Does the eye tremor provide the hyperacuity phenomenon?

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Fluctuations in the retina

Retina = map of sensors subjected to noise :

- Neuronal noise
- Random sampling by the photoreceptors
- Random distribution of rods and cones
- Saccadic movement of the eye (tremor)

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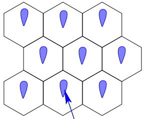
Aim :
investigation of this fluctuations on the visual process, through noise-enhanced processing point of view (e.g. Hyperacuity)

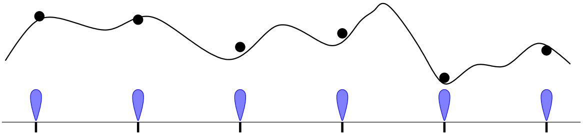
Henning et al. *Neurocomputing* 2004

Hongler et al. *IEEE PAMI* 2003

Landolt & Mitros *Autonomous Robots* 2001

Model of Retina





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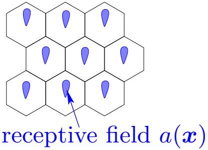
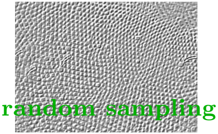
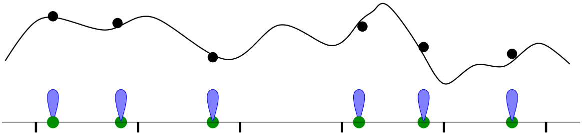


part of the fovea



Kolb et al.
webvision.edu.utah.edu



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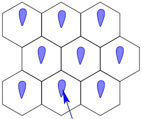

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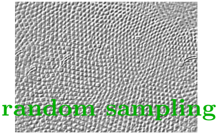

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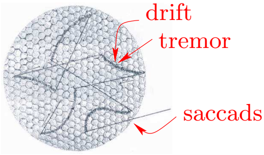
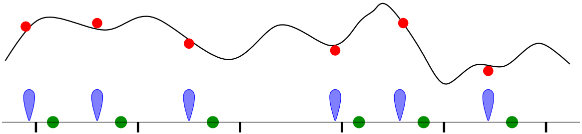


Martinez-conde et al., 2004



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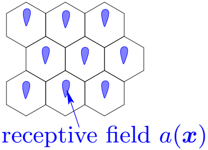

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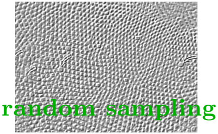

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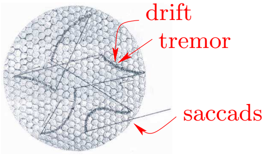

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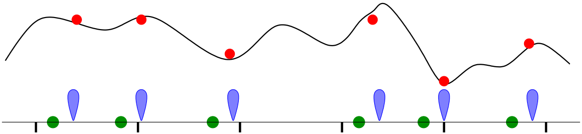

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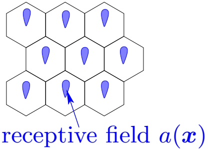

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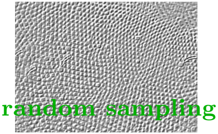

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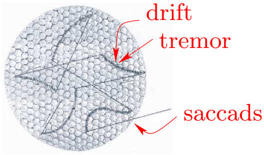

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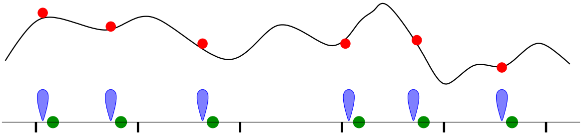

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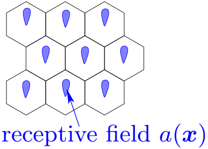

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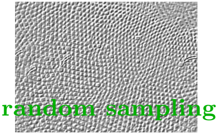

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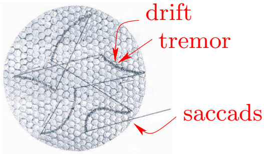

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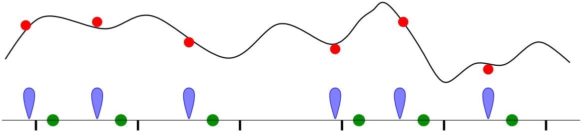

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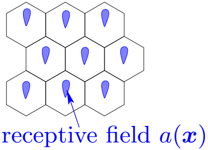

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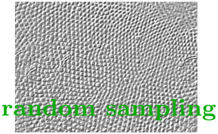

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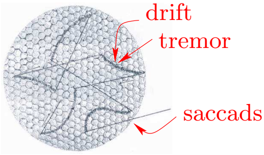

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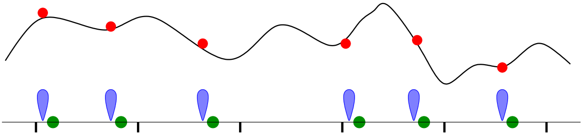

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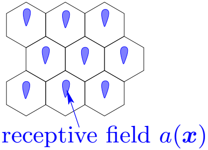

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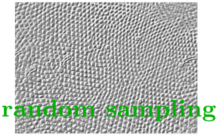

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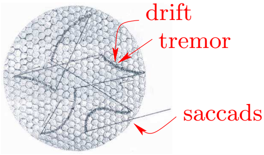

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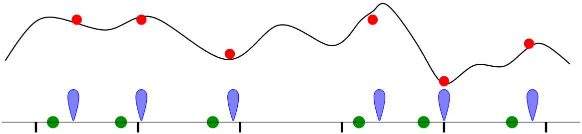

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Assumptions

$$S_r(\boldsymbol{x}_n, t) = \int S(\boldsymbol{x}_n + \gamma \boldsymbol{\epsilon}_n + \sigma \boldsymbol{\xi}_t + \boldsymbol{u})\, a(\boldsymbol{u})\, d\boldsymbol{u}$$

- Dimension $d \geq 1$
- S isotropic and homogeneous, $\Gamma_S(\boldsymbol{f}) \sim 1/\|\boldsymbol{f}\|^{2-\eta}$ (natural scene)

  $\Gamma_S(\boldsymbol{f}) = \beta^{-d}\Gamma_0(\|\boldsymbol{f}\|/\beta)$, "bandwidth" β

  $$\Gamma_0(f) \propto \begin{cases} \dfrac{1}{(1+4\pi^2 f^2)^{1-\eta/2}} & \text{Short Range Correlation} \\ (2\pi f)^{\frac{d+\eta-2}{2}} K_{1-\frac{\eta}{2}}(2\pi f) & \text{Long Range Correlation} \end{cases}$$

- $\gamma \boldsymbol{\epsilon}_n$ and $\sigma \boldsymbol{\xi}_t$: independent isotropic, of magnitude γ and σ
- receptive field $a(.)$ isotropic, "width" α: $a(\boldsymbol{u}) = \alpha^{-d} a_0 \left(\|\boldsymbol{u}\|/\alpha\right)$

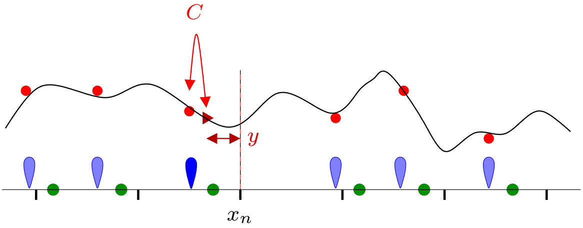
Measure of performance

Information acquired by the sensors, from scene S:

Shannon mutual information (rate) $I(S, S_r) \rightsquigarrow$ untractable calculus;

$\rightsquigarrow$ Similarity (info. acquired by) between $S_r(x_n, t)$ and (from) $S(x_n + y)$:

$$C(\boldsymbol{y}) = \frac{E[S_r(\boldsymbol{x}_n, t) S(\boldsymbol{x}_n + \boldsymbol{y})]}{\left(E[S_r(\boldsymbol{x}_n, t)^2] E[S(\boldsymbol{x}_n + \boldsymbol{y})^2]\right)^{\frac{1}{2}}}$$



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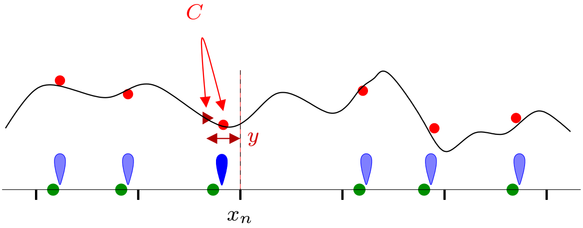

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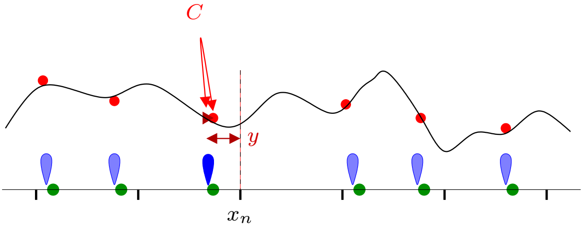

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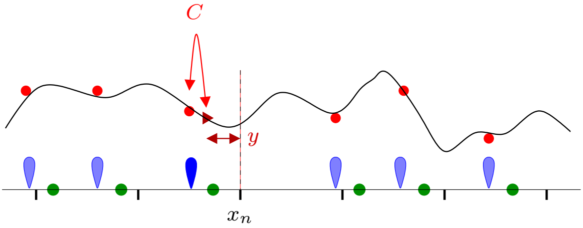

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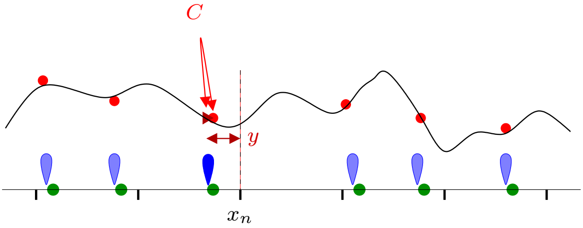

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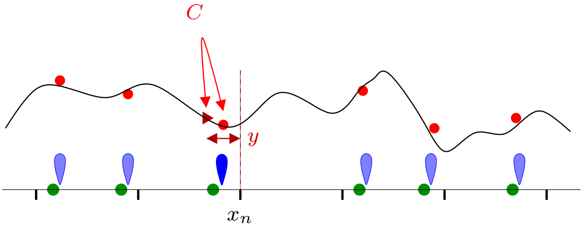


Measure of performance

General expression:

$$C(\boldsymbol{y}) \propto \int R_S(\boldsymbol{y} - \gamma\boldsymbol{\epsilon} - \sigma\boldsymbol{\xi} - \boldsymbol{u})\, a(\boldsymbol{u})\, f_{\epsilon}(\boldsymbol{\epsilon})\, f_{\xi}(\boldsymbol{\xi})\, d\boldsymbol{u}\, d\boldsymbol{\epsilon}\, d\boldsymbol{\xi}$$

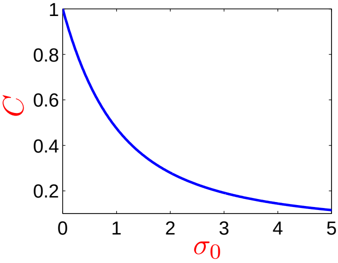
Using the isotropic properties

$$C(y_0) = \frac{y_0^{1-\frac{d}{2}} \displaystyle\int_0^{+\infty} f^{\frac{d}{2}}\, \Gamma_0\left(\frac{f}{2\pi}\right) A_0\left(\frac{\alpha_0 f}{2\pi}\right) \Phi_{\epsilon}(\gamma_0 f)\, \Phi_{\xi}(\sigma_0 f)\, J_{\frac{d}{2}-1}(y_0 f)\, df}{\left(\frac{2^{d+1}\pi^{\frac{3d}{2}}}{\Gamma\left(\frac{d}{2}\right)} \displaystyle\int_0^{+\infty} f^{d-1} \Gamma_0(f) |A_0(\alpha_0 f)|^2\, df\right)^{\frac{1}{2}}}$$

with $\{y, \alpha, \gamma, \sigma\}_0 = \beta \times \{\|\boldsymbol{y}\|, \alpha, \gamma, \sigma\}$

1D cable, uniform saccades  & filter  – Short Range Corr.

$\rightsquigarrow$ analytical study (for $\eta = 0$)



$\alpha_0 = 0.0$ $\quad$ $y_0 = 0$

Noise-enhanced correlation

1D cable, uniform saccades & filter – Short Range Corr.

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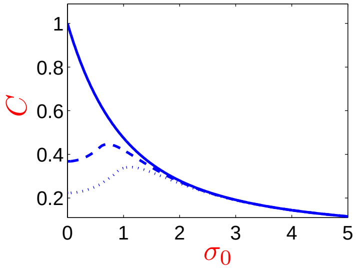

$\alpha_0 = 0.0$ $y_0 = 0, 1, 1.5$

- Existence of an *optimal stochastic control*

Noise-enhanced correlation

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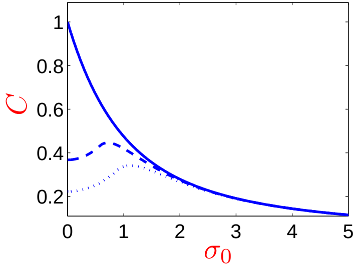

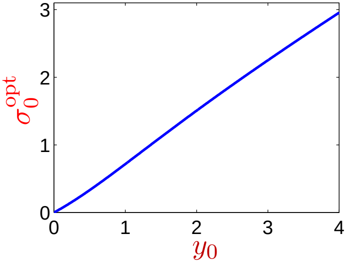

$\alpha_0 = 0.0$ $\qquad$ $y_0 = 0, 1, 1.5$

- Existence of an *optimal stochastic control*
- Linear-like optimal noise amplitude versus y

1D cable, uniform saccades & filter – Short Range Corr.

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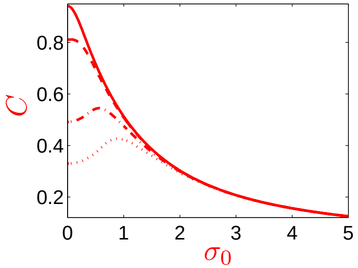

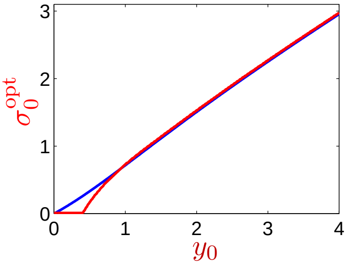

$\alpha_0 = 0.0, 0.3$ $y_0 = 0.1, 0.3, 0.8, 1.2$

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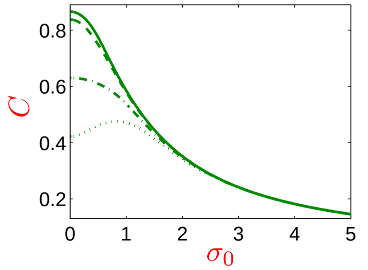
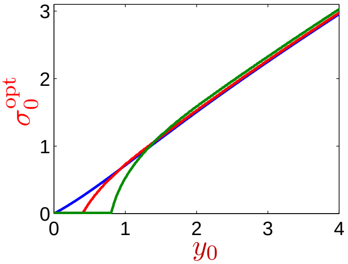
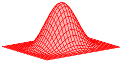
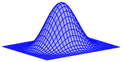


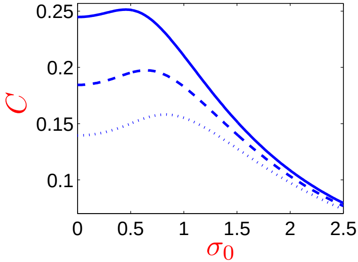


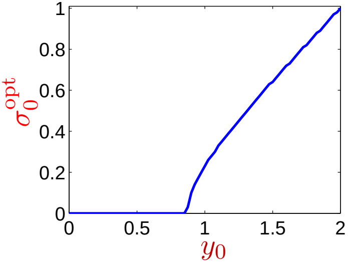
$\alpha_0 = 0.0, 0.3, 0.8$ $y_0 = 0.1, 0.3, 0.8, 1.2$

- Existence of an *optimal stochastic control*
- Linear-like optimal noise amplitude versus y when y_0 is higher than the visual width α_0

Noise-enhanced correlation

Student-r saccades  & filter , Short Range Correlation

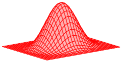
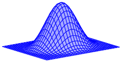


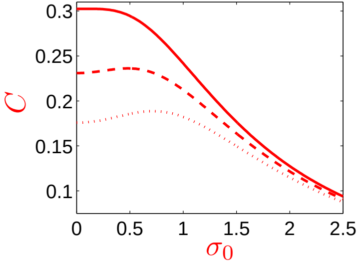


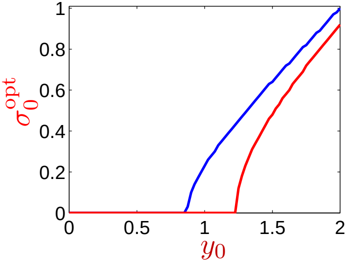
$\alpha_0 = 1.0$ $\quad$ $y_0 = 1.25, 1.5, 1.75$ $\quad$ $\eta = -0.5$

Same conclusions than in 1-D

Noise-enhanced correlation

Student-r saccades  & filter , Short Range Correlation

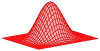
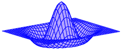


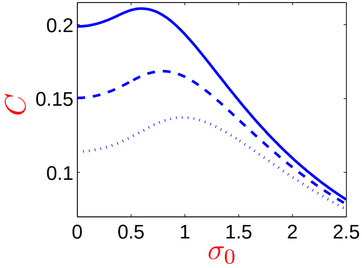


$\alpha_0 = 1.0, 1.7$ $\qquad$ $y_0 = 1.25, 1.5, 1.75$ $\qquad$ $\eta = -0.5$

Same conclusions than in 1-D

Noise-enhanced correlation

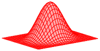
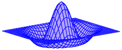
Student-r saccades  Hermite-Student-r filter  , Short Range Corr.

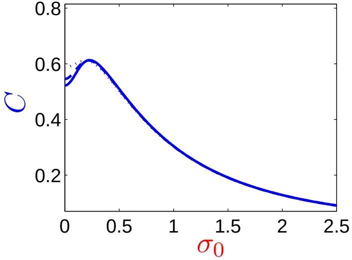


$\alpha_0 = 0.7$ $y_0 = 1.250, 1.50, 1.750$ $\eta = -0.5$

- Optimal stochastic control provided that y > receptive width, but...

Noise-enhanced correlation

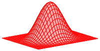
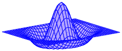
Student-r saccades  Hermite-Student-r filter  , Short Range Corr.

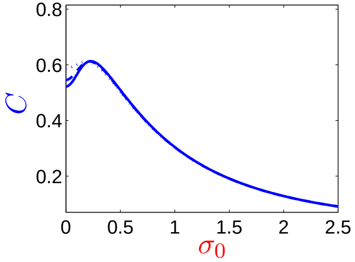


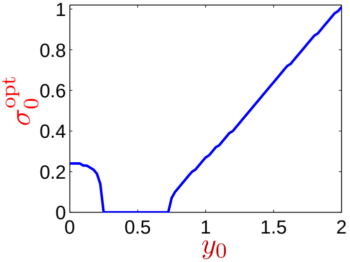
$\alpha_0 = 0.7$ $\quad$ $y_0 = 0.050, 0.100, 0.175$ $\quad$ $\eta = -0.5$

- Optimal stochastic control provided that y > receptive width, but...
- also exist when y small... due the inhibition:
  fluctuations may compensate the inhibition; effect no more "local";
  hyperacuity effect?

Noise-enhanced correlation

Student-r saccades  Hermite-Student-r filter , Short Range Corr.





$\alpha_0 = 0.7$ $\quad y_0 = 0.050, 0.100, 0.175$ $\quad \eta = -0.5$

- Optimal stochastic control provided that y > receptive width, but...
- also exist when y small... due the inhibition:
  fluctuations may compensate the inhibition; effect no more "local";
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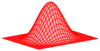
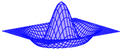
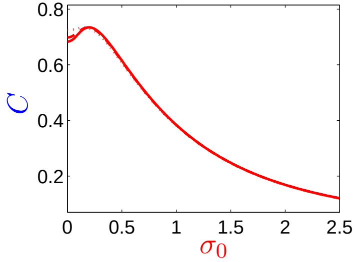
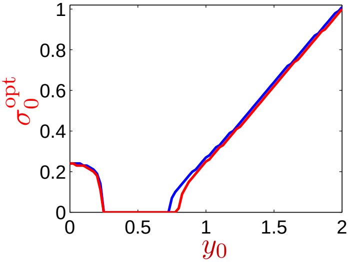
Noise-enhanced correlation

Student-r saccades  Hermite-Student-r filter  , Short Range Corr.

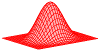
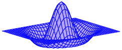


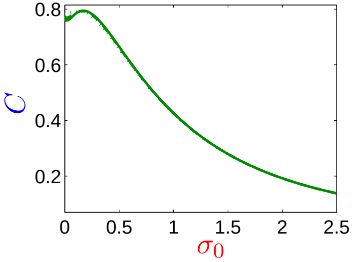


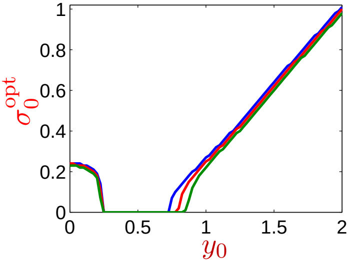
$\alpha_0 = 0.7$ $\qquad$ $y_0 = 0.050, 0.100, 0.175$ $\qquad$ $\eta = -0.5, -0.8$

- Optimal stochastic control provided that y $>$ receptive width, but…
- also exist when y small…due the inhibition:
  fluctuations may compensate the inhibition; effect no more “local”;
  hyperacuity effect?

Noise-enhanced correlation

Student-r saccades  Hermite-Student-r filter  , Short Range Corr.

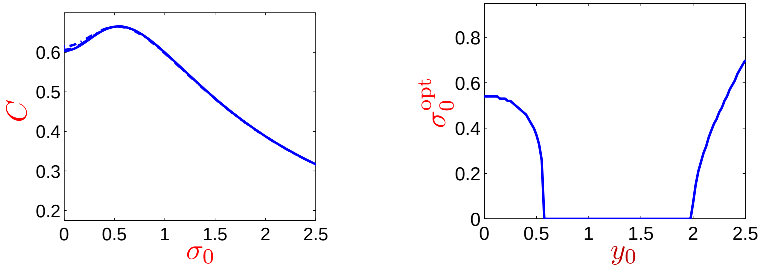




$\alpha_0 = 0.7$ $\quad$ $y_0 = 0.050, 0.100, 0.175$ $\quad$ $\eta = -0.5, -0.8, -1.0$

- Optimal stochastic control provided that y > receptive width, but...
- also exist when y small... due the inhibition:
  fluctuations may compensate the inhibition; effect no more "local";
  hyperacuity effect?
- Robustness against η, noise distribution, γ_0 (spatio-temp. $\gamma_0^2 + \sigma_0^2$),...

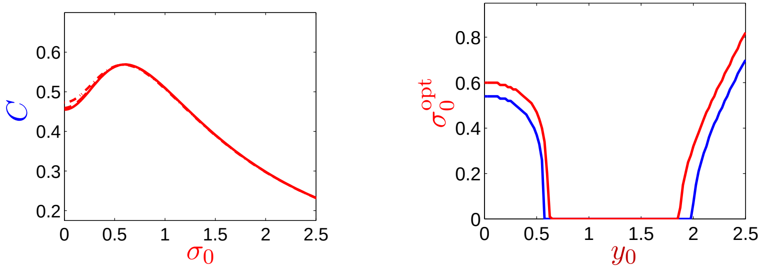
Long Range Correlation



$\alpha_0 = 0.5$ $y_0 = 0, 0.1, 0.2$ $\eta = 1.5$

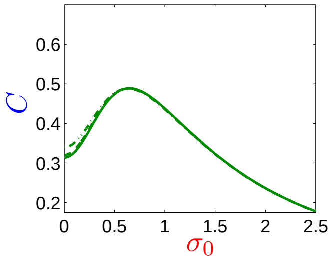
- For large y, only for $y \gg \alpha$ due to long range correlation
- Same conclusion than in the short range correlation for small y: important since each sensor may be devoted to a neighbourhood of it

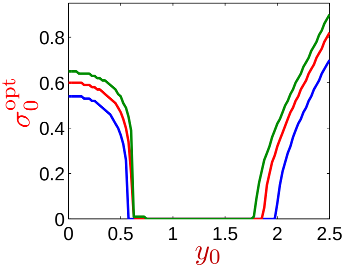
Long Range Correlation



$\alpha_0 = 0.5$ $\quad$ $y_0 = 0, 0.1, 0.2$ $\quad$ $\eta = 1.5, 2.0$

- For large y, only for $y \gg \alpha$ due to long range correlation
- Same conclusion than in the short range correlation for small y: important since each sensor may be devoted to a neighbourhood of it





$\alpha_0 = 0.5$ $\qquad$ $y_0 = 0, 0.1, 0.2$ $\qquad$ $\eta = 1.5, 2.0, 2.5$

- For large y, only for $y \gg \alpha$ due to long range correlation
- Same conclusion than in the short range correlation for small y: important since each sensor may be devoted to a neighbourhood of it

Summary

- Role of the saccades under the noise-enhanced processing point of view
- Hyperacuity effect? In the sense that the acquisition of information is “finer” than the sampling period
- Symmetry between random sampling and tremor: alias-free sampling & tremor + time-integration saccades as a way to perform the ensemble average?

Open questions

- Hyperactuity induced by noise? $\rightsquigarrow$ taking the **whole** retina, **as time goes on** is necessary
- Mutual Shannon information: if $S \sim \mathcal{N}$, conditionally to the fluctuations, $S_r \sim \mathcal{N}$ and $(S, S_r) \sim \mathcal{N}$
  - Fixed time, $S_r \sim \mathcal{N}$, but (S, S_r) is a mixture of Gaussians
  - As time goes on, process S_r is itself a mixture of Gaussians
- What about the time effet, spatio-temporal filtering, the nonlinearities, posterior neural processes...
- Comparisons with real retinal process?
  - Fluctuation magnitude – spectral characteristics of the scene (e.g. $\sigma_0^{\text{opt}} = \beta\, \sigma^{\text{opt}}$)?
  - Psychophysics experiments for validation... or not