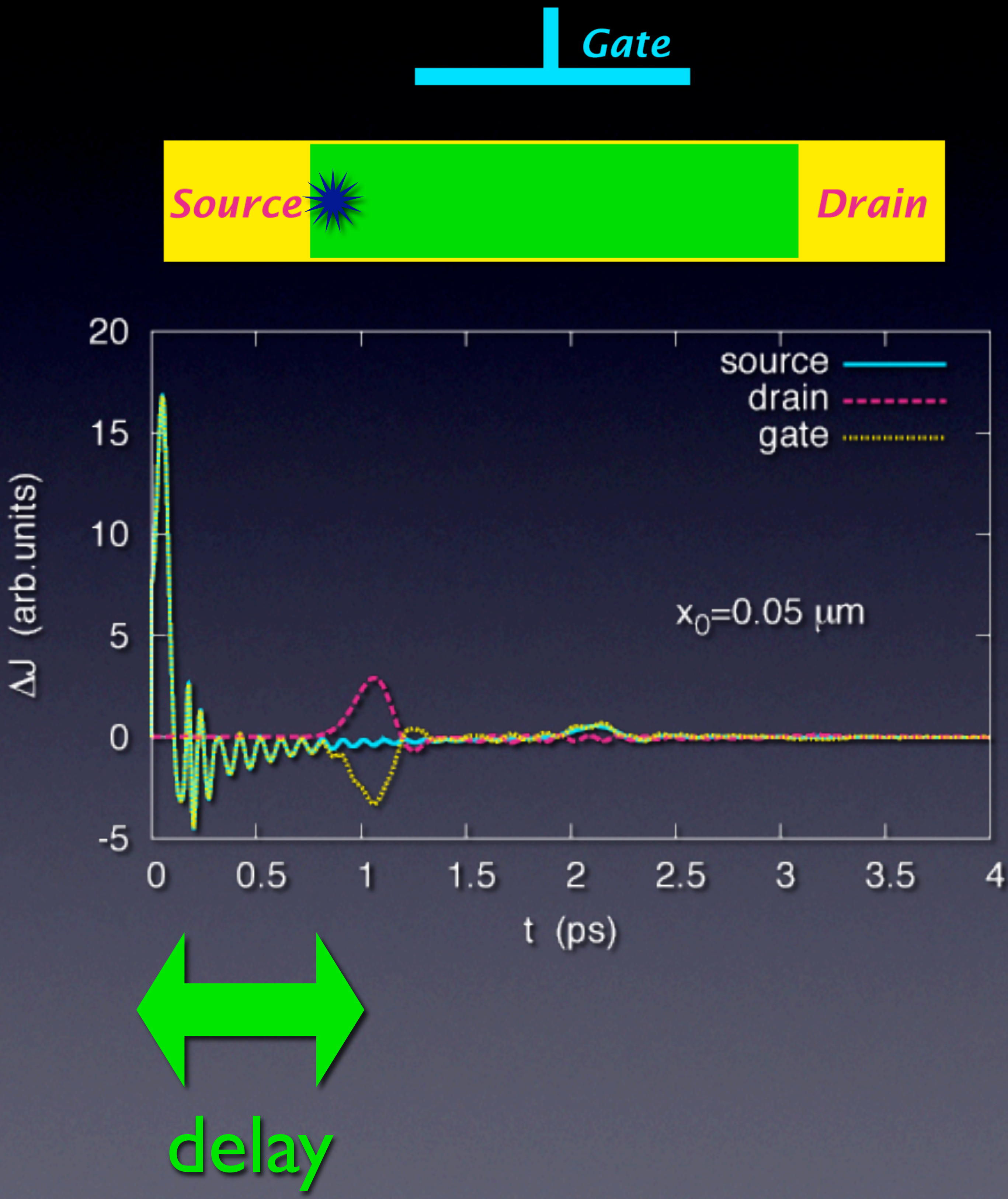
$= 4 k_B T \nu / m^*$

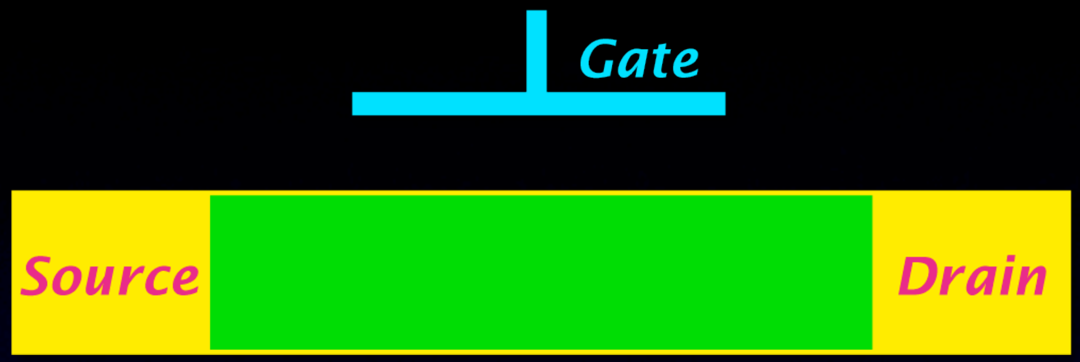
Current response at equilibrium



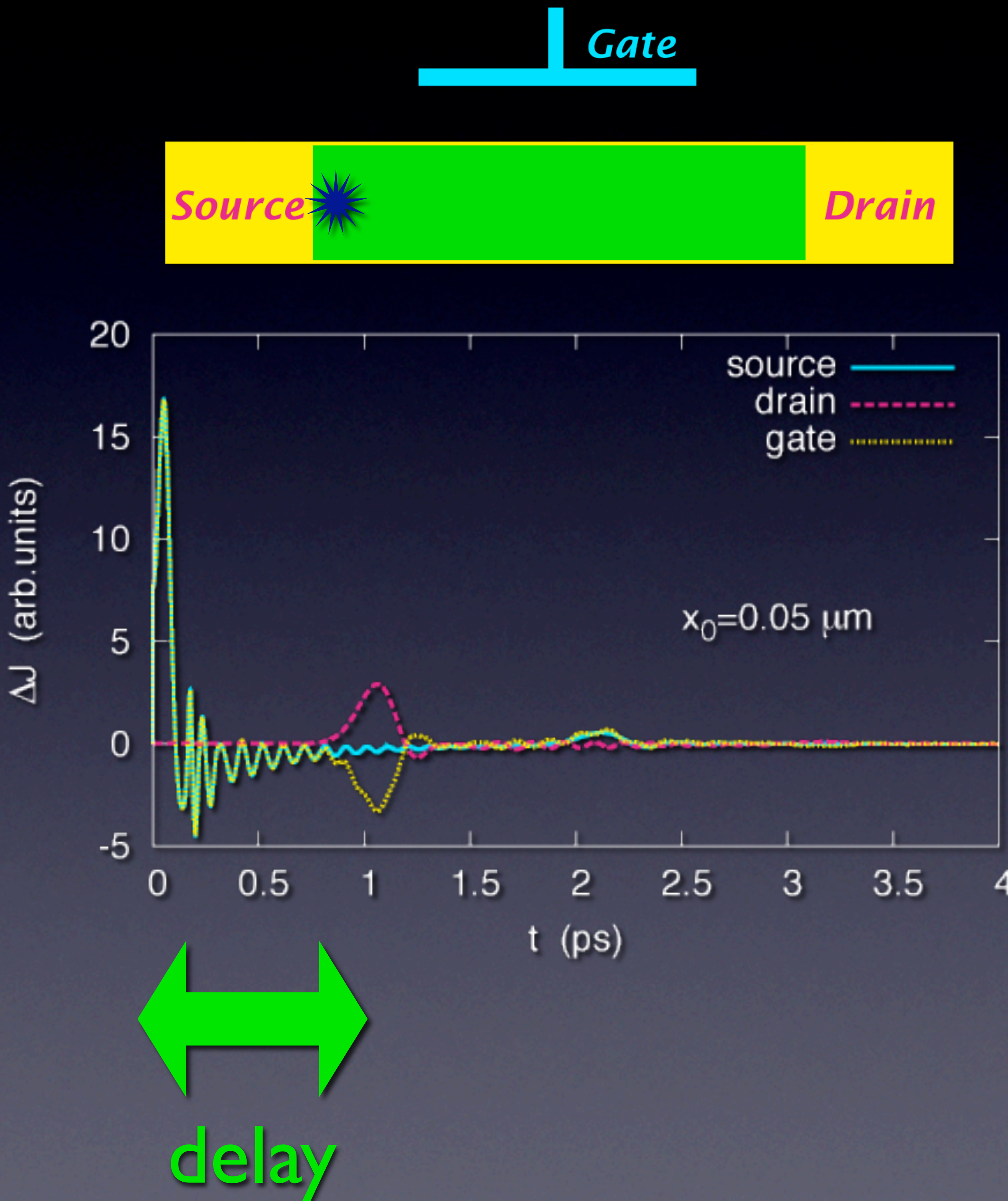


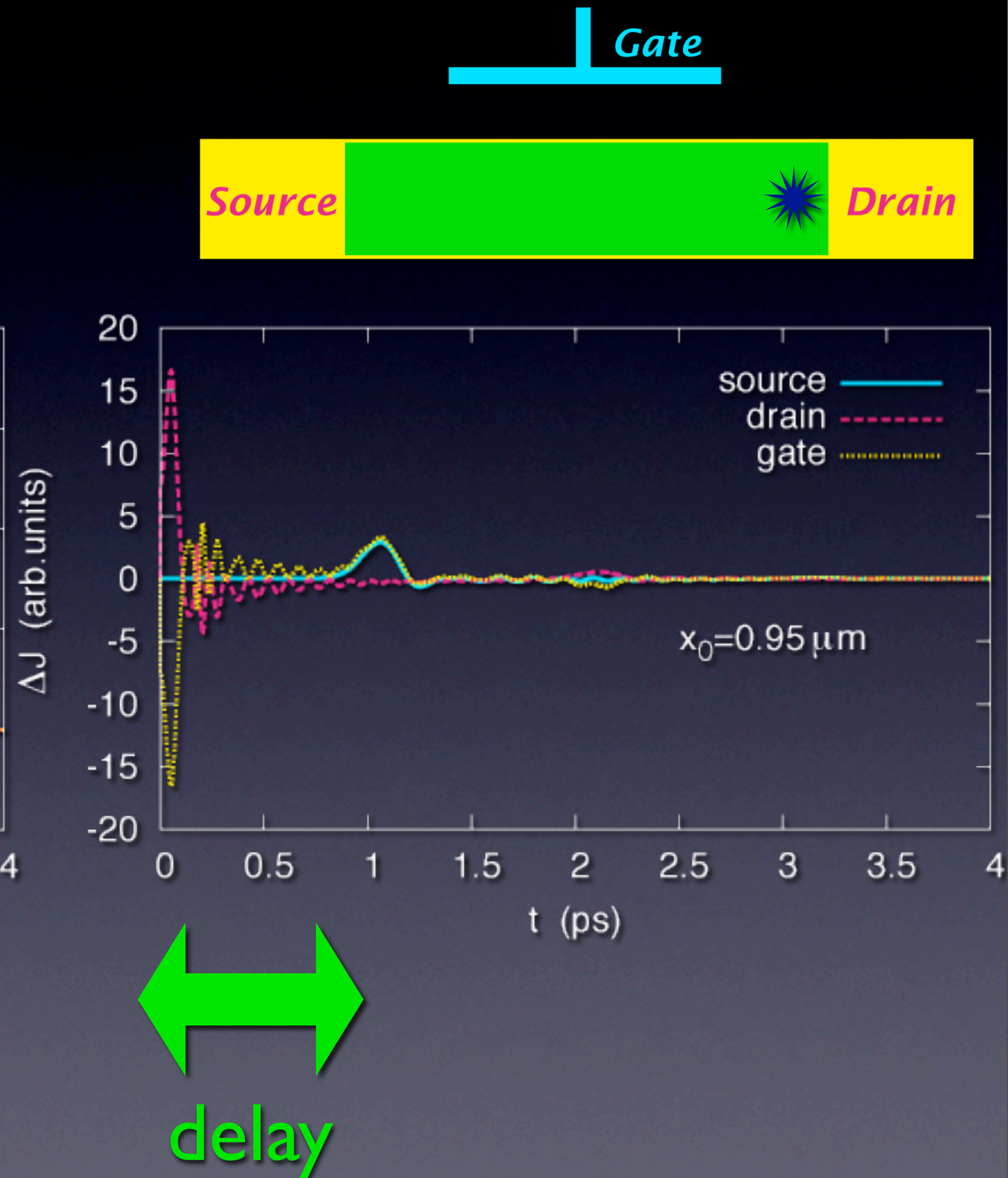
Current response at equilibrium





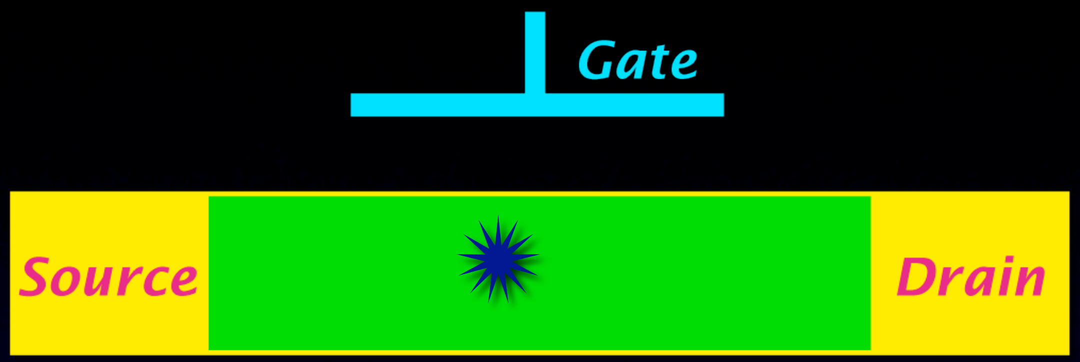
Current response at equilibrium

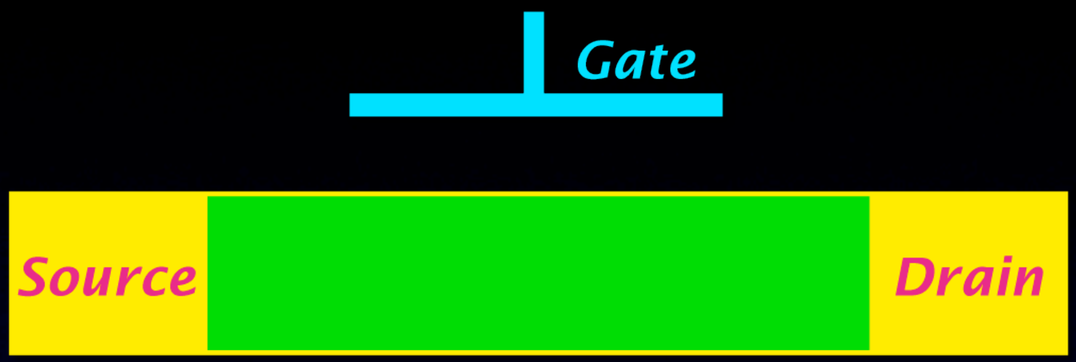


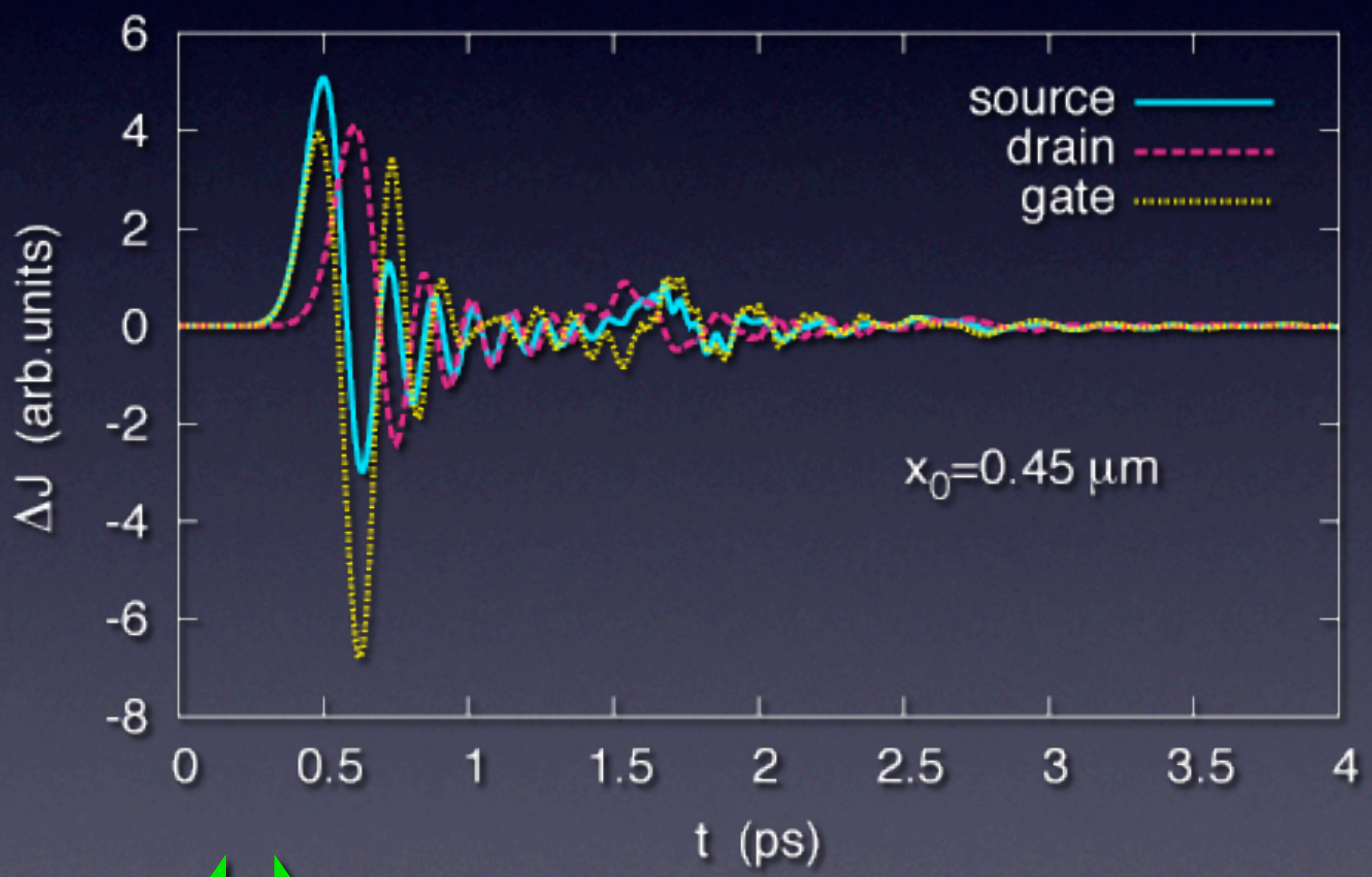


Current response at equilibrium

Current response at equilibrium

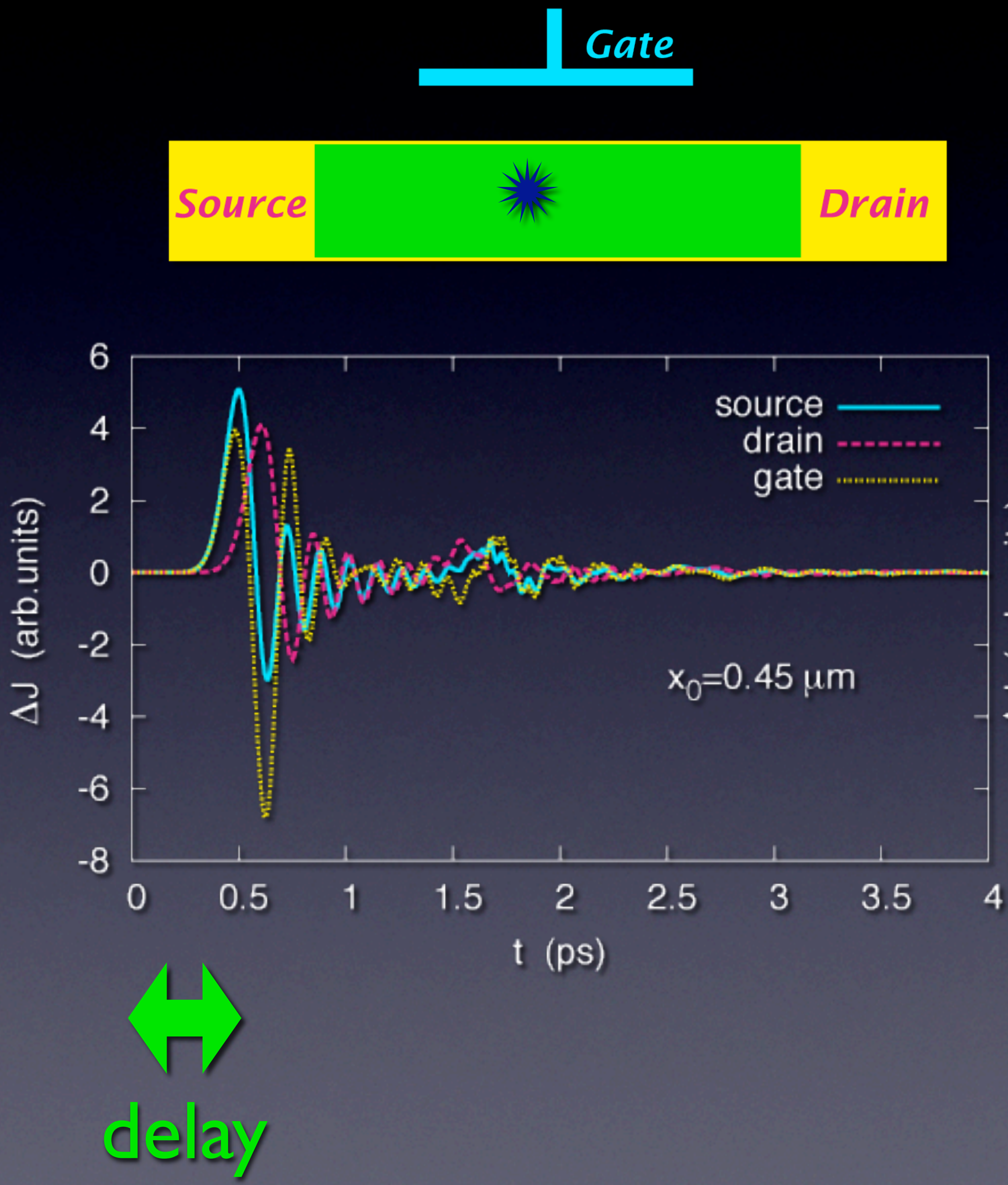


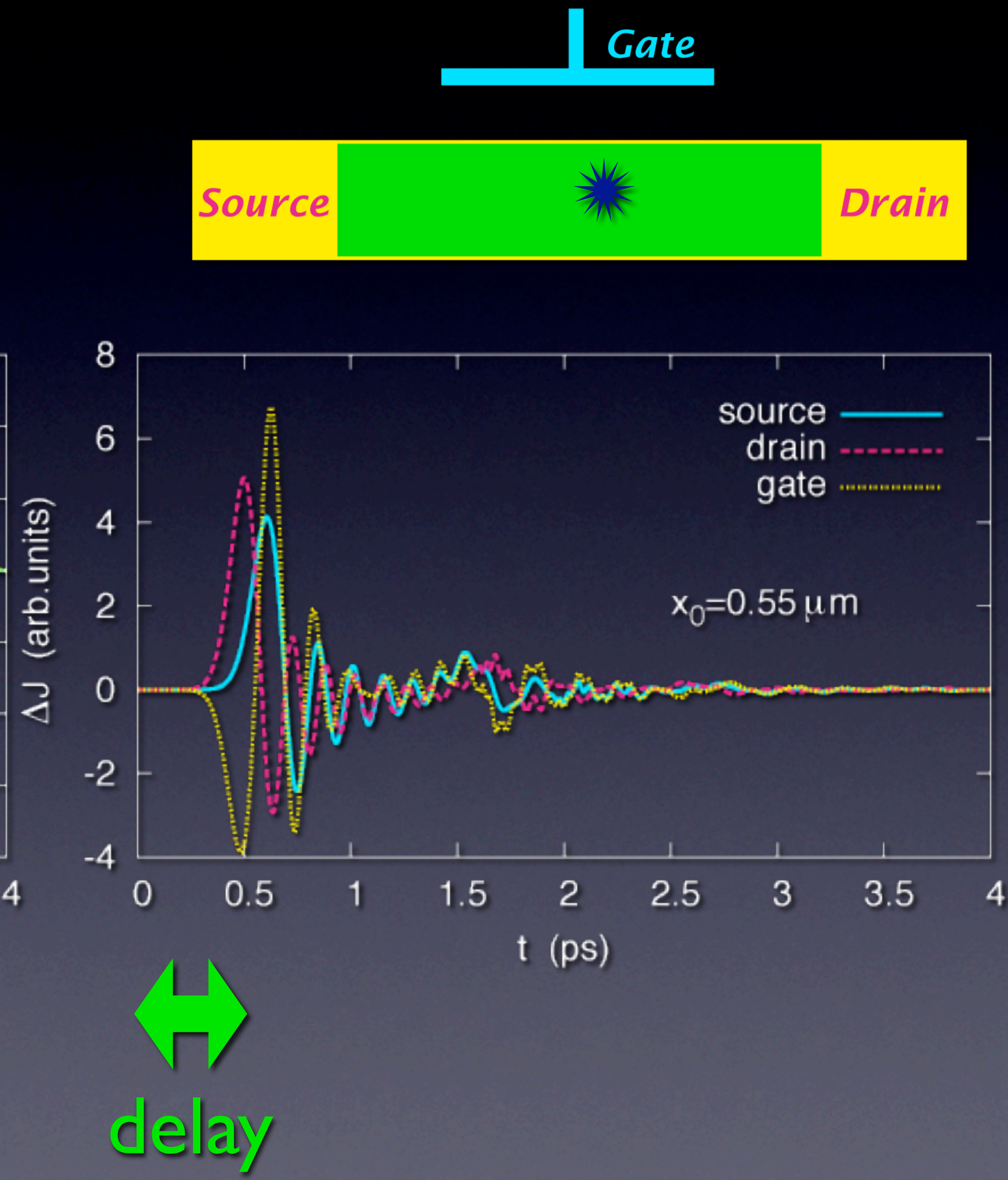




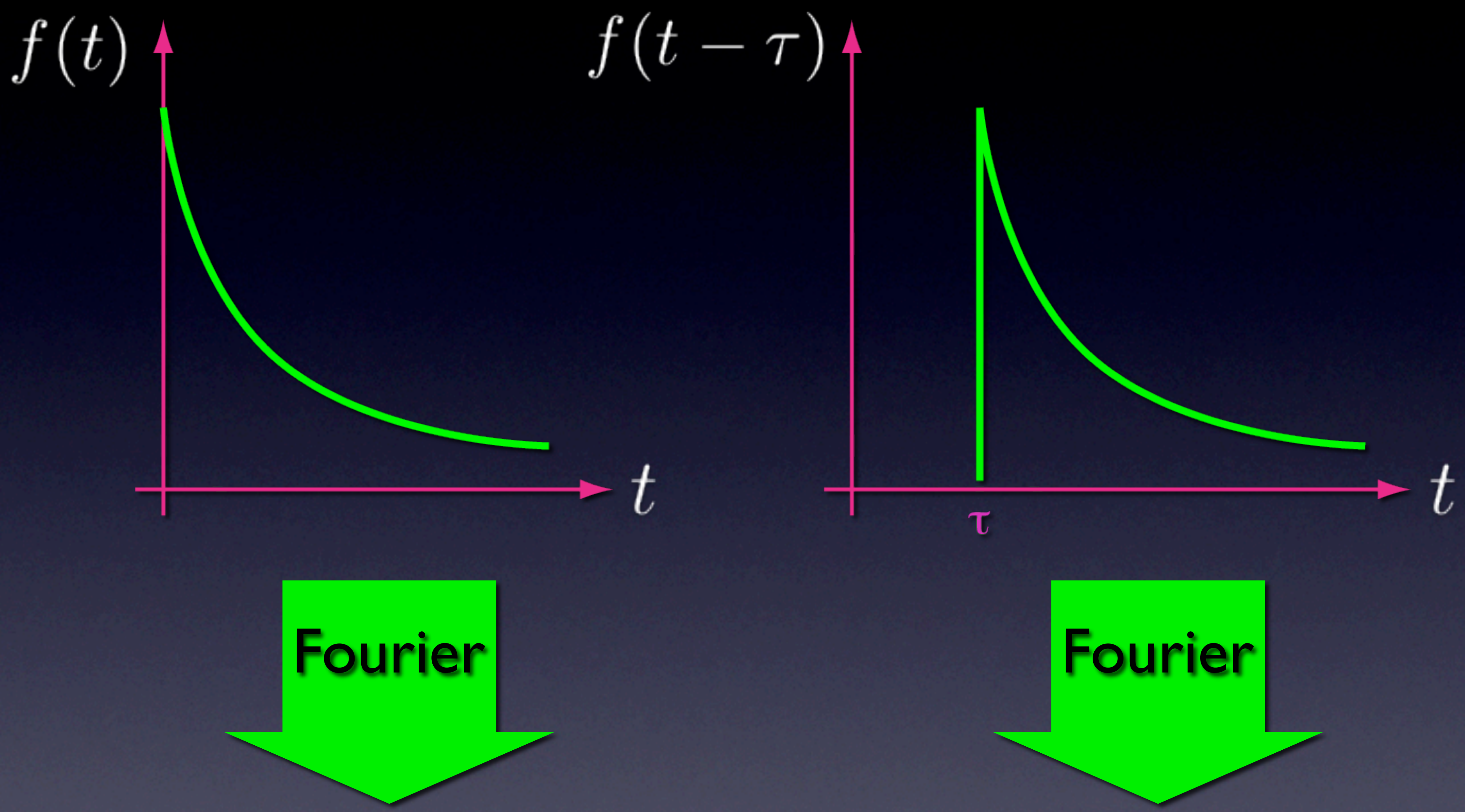
delay

Current response at equilibrium



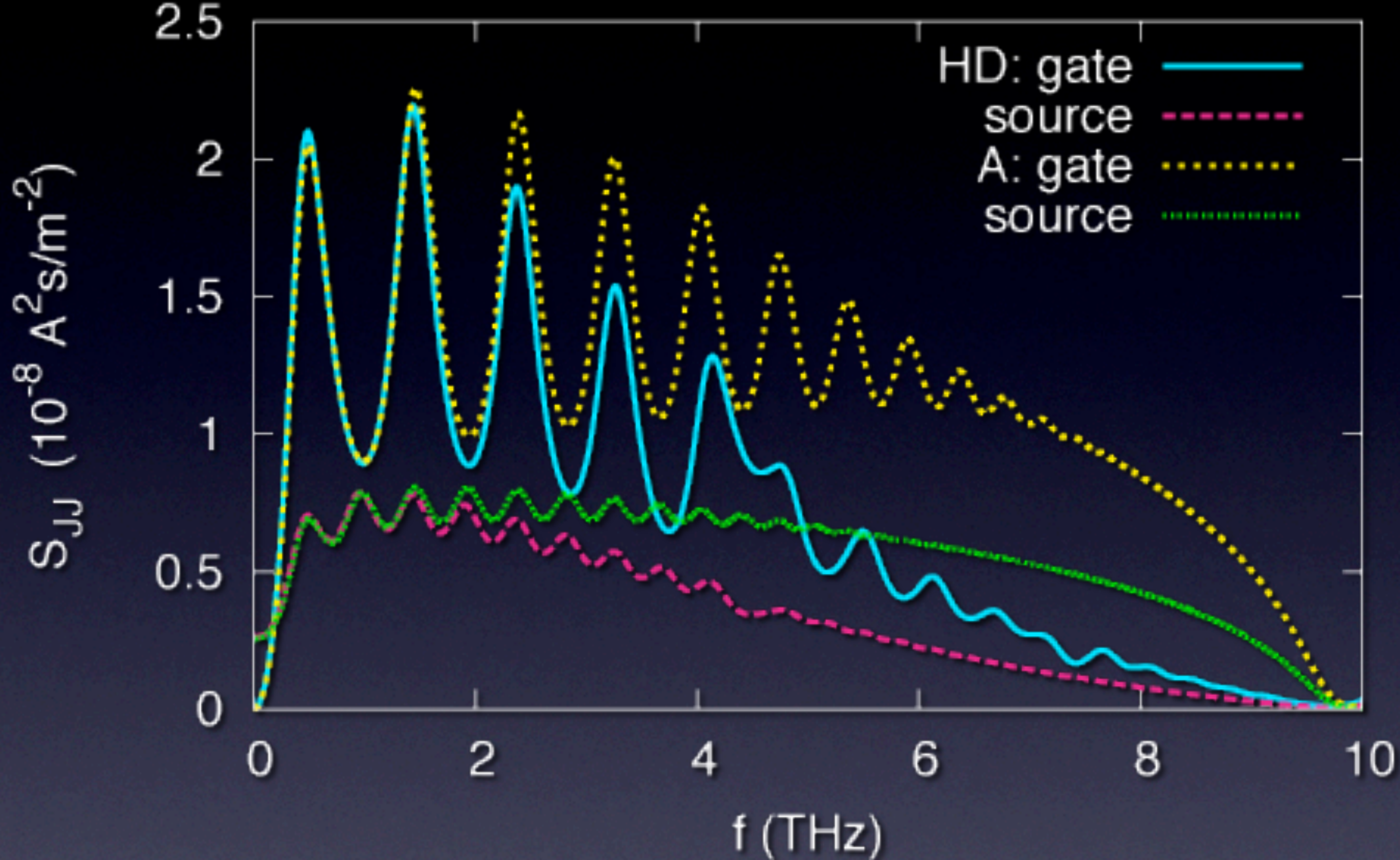


Effect of time delay



$F(\omega)$ $\qquad$ $e^{-i\omega\tau}F(\omega)$

Spectra of current fluctuations



- Resonances in the THz domain
- Ungated regions suppresses HF peaks
- Different resonances at source and gate

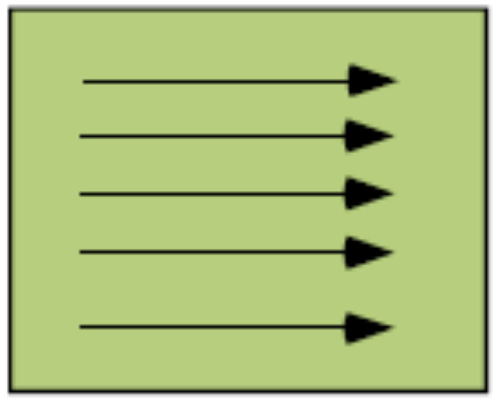
Unsolved problem #1

Is it possible to measure electronic noise at TeraHertz frequencies?

Challenge for experiments

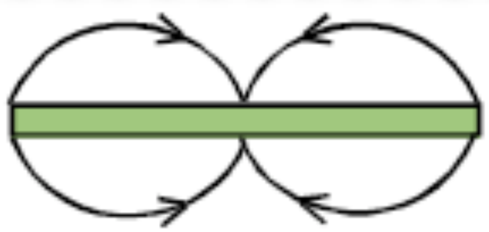
Plasma waves in semiconductors

3D

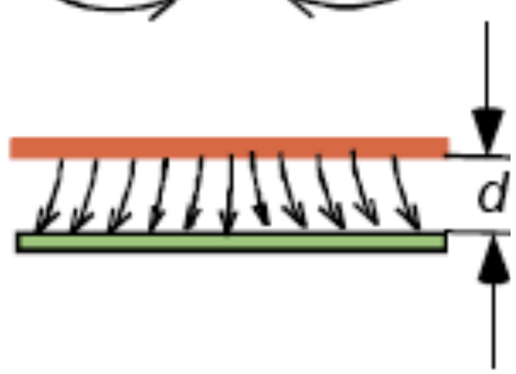


concentration $\omega_p^{3D} = \sqrt{\dfrac{e^2 n}{\epsilon_c \epsilon_0 m}}$

ungated
2D



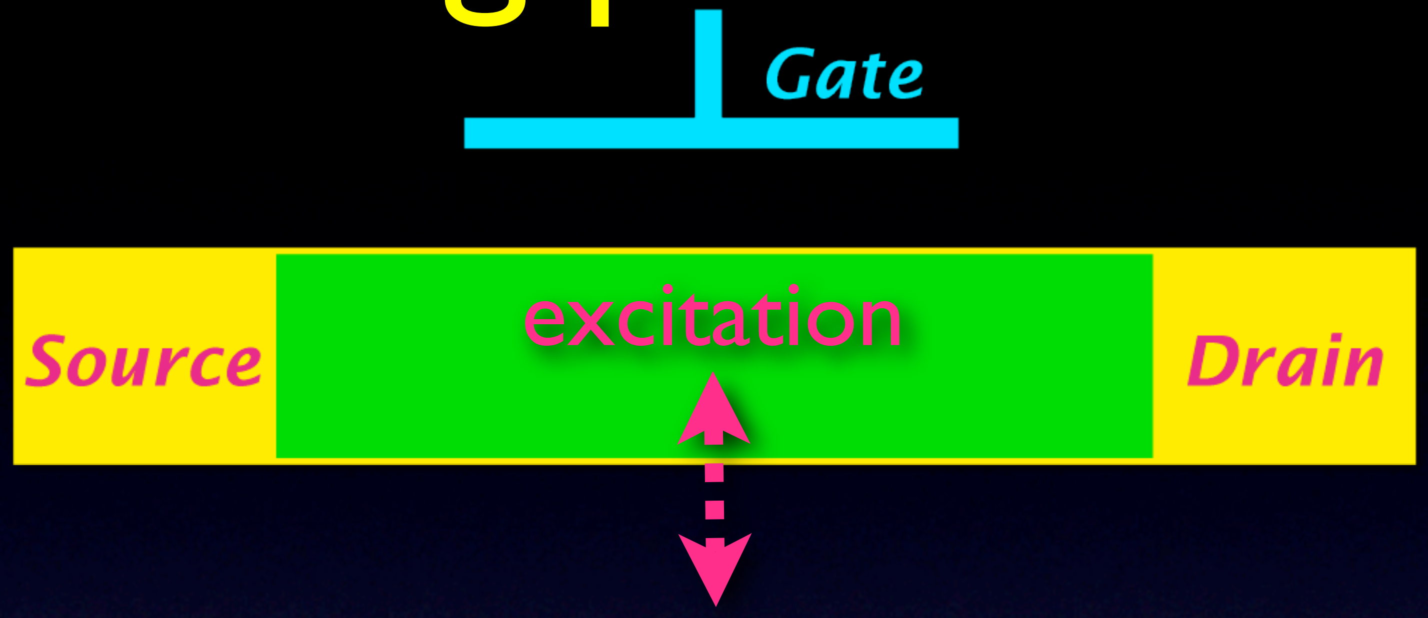
gated
2D



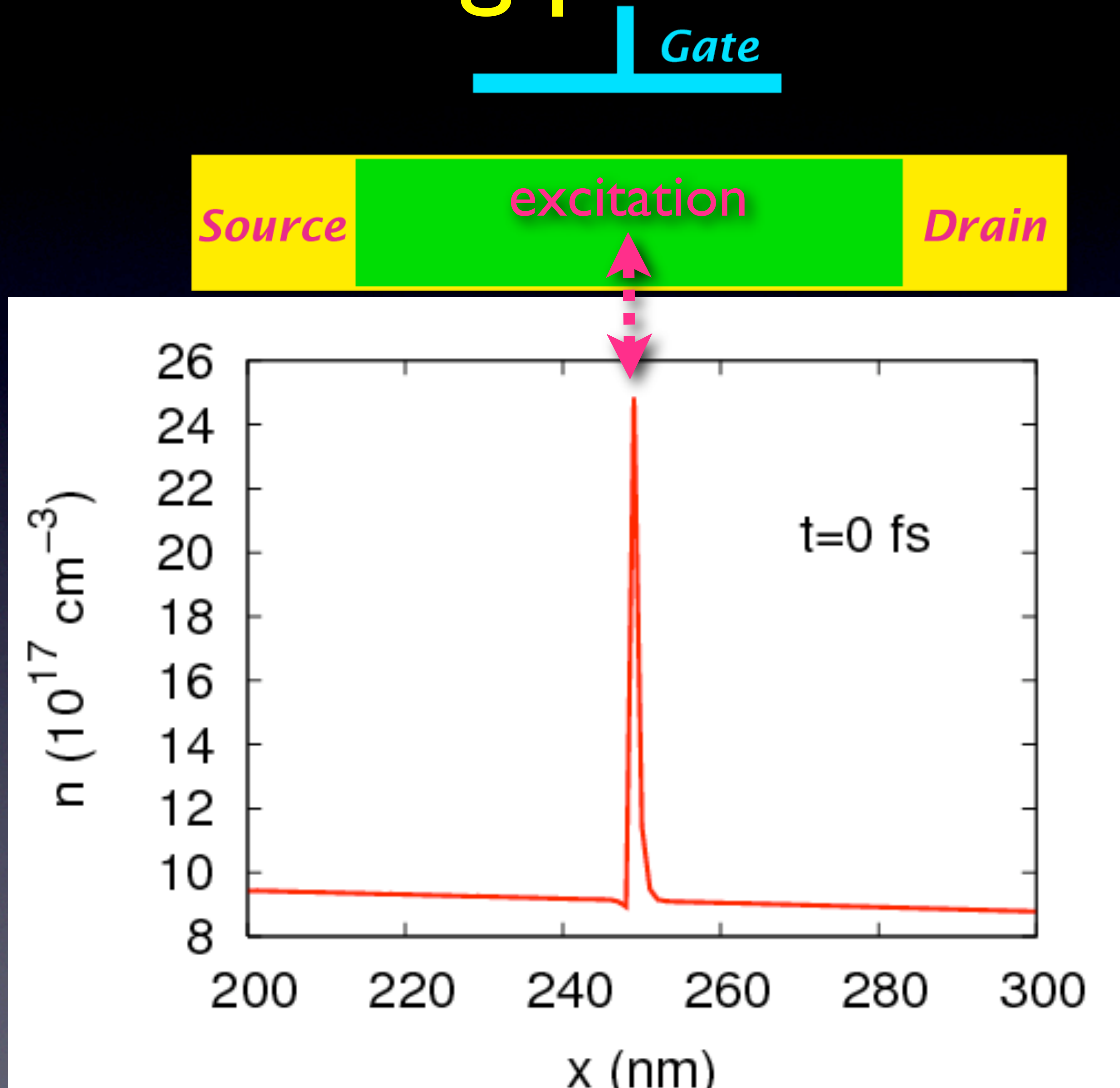
1D

concentration
+
geometry

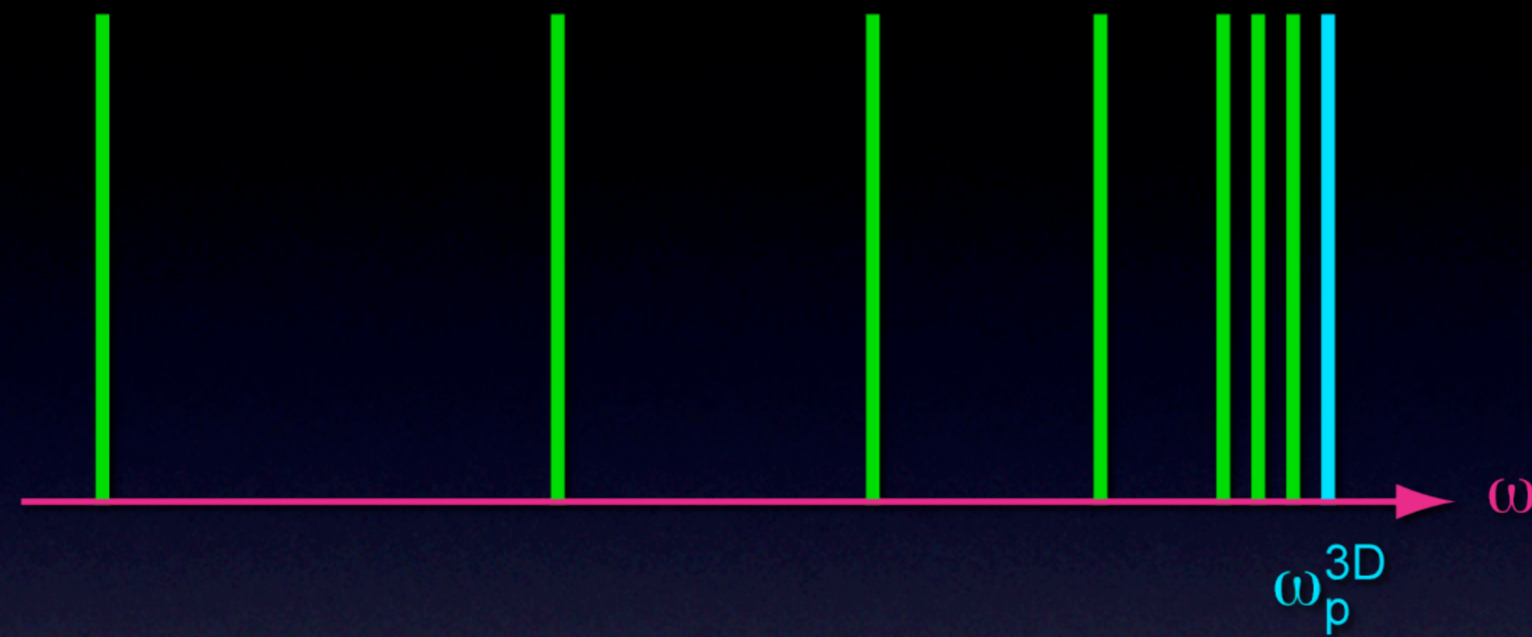
Travelling plasma waves



Travelling plasma waves



Plasma modes in field-effect transistors



$$\omega^{i}_{res}(n) = \omega_p^{3D} \frac{n}{\sqrt{(\lambda L/\pi)^2 + n^2}} \begin{cases} n = 0, 1, 2, \ldots & \text{, i=d} \\ n = 1, 3, 5, \ldots & \text{, i=g} \end{cases}$$

3D plasma frequency

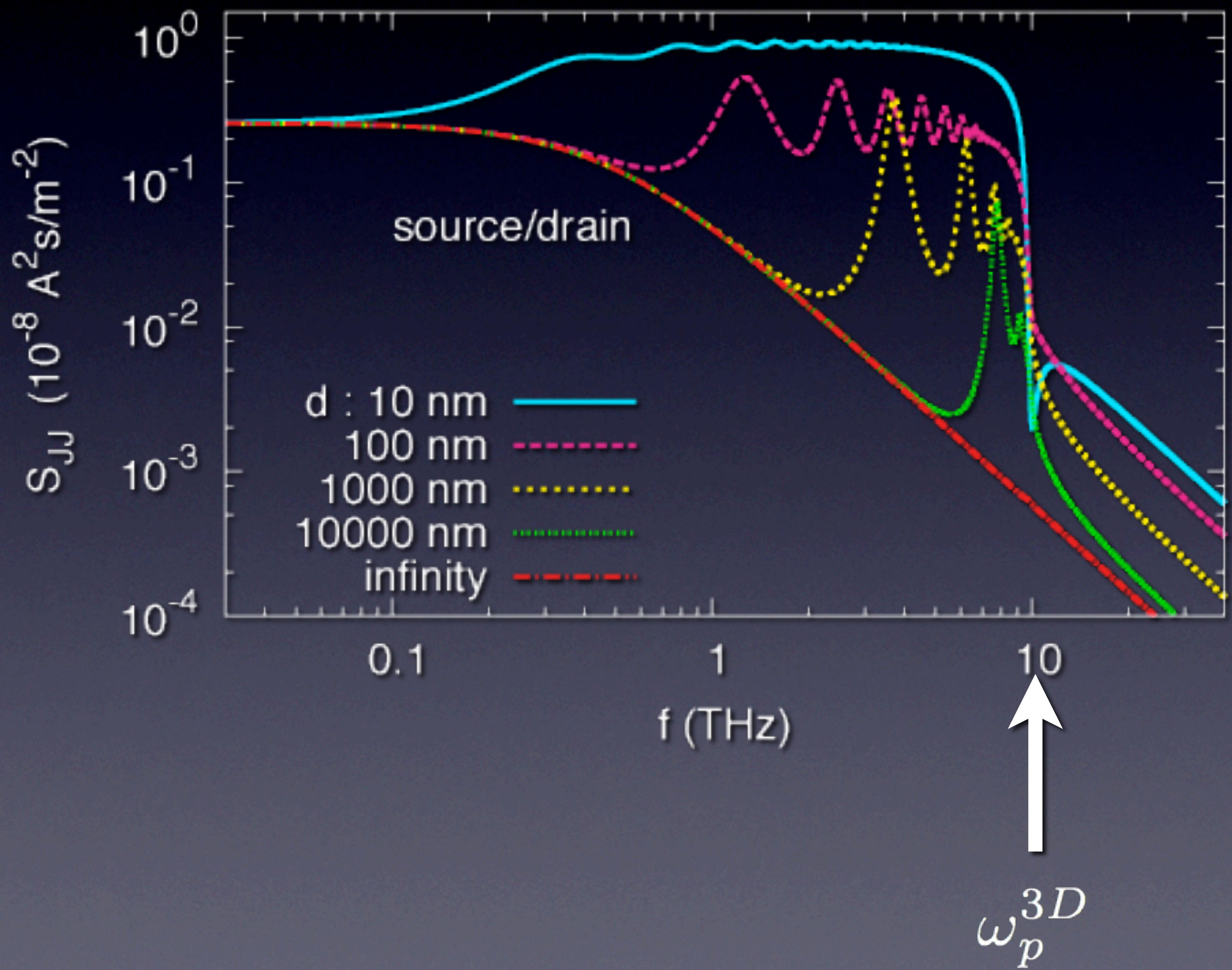
geometry factor $\lambda = \sqrt{\epsilon_d/\epsilon_c d\delta}$

Unsolved problem #2

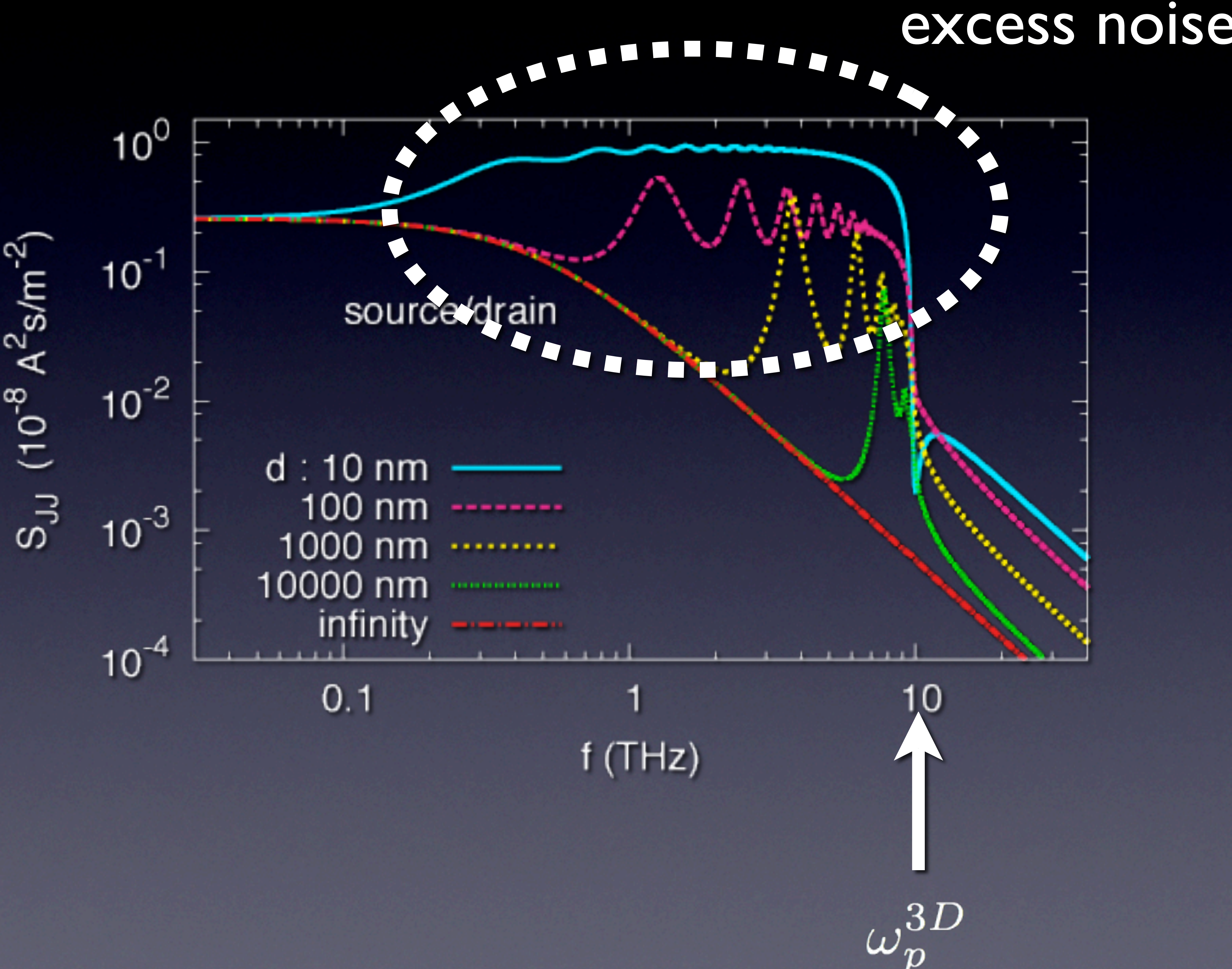
Is it possible to exploit plasma waves to obtain a detector or emitter of TeraHertz radiation?

Huge number of potential applications

From ungated to gated transistor



From ungated to gated transistor

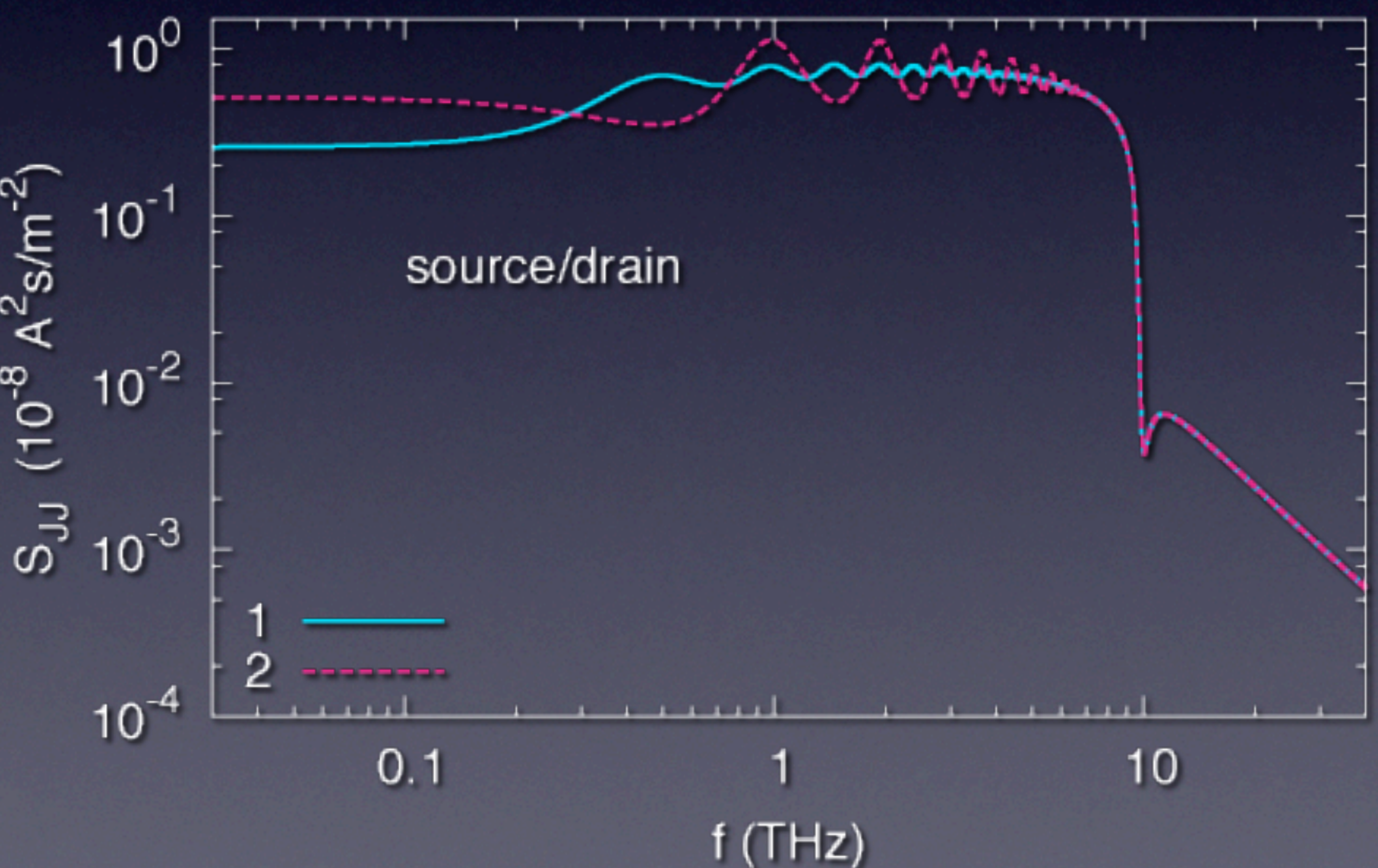


Unsolved problem #3

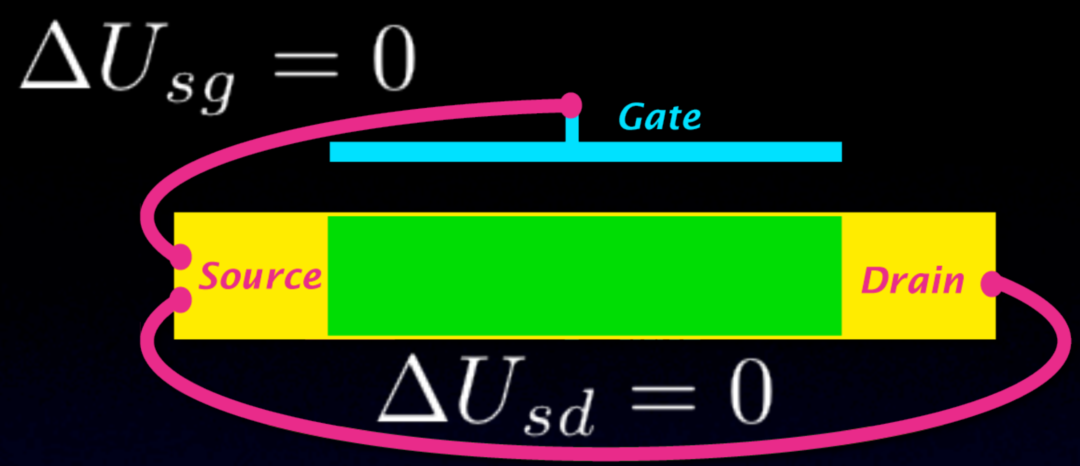
Which is the physical origin of the excess noise?

The system is at equilibrium and total noise power increases (not simple noise redistribution)

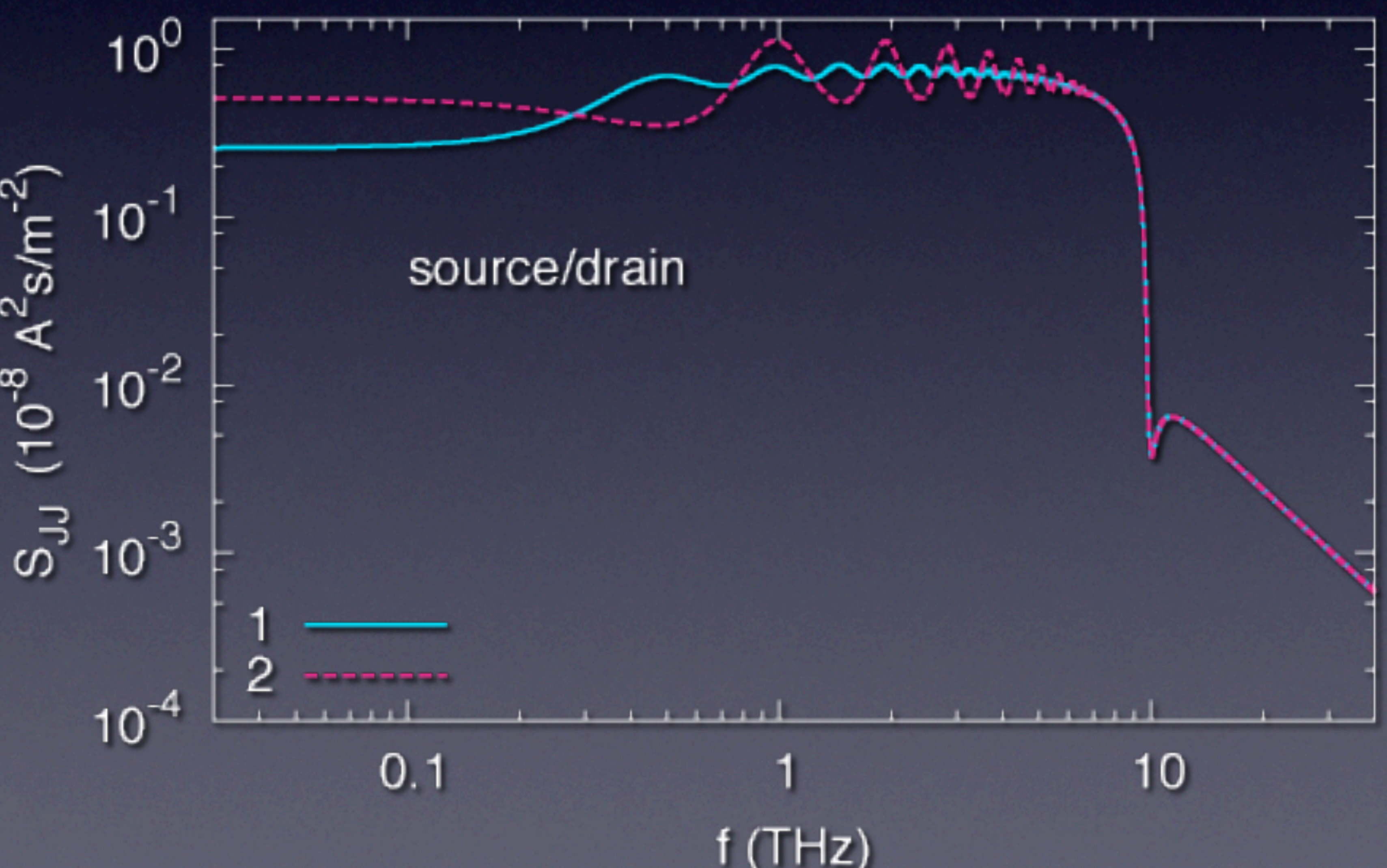
Noise vs operation mode



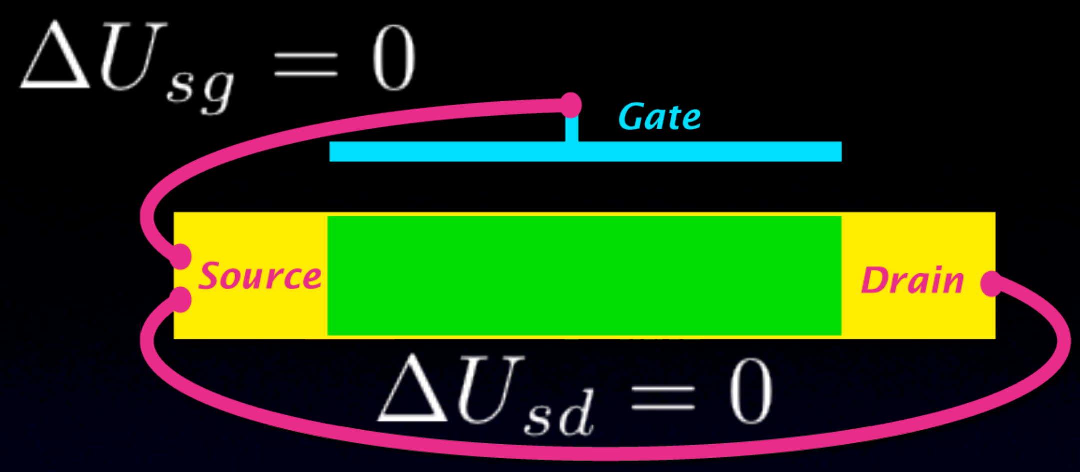
Noise vs operation mode



closed



Noise vs operation mode



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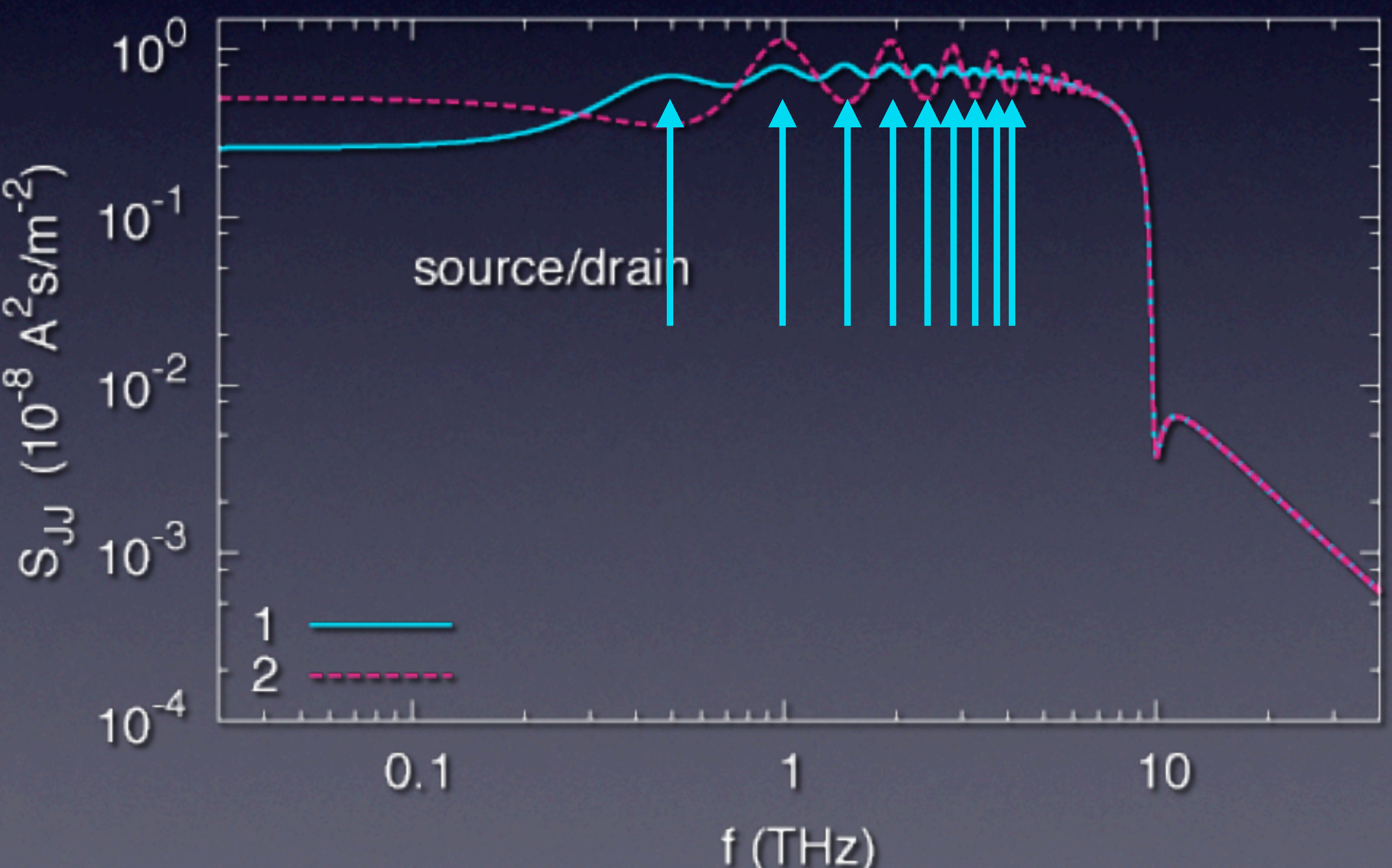
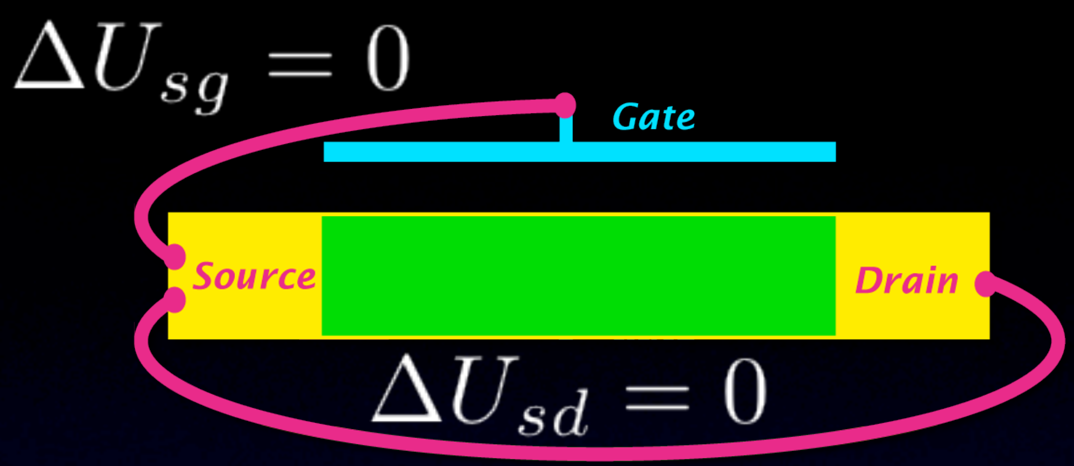
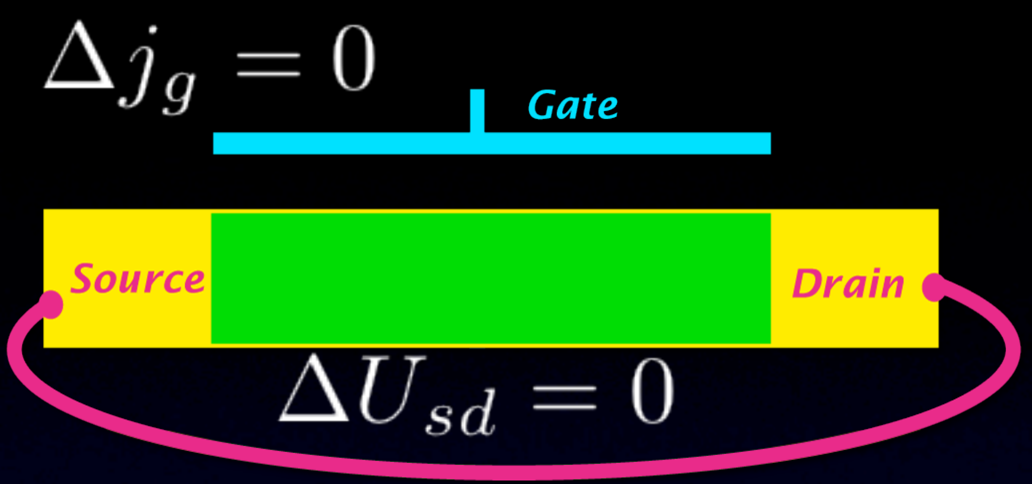


Noise vs operation mode





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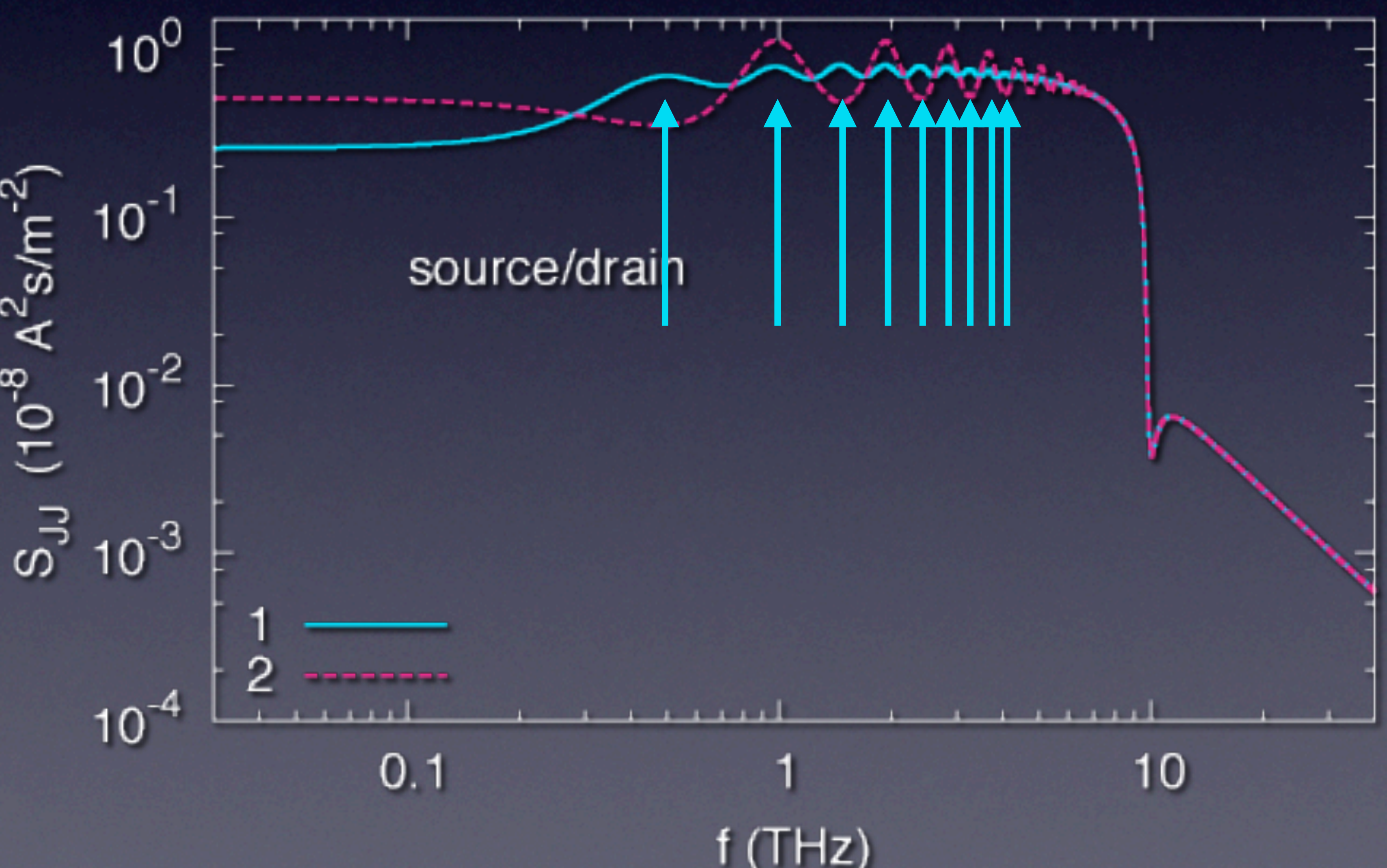
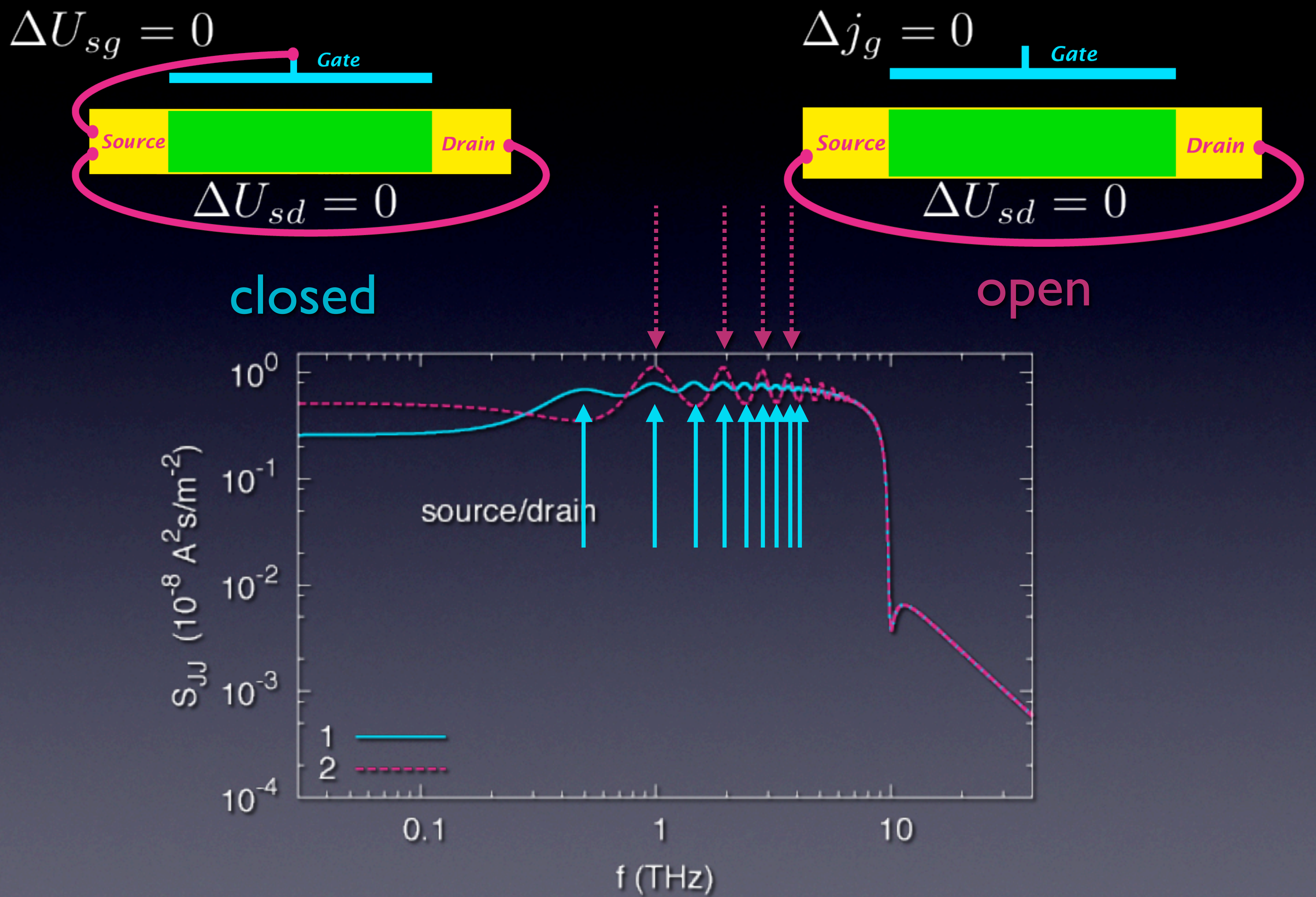
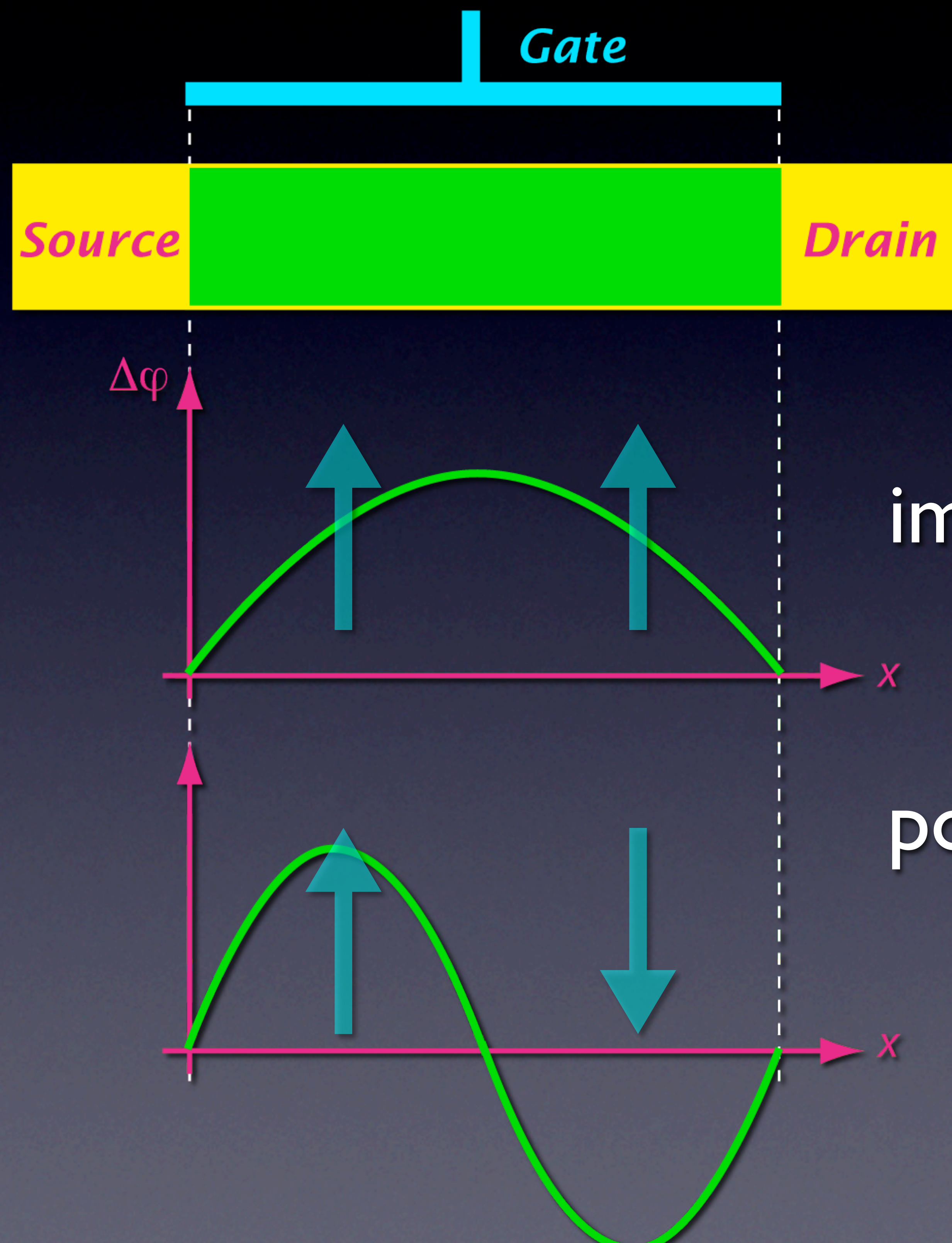


Noise vs operation mode



Suppression of odd harmonics

$$\Delta j_g = 0$$



impossible (n=1)

possible (n=2)

Unsolved problem #4

Which is the intrinsic noise of the transistor?

Noise spectra depend on operating conditions
Failure of Thevenin/Norton equivalence?

Conclusions

- Novel circuit + analytical model
- Thermal fluctuations excite standing plasma waves at THz frequencies
- Current branching strongly influences intrinsic noise spectra

unsolved problem on branching