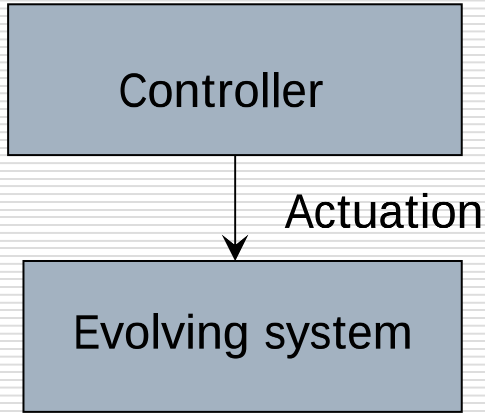
Open problems on information and feedback control of stochastic systems

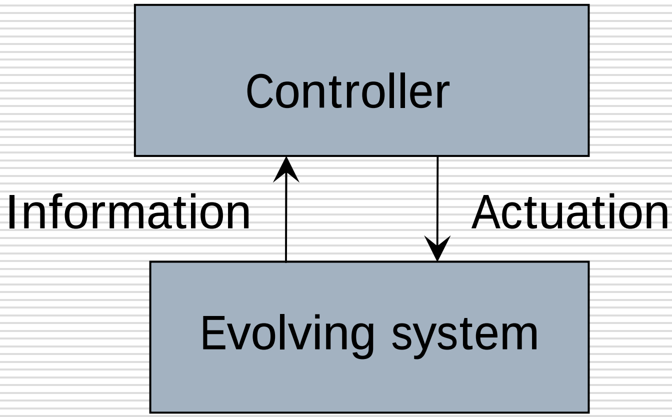
Francisco J. Cao (UCM, Spain)

Open-loop and closed-loop control

- **Open-loop control**: the controller actuates on the system **independently** of the system state.



- **Closed-loop or feedback control**: the controller actuates on the system using **information** of the system state.



J. Bechhoefer, Rev. Mod. Phys. 77, 783 (2005)

Information and feedback control

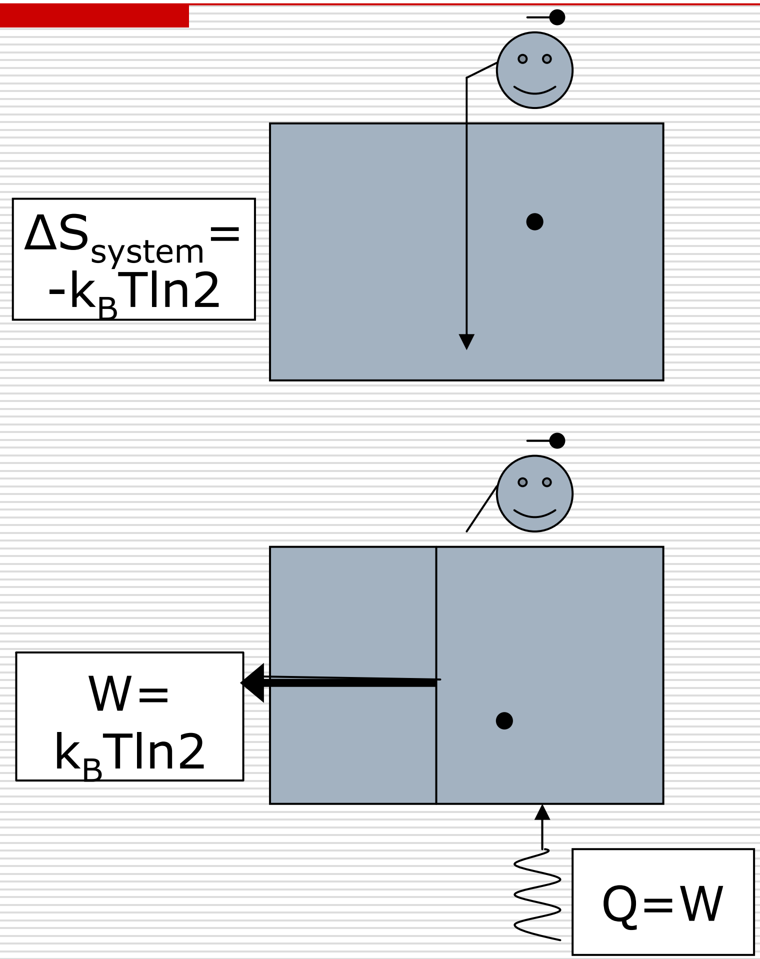
- The role of information in the feedback controlled system is still not completely understood
- Thermodynamics of feedback control is incomplete
- Much of the progress has come from the study of the Maxwell's demon, and mainly from a computation theory point of view.

Overview

- Maxwell's demon understanding
- Open questions
- Feedback flashing ratchets
- Additional entropy reduction due to information
- Thermodynamics of feedback controlled systems (NEW RESULTS)

Maxwell’s demon: Szilard’s engine

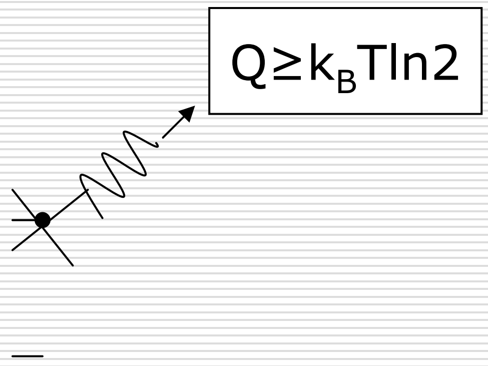
- Once the demon knows in which side is the particle, it can put a wall in the middle
- After the wall is attached to a piston in the correct side to do work W
- Apparently the efficiency is $\eta = W/Q = 1???$
- Thus, second law??? $\Delta S_{system} < 0$



J.C.Maxwell, Theory of heat (1871)
L.Szilard, Behav.Sc.9, 301 (1964)

Landauer's principle

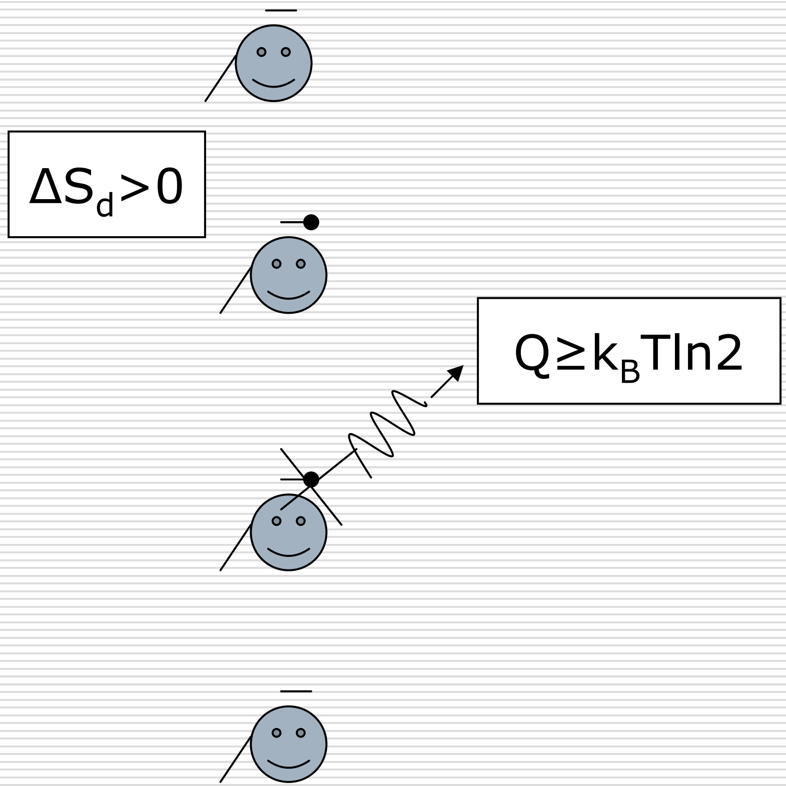
- The erasure of one bit of information at temperature T implies an energetic cost of at least $k_B T \ln 2$
- It is obtained from the second law of thermodynamics



R. Landauer, IBM J. Res. Dev. 5, 183 (1961)

Maxwell's demon "solution" from the system + demon view

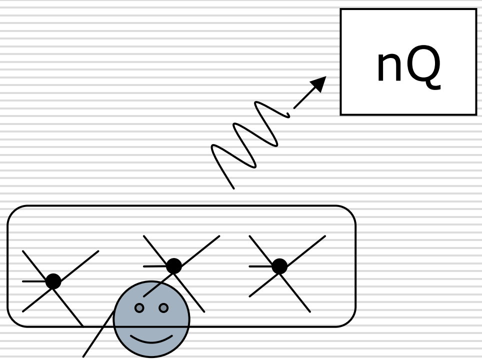
- Applying the Landauer's principle to the demon we have $\Delta S_d = -Q/T \geq k_B T \ln 2$
- Therefore, at the measurement $\Delta S_{system} + \Delta S_d \geq 0$



C.H.Bennett, Int.J.Theor.Phys. 21, 905 (1982)

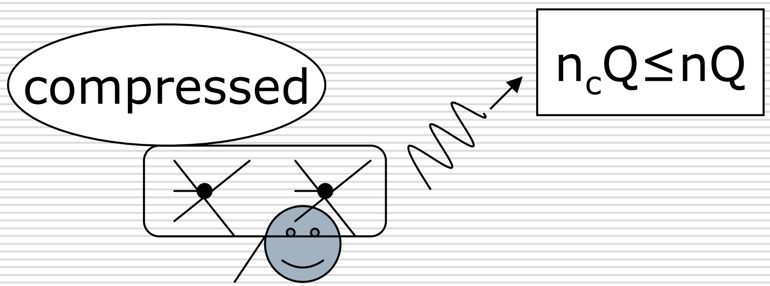
Many measurements from the system + controller view

- Zurek's algorithmic complexity approach showed how to minimize the erasure cost



- Clever demons compress the information

  (less bits = less erasure cost)



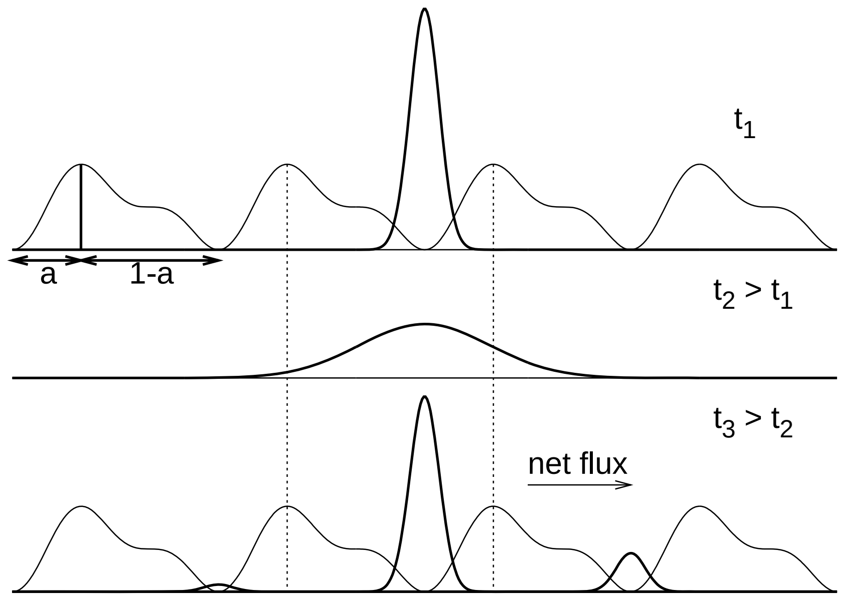
W.H.Zurek, Phys.Rev.A 40, 4731 (1989)

Open questions on information and feedback

- The understanding is more advanced in the system + demon view
- **The understanding from the point of view of system is still incomplete**
- **Thermodynamics of feedback controlled systems still incomplete**
- **Relations between information and performance still need study**

Feedback flashing ratchets

- Brownian particle
- Asymmetric periodic potential
- *Open-loop flashing ratchet: Periodic switching*
- Induced flux $\rightarrow$
- *Feedback flashing ratchet: State dependent switching*

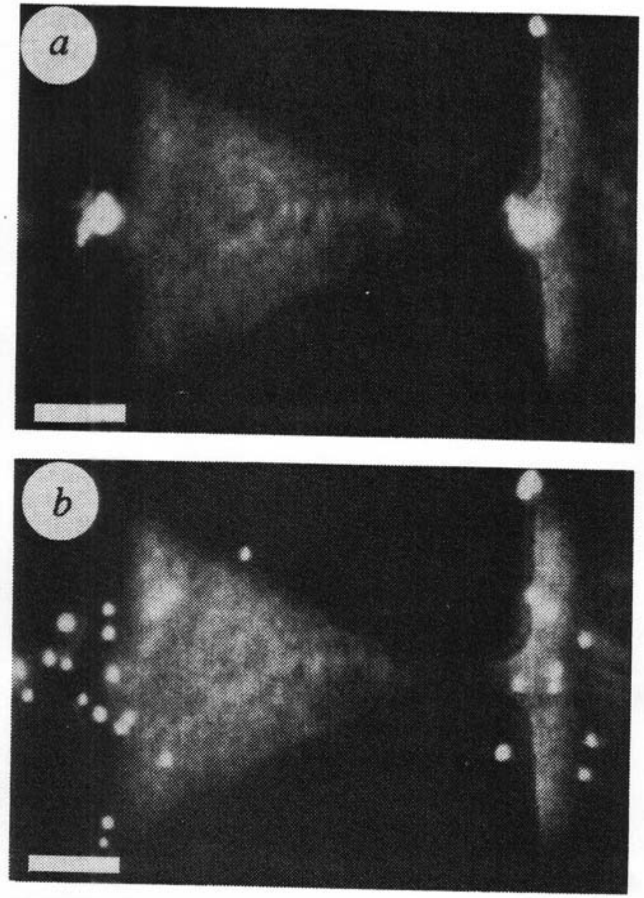


PRL 93, 040603 (2004)

Implementing feedback control in ratchets

- Feedback control could be implemented in experimental setups where particles are monitored
- Example: colloidal particles

  J. Rousselet, L. Salome, A. Ajdari, J. Prost, Nature 370, 446 (1994)

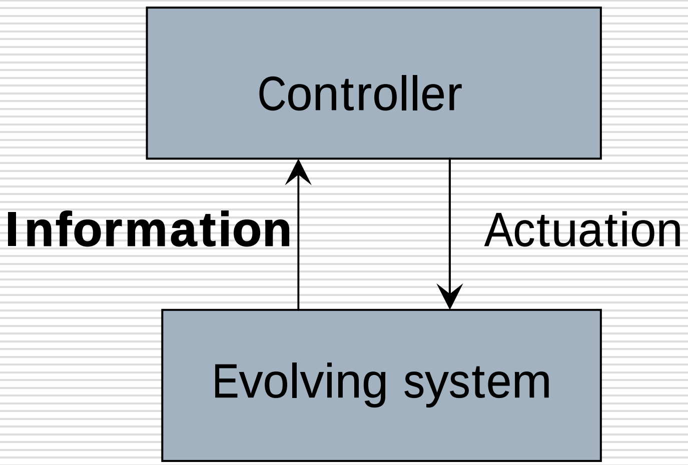


Information quantification

- Instant maximization of the CM velocity: controller knows the sign of the net force
- Error p in the estimation of the net force received by the controller implies

$$1 - 2p \approx \sqrt{\frac{I \ln 2}{2b(1-b)}}$$

with I info and b = b(a,N)



cond-mat/0703492 & EPJB 59, 63 (2007)

Information and performance of feedback flashing ratchets

- Flux performance (one or few particles)

$$\left\langle \dot{x}_{CM} \right\rangle_{st} \leq C_N \sqrt{I}$$

- Power performance (one or few particles)

$$P \leq E_N I$$

- C_N and E_N depend only on the system parameters
- Approximate expressions for small potentials (V_0 ≤kT) and upper bounds for all potential heights (any V_0 value)

Additional entropy reduction due to information

- Point of view of the controlled system
- Maximum entropy that can be extracted with a closed-loop controller is

$$\Delta H_{closed} \leq \Delta H_{open} + I(X;C)$$

ΔH_{open}: maximum entropy decrease with an open-loop control

$I(X;C)$: mutual information gathered by the controller upon observation of the system

H.Touchete, S.Lloyd, PRL 84, 1156 (2000) & Physica A 331, 140 (2004)

Thermodynamics of feedback controlled systems (NEW RESULTS)

From statistical mechanics and from the point of view of the system:

Entropy *before* measurement

$$S_1^b = -\sum_{x\in X} p_{X_1}(x)\ln p_{X_1}(x) =: H(X_1)$$

If the measurement implies $C_1 = c$, the entropy decreases to

$$H(X_1|C_1 = c) := -\sum_{x\in X} p_{X_1|C_1}(x|c)\ln p_{X_1|C_1}(x|c)$$

Average entropy *after* measurement

$$S_1^a = -\sum_{c\in C} p_{C_1}(c) H(X_1|C_1 = c) =: H(X_1|C_1)$$

Average variation of entropy at the first measurement

$$\Delta S_1 = S_1^a - S_1^b = H(X_1|C_1) - H(X_1) =: -I(X_1;C_1)$$

arxiv:0805.4824 (2008)

Thermodynamics of feedback controlled systems (NEW RESULTS)

From statistical mechanics and from the point of view of the system:

- Average reduction of entropy at the first measurement

$$\Delta S_1 = -I(X_1;C_1) = -\sum_{x \in X, c \in C} p_{X_1C_1}(x,c) \ln \frac{p_{X_1C_1}(x,c)}{p_{X_1}(x)\, p_{C_1}(c)}$$

- It can be computed in terms of joint probabilities

Thermodynamics of feedback controlled systems (NEW RESULTS)

From statistical mechanics and from the point of view of the system (many measurements case):

- The **maximum entropy reduction** for the controlled system can be computed just from the knowledge of the system and control actions history (no details of the controller are needed).

$$\Delta S_{\text{info}} = -\sum_{k=1}^{M} I(C_k ; X_k | C^{k-1}) = -H(C^M) + \sum_{k=1}^{M} H(C_k | C^{k-1}, X_k)$$

with $C^{k-1} = C_{k-1}, C_{k-2}, \ldots, C_1$

See arxiv:0805.4824 and/or ask me for details

Thermodynamics of feedback controlled systems (NEW RESULTS)

$$\Delta S_{\text{info}} = -\sum_{k=1}^{M} I(C_k; X_k \middle| C^{k-1}) = -H(C^M) + \sum_{k=1}^{M} H(C_k \middle| C^{k-1}, X_k)$$

with $C^{k-1} = C_{k-1}, C_{k-2}, \ldots, C_1$

- $-H(C^M)$ indicates that only **non-redundant information** is useful to reduce the system entropy (corresponds to Zurek's compression)
- The second term indicates that non-deterministic control limits the attainable entropy reduction.

arxiv:0805.4824 (2008)

Thermodynamics of feedback controlled systems (NEW RESULTS)

- Example: implications for **isothermal** feedback controlled systems

$$W \leq -\Delta U_{cont} + T\Delta S_{cont} = -\Delta F_{cont}$$

as $\Delta S_{cont} \geq -\Delta S_{\mathrm{info}}$, the maximum efficiency is

$$\eta = \frac{\mathrm{W}}{-\Delta U_{cont} - T\Delta S_{\mathrm{info}}}$$

arxiv:0805.4824 (2008)

Conclusions

- The **thermodynamics** from the point of view of the system *only very recently* has been clarified
- Relations between **information and performance** are still an *open* question for the general case
- Relations between **information and feedback** from the point of view of control theory are still an open question (see Bechhoefer review)